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Gain scheduled observer state feedback controller for rational LPV systems

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Abstract: This paper addresses the design of gain scheduled observer-based controllers for rational linear parameter varying systems (LPV). Such systems are equivalently recast as affine descriptor LPV systems. Based on this new realization a descriptor observer-based controller is designed by means of some new sufficient conditions given as LMIs. The stability of the descriptor closed-loop system is proved. A state space rational controller, with an observer-based structure, is then derived. The stability of the obtained rational closed-loop is also proved. A numerical example is presented to illustrate the efficiency of the method.

1. INTRODUCTION

Analysis and control of linear parameter varying systems have been a very popular topic during the last two decades since this class of systems covers a large scale of practical systems, including some non linear systems as illustrated for instance in (Apkarian et al. 1994), (Leith et al., 2000) and many other papers.

Several techniques can be found in the literature about gain scheduling control of LPV systems. One of these approaches consists in interpolating several invariant controllers tuned for different operating points. This classical gain scheduling design is obtained in three steps: define the operating points for the LPV system, design a linear time invariant controller for each one of them and finally build the gain scheduled controller. The last step of the design is based on interpolation techniques. Numerous interpolation schemes can be used, as for instance the interpolation of the state space matrices, the poles, zeros and gains of the controllers...(see for example (Stillwell et al., 1999)). This simple scheme has the drawback that in most cases no theoretical proof of the stability of the closed-loop is given. However in (Stilwell et al., 2000) the stability of the closed-loop is ensured thanks to some additional constraints. Furthermore, interpolation of observer and state feedback gains has been tackled with in several papers as for instance in (Raharijoana et al., 2006) and (Berriri et al., 2006). Another way to design gain scheduled controllers is based on the Lyapunov's theory and the extension of the LMI conditions known in the LTI case. The main advantage of this approach is that the stability of the closed-loop system is ensured. Recently, significant progress has been made in this area by using some special representations for LPV systems: LFT (linear fractional transformation), affine, polytopic, representations. In the case of polytopic systems, the design of an observer-based controller was addressed by (Bara et al., 2002), (Bara et al., 2001). Our objective is to extend these results to the case of rational LPV systems. Numerous results on the control of LPV rational system can be found in the literature. Most of them are based on the use of a LFT and provide sufficient conditions for the design of stabilizing controllers (see for example (Scherer et al., 2001)). In this paper the design of observer-based controllers for rational LPV systems is tackled with via an equivalent descriptor realization with an affine dependency on the varying parameter. This idea was used in (Bouali et al., 2006) and (Bouali et al., 2007a) in the state feedback synthesis case.

This paper is organized as fallows. We present the problem under consideration in Section 2. Some preliminary results concerning the stability of LPV descriptor systems are presented in Section 3. In section 4, we consider the design of an observer-based controller for the equivalent LPV descriptor system. The obtained descriptor controller is used in Section 5 to derive a state space rational controller with an observer-based structure. Finally, a numerical example is given, in Section 6, to illustrate the efficiency of the proposed method before concluding.

Notation: The notation $A > 0$ (respectively $A \geq 0$) stands for A definite positive (respectively semi-definite positive). The notation $Bdiag(A_1, A_2)$ denotes for a bloc diagonal matrix with A_1 and A_2 on the principal diagonal. $He\{A\}$ stands for $A^T + A$ and $\bullet$ for terms that are induced by symmetry.

2. PROBLEM FORMULATION

The considered class of LPV systems is described by

$$(\Sigma_r):\begin{cases}\dot{x}_1(t)=A_r(\theta(t))x_1(t)+B_r(\theta(t))u(t)\\ y(t)=C_r(\theta(t))x_1(t)+D_r(\theta(t))u(t)\end{cases} \tag{1}$$

where the state matrices are assumed to be rational functions of the time varying vector $\theta(t)$. The state vector is given by $x_1(t)\in\mathbb{R}^{n_1}$. The input vector is $u(t)\in\mathbb{R}^{n_u}$ and $y(t)\in\mathbb{R}^{n_y}$ is the output vector. The vector $\theta(t)=\begin{bmatrix}\theta_1(t) & \theta_2(t) & \dots & \theta_q(t)\end{bmatrix}^T$ is assumed to be real, continuous time varying and satisfying the following constrains:

-Each parameter $\theta_i(t)$ is real time measurable and ranges between known extremal values $\theta_i(t)\in\begin{bmatrix}\underline{\theta}_i & \overline{\theta}_i\end{bmatrix}$.

-The variation rate of each parameter $\dot{\theta}_i(t)$ is limited by known upper and lower bounds, that is $\dot{\theta}_i(t)\in\begin{bmatrix}\underline{\tau}_i & \overline{\tau}_i\end{bmatrix}$.

As a consequence, the varying parameter and its rate evolve both in hyper rectangles with vertices defined by

$$\Xi=\left\{(\omega_1,\ldots\omega_{2^q})\setminus\omega_i\in\left\{\underline{\theta}_i \quad \overline{\theta}_i\right\}\right\}$$

and $\Omega=\left\{(\tau_1,\ldots\tau_{2^q})\setminus\tau_i\in\left\{\underline{\tau}_i \quad \overline{\tau}_i\right\}\right\}$.

The set of possible trajectories $\theta(.)$ is noted Θ.

The main objective of this paper is to design an observer state feedback controller which is rationally dependent on the time varying parameter. The sought controller can be given by

$$(K_r):\begin{cases}\dot{\hat{x}}_1=A_r(\theta)\hat{x}_1+B_r(\theta)u+L_r(\theta)(\hat{y}-y)\\ \hat{y}=C_r(\theta)\hat{x}_1+D_r(\theta)u\\ u=F_r(\theta)\hat{x}_1+G_r(\theta)(\hat{y}-y)\end{cases} \tag{2}$$

where $L_r(\theta)$ and $F_r(\theta)$ are respectively the parameter varying observer and state feedback gains which are rationally dependant on the varying parameter.

In order to design a stabilizing controller as given by (2) we propose to use a descriptor realization. Indeed, it has been demonstrated in (Bouali et al., 2006) that the rational LPV system given by (1) can be equivalently recast into a descriptor realization as follows

$$(\Sigma_d):\begin{cases}\begin{pmatrix}I_{n_1} & 0\\ 0 & 0\end{pmatrix}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{pmatrix}A_1(\theta) & A_2(\theta)\\ A_3(\theta) & A_4(\theta)\end{pmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{pmatrix}B_1\\ B_2\end{pmatrix}w\\ y(t)=\begin{pmatrix}C_1 & C_2\end{pmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\end{cases},\quad \begin{bmatrix}x_1\\ x_2\end{bmatrix}\in\mathbb{R}^n \tag{3}$$

where $(A_i(\theta))_{i\in\{1,2,3,4\}}$ are affine functions of the parameter $\theta(t)$ and matrices B_1, B_2, C_1 and C_2 are all constant. Matrix $A_4(\theta)$ is non singular for all trajectories $\theta(.)\in\Theta$.

The following equations describe the relation between realization (1) and (3)

$$A_r(\theta)=A_1(\theta)-A_2(\theta)A_4(\theta)^{-1}A_3(\theta) \tag{4.a}$$

$$B_r(\theta)=B_1-A_2(\theta)A_4(\theta)^{-1}B_2 \tag{4.b}$$

$$C_r(\theta)=C_1-A_4(\theta)^{-1}A_3(\theta)C_2 \tag{4.c}$$

$$D_r(\theta)=-C_2A_4(\theta)^{-1}B_2 \tag{4.d}$$

The design of the controller given by (3) can be done thanks to the synthesis of an observer state feedback controller for the equivalent affine descriptor realization (3). This controller can be given by

$$(K_d):\begin{cases}E\dot{\hat{x}}=A(\theta)\hat{x}+Bu+L(\theta)(\hat{y}-y)\\ \hat{y}=C\hat{x}\\ u=F(\theta)\hat{x}\end{cases} \tag{5}$$

where the generalized state vector is $\hat{x}=\begin{bmatrix}\hat{x}_1^T & \hat{x}_2^T\end{bmatrix}^T$ and the state matrices are partitioned as follows

$$E=\begin{pmatrix}I & 0\\ 0 & 0\end{pmatrix},\ A(\theta)=\begin{pmatrix}A_1(\theta) & A_2(\theta)\\ A_3(\theta) & A_4(\theta)\end{pmatrix},$$

$$B=\begin{pmatrix}B_1\\ B_2\end{pmatrix},\ L(\theta)=\begin{pmatrix}L_1(\theta)\\ L_2(\theta)\end{pmatrix},$$

$$C=\begin{pmatrix}C_1 & C_2\end{pmatrix},\ F(\theta)=\begin{pmatrix}F_1(\theta) & F_2(\theta)\end{pmatrix}.$$

The design of the controller (2) thanks to classical LMIs as those proposed in (Bara et al., 2002) is interesting when state matrices are polytopic. Due to the rational dependency of the state matrices in (2), we can no longer use a finite set of LMIs as done in (Bara et al., 2002). The use of realization (3) is an intermediary step. In fact, we propose here new sufficient conditions allowing to design a descriptor observer-based controller by means of a finite set of LMIs. Since (3) is affinely dependant on the varying parameter, it seems natural to choose the same structure for the descriptor controller. A rational controller, of the form (2), can then be easily extracted.

3. PRELIMINARY RESULTS

3.1 Admissibility conditions of LPV descriptor systems

In this paper, the admissibility of LPV descriptor systems as defined in (Masubuchi et al., 2003) is considered. In fact, consider the following LPV descriptor system

$$E\dot{x}=A(\theta)x \tag{6}$$

with $x(t)\in\mathbb{R}^n$ and $rank(E)=n_1\leq n$.

We assume, without loss of generality, that

$$E=\begin{pmatrix}I_{n_1} & 0\\ 0 & 0\end{pmatrix}$$

since it is always possible to consider a singular value decomposition (SVD) of matrix E. Moreover, this particular structure is natural when considering the LPV descriptor system (Σ_d).

Theorem 1: (Masubuchi et al., 2003)

The descriptor system given by (6) is admissible if there exists a continuously differentiable function $P:\Theta\rightarrow\mathbb{R}^{n\times n}$ such that for all $\theta(.)\in\Theta$

$$E^TP(\theta)=P(\theta)^TE\geq 0 \tag{7.a}$$

$$He\left\{A(\theta)^TP(\theta)\right\}+E^T\frac{d}{dt}(P(\theta))<0 \tag{7.b}$$

An equivalent strict LMI condition has been proposed in (Bouali et al., 2006a) and is reminded next.

Theorem 2: (Bouali et al., 2007a)
The descriptor system given by (6) is admissible if there exist continuously differentiable functions $P:\Theta \to \mathbb{R}^{n\times n}$ and $S:\Theta \to \mathbb{R}^{(n-n_1)\times(n-n_1)}$ such that for all $\theta(.)\in\Theta$

$$P(\theta) = P(\theta)^T > 0 \tag{8.a}$$

$$He\left\{A(\theta)^T\left(P(\theta)E + US(\theta)V^T\right)\right\} + E^T\frac{d}{dt}(P(\theta))E < 0 \tag{8.b}$$

where $V,U \in \mathbb{R}^{n\times(n-n_1)}$ are matrices of full column rank and composed of bases of $\ker E$ and $\ker E^T$ respectively.

Remark 1: Conditions given by (7) or (8) are hardly exploitable for the design of an observer state feedback controller without avoiding the use of gridding techniques. For this reason, a new condition is proposed next in terms of an extended LMI.

Theorem 3: The descriptor system (6) is admissible if there exist continuously differentiable functions $P:\Theta\to\mathbb{R}^{n\times n}$, $S:\Theta\to\mathbb{R}^{(n-n_1)\times(n-n_1)}$ and a matrix $W\in\mathbb{R}^{n\times n}$ such that for all $\theta(.)\in\Theta$

$$P(\theta) = P(\theta)^T > 0 \text{ and } S(\theta) = S(\theta)^T > 0 \tag{9.a}$$

$$\begin{pmatrix} -He\{W\} & W^TA^T(\theta)+\Phi(\theta) & W^T \\ \bullet & -\Phi(\theta)+Bdiag\left(\frac{dP(\theta)}{dt},0\right) & 0 \\ \bullet & \bullet & -\Phi(\theta)\end{pmatrix} < 0 \tag{9.b}$$

with $\Phi(\theta) := Bdiag(P(\theta), S(\theta))$

Proof. For brevity reasons only the main lines of the proof are presented here. Let assume that conditions (9) hold. Considering that null spaces of $\begin{pmatrix} I & 0 & 0\end{pmatrix}$ and $\begin{pmatrix} -I & A(\theta) & I\end{pmatrix}$ are $\begin{pmatrix} 0 & I & 0 \\ 0 & 0 & I\end{pmatrix}^T$ and $\begin{pmatrix} A(\theta)^T & I & 0 \\ I & 0 & I\end{pmatrix}^T$, we can drop matrix W by applying projection lemma and Schur complement. This leads to conditions (8). As a consequence the descriptor system given by (6) is admissible.

Remark 2: When applying conditions presented in Theorem 3 in (Bouali et al., 2006a) for a state feedback design it appears that an additional structure constraint on the matrix W is necessary to solve the LMIs. This is obviously not the case when using Theorem 3. One of the advantages of this extended LMI condition is that there is no longer products between the state matrices and the unknown functions $P(.)$ and $S(.)$. The parameterized LMIs proposed in Theorem 3 can be solved thanks to a finite set of LMIs when dealing with polytopic descriptor systems. A dual form of the previous Theorem can easily be derived.

3.2 Strong equivalence and admissibility

When two LTI realizations are equivalent, the stability of the first one implies the stability of the second. This simple result is no longer true when considering time varying systems as illustrated in (Cobb 2006). In this paper, we introduce a new characterization of equivalence of two realizations in the descriptor LPV case.

In fact, consider two LPV descriptor realizations given by the pairs

$$(E, A(\theta)),\ \left(\bar{E}, \bar{A}(\theta)\right) \tag{10}$$

Definition 1: The two realizations (10) are said *strongly equivalent* if there exist two continuously differentiable functions $M:\Theta\to\mathbb{R}^{n\times n}$, $N:\Theta\to\mathbb{R}^{n\times n}$ such that for all $\theta(.)\in\Theta$

i) $M(\theta)$ and $N(\theta)$ are non singular matrices

ii) $M^{-1}(\theta)$, $N^{-1}(\theta)$ are continuously differentiable

and the following equations hold

$$M(\theta)EN(\theta) = \bar{E} \tag{11.a}$$

$$M(\theta)A(\theta)N(\theta) = \bar{A}(\theta) \tag{11.b}$$

$$M(\theta)E\frac{d}{dt}\left(N(\theta)\right) = 0 \tag{11.c}$$

This property is used in the next result.

Theorem 4: (Bouali et al., 2007b)
Consider the two strongly equivalent descriptor realizations $(E, A(\theta))$ and $\left(\bar{E}, \bar{A}(\theta)\right)$. The following statements are equivalent

i) There exists a continuously differentiable function $P:\Theta\to\mathbb{R}^{n\times n}$ such that for all $\theta(.)\in\Theta$

$$E^TP(\theta) = P(\theta)^TE \geq 0$$

$$He\left\{A(\theta)^TP(\theta)\right\} + E^T\frac{d}{dt}(P(\theta)) < 0$$

ii) There exists a continuously differentiable function $\bar{P}:\Theta\to\mathbb{R}^{n\times n}$ such that for all $\theta(.)\in\Theta$

$$\bar{E}^T\bar{P}(\theta) = \bar{P}(\theta)^T\bar{E} \geq 0$$

$$He\left\{\bar{A}(\theta)^T\bar{P}(\theta)\right\} + \bar{E}^T\frac{d}{dt}\left(\bar{P}(\theta)\right) < 0\,.$$

3.3 Admissibility of descriptor systems with a special structure

In this section we consider two descriptor polytopic pairs given by $(E, A_1(\theta))$ and $(E, A_2(\theta))$.

Lemma 1: If there exist affine functions $X_1:\Theta\to\mathbb{R}^{n\times n}$ and $X_2:\Theta\to\mathbb{R}^{n\times n}$ such that for all $\theta(.)\in\Theta$ the following conditions hold

$$E^TX_i(\theta) = X_i(\theta)^TE \geq 0 \tag{12a}$$

$$He\left\{A_i(\theta)^TX_i(\theta)\right\} + E^T\frac{d}{dt}(X_i(\theta)) < 0 \tag{12b}$$

for $i \in \{1,2\}$, then for any bounded matrix $A_3(\theta)$, the descriptor pair given by $\left(\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}, \begin{pmatrix} A_1(\theta) & 0 \\ A_3(\theta) & A_2(\theta) \end{pmatrix}\right)$ is admissible.

Proof. Assume that conditions (12) hold for the pairs $(E, A_1(\theta))$ and $(E, A_2(\theta))$. Thus, there exist positive real numbers r_1 and r_2 such that

$$He\{A_1(\theta)^T X_1(\theta)\} + E^T \dot{X}_1(\theta) \leq -r_1 I$$
$$-r_2 I \leq He\{A_2(\theta)^T X_2(\theta)\} + E^T \dot{X}_2(\theta) < 0$$

Since $X_2(\theta)$ and $A_3(\theta)$ are bounded for all $\theta(.) \in \Theta$ there exists a real positive number r_3 such that $X_2(\theta) A_3(\theta) \leq r_3 I$.

Let us consider now the matrix

$$X(\theta) = Bdiag(X_1(\theta), \lambda X_2(\theta))$$

with $\lambda > 0$. First, it is easy to see that $X(\theta)$ is a continuously differentiable function such that for all $\theta(.) \in \Theta$, $Bdiag(E,E)^T X(\theta) = X(\theta)^T Bdiag(E,E) \geq 0$ holds. We show next that the inequality included in conditions (7) holds for the pair

$$\left(\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}, \begin{pmatrix} A_1(\theta) & 0 \\ A_3(\theta) & A_2(\theta) \end{pmatrix}\right)$$

Indeed by noting that

$$\begin{aligned} & He\{A_1(\theta)^T X_1(\theta)\} + E^T \dot{X}_1(\theta) \\ & -\lambda A_3(\theta)^T X_2(\theta)\left(He\{A_2(\theta)^T X_2(\theta)\} + E^T \dot{X}_2(\theta)\right)^{-1} X_2(\theta)^T A_3(\theta) \\ & \leq -r_1 I + \lambda r_2 A_3(\theta)^T X_2(\theta) X_2(\theta)^T A_3(\theta) \leq -r_1 I + \lambda r_2 (r_3)^2 I \end{aligned}$$

and by choosing $0 < \lambda < r_1 r_2^{-1} (r_3)^{-2}$ it comes that

$$\begin{aligned} & He\{A_1(\theta)^T X_1(\theta)\} + E^T \dot{X}_1(\theta) \\ & -\lambda A_3(\theta)^T X_2(\theta)\left(He\{A_2(\theta)^T X_2(\theta)\} + E^T \dot{X}_2(\theta)\right)^{-1} X_2(\theta)^T A_3(\theta) < 0 \end{aligned}$$

and $\lambda\left(He\{A_2(\theta)^T X_2\} + E^T \dot{X}_2(\theta)\right) < 0$.

Applying Schur complement leads finally to

$$He\left\{\begin{pmatrix} A_1(\theta) & 0 \\ A_3(\theta) & A_2(\theta) \end{pmatrix}^T X(\theta)\right\} + \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}^T \dot{X}(\theta) < 0$$

which implies that the pair $\left(\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}, \begin{pmatrix} A_1(\theta) & 0 \\ A_3(\theta) & A_2(\theta) \end{pmatrix}\right)$ is admissible according to Theorem 1.

4. DESCRIPTOR OBESERVER-BASED CONTROLLER DESIGN

In this section, the state feedback and the observer problems are addressed separately. Sufficient conditions for the synthesis of a stabilizing state feedback gain are proposed. Similarly, dual conditions will be given for the synthesis of the observer gain. Finally, the stability of the closed-loop with the descriptor observer-based controller is proved.

4.1 A new LMI-based condition for state feedback synthesis for descriptor systems

Let us consider the following descriptor LPV system given by

$$\begin{cases} E\dot{x} = A(\theta)x + Bu \\ y = Cx \end{cases}, \qquad (13)$$
$$x \in \mathbb{R}^n,\ rank(E) = n_1 \leq n$$

Theorem 5: There exists a descriptor state feedback controller such that the closed-loop pair given by $(E, A(\theta) + BF(\theta))$ is admissible if the there exist continuously differentiable functions $P : \theta \to P(\theta)$ $S : \theta \to S(\theta)$, $R : \theta \to R(\theta)$ of appropriate dimensions and a matrix $W \in \mathbb{R}^{n \times n}$ such that for all $\theta(.) \in \Theta$

$$P(\theta) = P(\theta)^T > 0 \text{ and } S(\theta) = S(\theta)^T > 0 \quad (14.a)$$

$$\begin{pmatrix} -He\{W\} & W^T A(\theta)^T + R(\theta) B^T + \Phi(\theta) & W^T \\ \bullet & -\Phi(\theta) - Bdiag\left(\frac{dP(\theta)}{dt}, 0\right) & 0 \\ \bullet & \bullet & -\Phi(\theta) \end{pmatrix} < 0 \quad (14.b)$$

with $\Phi(\theta) := Bdiag(P(\theta), S(\theta))$

The state feedback gain is then given by

$$F(\theta) = R(\theta)^T W^{-1}$$

Proof. By setting $R(\theta) = (F(\theta)W)^T$, this result can easily be proved using Theorem 3.

In the case of a polytopic descriptor system, previous condition can be expressed thanks to a finite set of LMIs as follows.

Corollary 1: There exists a descriptor state feedback controller such that the closed-loop pair given by $(E, A(\theta) + BF(\theta))$ is admissible if the there exist affinely dependant functions $P : \theta \to P(\theta)$ $S : \theta \to S(\theta)$, $R : \theta \to R(\theta)$ of appropriate dimensions and a matrix $W \in \mathbb{R}^{n \times n}$ such that $\forall (\omega_i, \tau_j) \in \Xi \times \Omega$

$$P(\omega_i) = P(\omega_i)^T > 0 \text{ and } S(\omega_i)^T = S(\omega_i)^T > 0$$

$$\begin{pmatrix} -He\{W\} & W^T A(\omega_i)^T + R(\omega_i) B^T + \Phi(\omega_i) & W^T \\ \bullet & -\Phi(\omega_i) + Bdiag(P(\tau_j) - P(0), 0) & 0 \\ \bullet & \bullet & -\Phi(\omega_i) \end{pmatrix} < 0$$

with $\Phi(\omega_i) := Bdiag(P(\omega_i), S(\omega_i))$

The state feedback gain is then given by

$$F(\theta) = R(\theta)^T W^{-1}.$$

4.2. A new LMI-based condition for observer synthesis of descriptor systems

New sufficient conditions similar to those proposed for the state feedback design are proposed next.

Theorem 6: There exist a descriptor observer gain such that the closed-loop pair given by $(E, A(\theta) + L(\theta)C)$ is admissible if the there exist continuously differentiable functions $P : \theta \to P(\theta)$, $S : \theta \to S(\theta)$, $H : \theta \to H(\theta)$ of appropriate dimensions and a matrix $W \in \mathbb{R}^{n\times n}$ such that for all $\theta(.) \in \Theta$

$$P(\theta) = P(\theta)^T > 0 \text{ and } S(\theta) = S(\theta)^T > 0 \quad (15.a)$$

$$\begin{pmatrix} -He\{W\} & W^T A(\theta) + H(\theta)C + \Phi(\theta) & W^T \\ \bullet & -\Phi(\theta) + Bdiag\left(\frac{dP(\theta)}{dt}, 0\right) & 0 \\ \bullet & \bullet & -\Phi(\theta) \end{pmatrix} < 0 \quad (15.b)$$

with $\Phi(\theta) := Bdiag(P(\theta), S(\theta))$

The observer gain is then given by $L(\theta) = W^{-T} H(\theta)$.

As in Corollary 1 those conditions can be turned into a finite set of LMIs in the case of affine LPV descriptor systems.

4.3. Stability of the closed-loop LPV system with the observer state feedback controller.

We consider the closed-loop descriptor system (2) with the controller (3). We assume that the state feedback $F(\theta)$ and the observer $L(\theta)$ have been synthesized thanks to Theorem 5 and Theorem 6. The closed-loop can be described by

$$\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{pmatrix} A(\theta) & BF(\theta) \\ -L(\theta)C & A(\theta) + BF(\theta) + L(\theta)C \end{pmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Let us define $e = x - \hat{x}$. The closed-loop system is equivalent to

$$\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} \begin{bmatrix} \dot{e} \\ \dot{\hat{x}} \end{bmatrix} = \begin{pmatrix} A(\theta) + L(\theta)C & 0 \\ -L(\theta)C & A(\theta) + BF(\theta) \end{pmatrix} \begin{bmatrix} e \\ \hat{x} \end{bmatrix} \quad (16)$$

According to the design of the state feedback and the observer, the pairs $(E, A(\theta) + BF(\theta))$ and $(E, A(\theta) + L(\theta)C)$ are both admissible. As a consequence, applying Lemma 1, lead to the admissibility of the closed-loop given by (16).

5. OBESERVER STATE FEEDBACK DESIGN FOR RATIONAL LPV SYSTEMS

In this section, we extract the rational observer state feedback controller thanks to the descriptor controller (5) designed in previous section. Once the state space rational LPV controller obtained we still need to prove the stability of the state space closed-loop. These results are formulated in next theorem.

Theorem 8: There exists a rational observer-based controller (2) stabilizing the rational LPV system given by (1) if there exists a descriptor observer-based controller given by (5) stabilizing the descriptor LPV system (3). The rational state feedback and observer gains are then given by

$$F_r(\theta) = \left(I - F_2(\theta)(A_4(\theta) + B_2F_2(\theta))^{-1} B_2\right) F_R(\theta) \quad (17.a)$$

$$L_r(\theta) = L_1(\theta) - A_2(\theta) A_4(\theta)^{-1} L_2(\theta) \quad (17.b)$$

with $\quad F_R(\theta) = F_1(\theta) - F_2(\theta) A_4(\theta)^{-1} A_3(\theta) \quad (17.c)$

and the matrix $G_r(\theta)$ is given by

$$G_r(\theta) = -F_2(\theta)(A_4(\theta) + B_2F_2(\theta))^{-1} L_2(\theta) \quad (17.d)$$

Proof: Realizations (1) and (3) are strongly equivalent (see Definition 1). Indeed, if we consider the two following continuously differentiable and non singular functions

$$M(\theta) = \begin{pmatrix} I & -A_2(\theta) A_4(\theta)^{-1} \\ 0 & A_4(\theta)^{-1} \end{pmatrix}, \; N(\theta) = \begin{pmatrix} I & 0 \\ -A_4(\theta)^{-1} A_3(\theta) & I \end{pmatrix}$$

it appears that

$$M(\theta) E N(\theta) = E, \; M(\theta) E \frac{d}{dt}(N(\theta)) = 0$$

$$M(\theta) \begin{pmatrix} A_1(\theta) & A_3(\theta) \\ A_2(\theta) & A_4(\theta) \end{pmatrix} N(\theta) = \begin{pmatrix} A_r(\theta) & 0 \\ 0 & I \end{pmatrix}$$

and that $M^{-1}(\theta), N^{-1}(\theta)$ are non singular and continuously differentiable functions.

Based on this, a strongly equivalent system to the descriptor controller (5) can be given by

$$\begin{cases} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{X}}_2 \end{bmatrix} = \begin{pmatrix} A_r(\theta) & 0 \\ 0 & I \end{pmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{X}_2 \end{bmatrix} \\ + \begin{pmatrix} B_r \\ A_4(\theta)^{-1} B_2 \end{pmatrix} u + \begin{pmatrix} L_r(\theta) \\ A_4(\theta)^{-1} L_2(\theta) \end{pmatrix} (\hat{y} - y) \\ u = \begin{pmatrix} F_R(\theta) & F_2(\theta) \end{pmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{X}_2 \end{bmatrix} \\ \hat{y} = C_r(\theta) \hat{x}_1 + C_2 X_2 \end{cases} \quad (18)$$

where $\quad \hat{X}_2 = A_4(\theta)^{-1} A_3(\theta) \hat{x}_1 + \hat{x}_2$

$$L_r(\theta) = L_1(\theta) - A_2(\theta) A_4(\theta)^{-1} L_2(\theta)$$

and $\quad F_R(\theta) = F_1(\theta) - F_2(\theta) A_4(\theta)^{-1} A_3(\theta)$

Eliminating $\hat{X}_2$ from (18) leads to the following state space realization for the controller

$$(K_r): \begin{cases} \dot{\hat{x}}_1 = A_r(\theta) \hat{x}_1 + B_r(\theta) u + L_r(\theta)(\hat{y} - y) \\ \hat{y} = C_r(\theta) \hat{x}_1 + D_r(\theta) u \\ u = F_r(\theta) \hat{x}_1 + G_r(\theta)(\hat{y} - y) \end{cases}$$

with $F_r(\theta) = \left(I - F_2(\theta)(A_4(\theta) + B_2F_2(\theta))^{-1} B_2\right) F_R(\theta)$

and $\quad G_r(\theta) = -F_2(\theta)(A_4(\theta) + B_2F_2)^{-1} L_2(\theta)$.

Furthermore, the admissibility of the descriptor LPV system (16) proves that $\left[(x - \hat{x})^T \quad \hat{x}^T\right]^T \to 0$ which implies that $\left[(x_1 - \hat{x}_1)^T \quad \hat{x}_1^T\right]^T \to 0$. This means that the rational LPV closed-loop system is stable.

Finally, the rational controller is entirely described by equations (17) and stabilizes the state space rational LPV system (1).

6. NUMERICAL EXAMPLE

Let us consider the following rational LPV system

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{pmatrix} \frac{\theta^2 + \theta}{\theta + 2} & \frac{3\theta + 4}{\theta + 2} \\ 1 & -1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} u \\ y = x_1 + x_2 \end{cases}$$

with $\theta \in \left[-1.5 \quad 1.5\right]$ and $\dot{\theta} \in \left[-1 \quad 1\right]$.

The simple change of variables $x_3 = \dfrac{\theta}{2+\theta}(x_1 - x_2)$ leads to the following affine LPV descriptor realization

$$\begin{cases} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{pmatrix} \theta & 2 & 1 \\ 1 & -1 & 0 \\ \theta & -\theta & -(2+\theta) \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} u \\ y = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{cases}$$

Applying Theorem 5 and Theorem 6 leads to the following gains

$$F(\theta) = \begin{pmatrix} -1.95+0.40\theta & -0.723+0.053\theta & -0.199-0.013\theta \end{pmatrix}$$

$$L(\theta)^T = \begin{pmatrix} -2.961-0.41\theta & -1.202-0.16\theta & -2.647-0.16\theta \end{pmatrix}^T$$

The rational LPV controller is then derived thanks to equations (17). We obtain the following rational matrices

$$F_r(\theta) = \begin{pmatrix} \dfrac{0.387\theta^2 - 0.951\theta - 3.9}{\theta+2} & \dfrac{0.06\theta^2 - 0.816\theta - 1.446}{\theta+2} \end{pmatrix}$$

$$L_r(\theta)^T = \begin{pmatrix} \dfrac{-0.41\theta^2 - 3.941\theta - 8.3890}{\theta+2} & -1.2021 - 0.16\theta \end{pmatrix}^T$$

$$G_r(\theta) = \frac{-0.002\theta^2 - 0.558\theta - 0.527}{\theta+2}$$

To see the influence of the chosen parameter variation, the eigenvalues domain of the estimation state error and the state feedback is given in Figure 1.

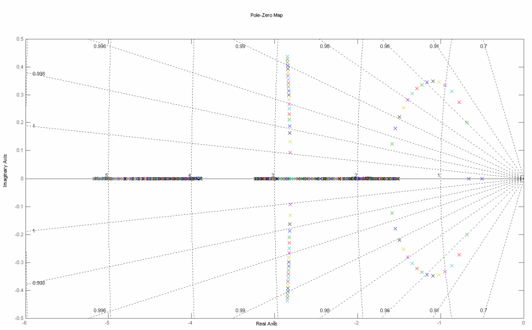

Figure 1: The eigenvalues domain for matrices $A_r(\theta) + L_r(\theta)C_r(\theta)$ and $A_r(\theta) + B_r(\theta)F_r(\theta)$

7. CONCLUSION

In this paper, a method for the design of gain scheduled observer-based controllers for rational LPV systems is presented. Based on an equivalent descriptor affine LPV realization, a descriptor observer-based controller is designed by means of some new sufficient conditions given as LMIs. A rational LPV controller, with an observer-based structure, is then derived. The stability of the obtained rational closed-loop is proved. The effectiveness of the proposed method has been tested on a numerical example.

REFERENCES

Apkarian P., Gahinet P. and Becker G., (1994), Self scheduled H_∞ control of linear parameter varying systems, *Proceeding of the American Control Conference on Control, pp. 856-860.*

Bara G. I., Daafouz J., Ragot J., (2002), Gain Scheduling techniques for the design of observer state feedback controllers, *15th IFAC World Congress, Barcelona, Spain.*

Bara G. I., Daafouz J., Kratz F., (2001), Advanced gain scheduling techniques for the design of parameter-dependent observers, *Proceeding of the 40th IEEE Conference on Decision and Control*, *Orlando, Florida, USA.*

Berriri M., Chevrel P., Yagoubi M., Claveau F., (2006), Observer-based gain scheduling H_∞ controller for a 3 motors web transport system, *5th IFAC Symposium on Robust Control Design, Toulouse, France.*

Bouali A., Chevrel P., Yagoubi M., (2006), About gain scheduled state feedback controllers for rational LPV systems, *9th International Conference on Automation Robotics Control and Vision, Singapore.*

Bouali A., Yagoubi M., Chevrel P., (2007a), New LMI-based conditions for stability, H_∞ performance analysis and state-feedback control of rational LPV systems, *European Conference on Control, Kos, Greece.*

Bouali A., Yagoubi M., Chevrel P., (2007b), Gain scheduled state feedback design for a class of LPV descriptor systems, *Conference on Systems and Control, Marrakech, Morocco.*

Cobb J. D., (2006), State Feedback Impulse Elimination for Singular Systems over a Hermite Domain, *SIAM Journal on Control and Optimization*, *Vol. 44, N° 6, pp. 2189-2209.*

Leith D., Leithead W., (2000), Survey of gain scheduling analysis and design, *International Journal of Control, Vol. 73, N°11, pp. 1001-1025.*

Masubuchi I., Kato J., Saeki M. and Ohara A., (2003), Synthesis of Output Feedback Gain Scheduling Controllers Based on Descriptor System Representation, *IEEE Conference on Decision and Control, pp. 6115-6120 Hawaii USA.*

Raharijoana T., Chevrel P., Duc G., (2006), Reduced order and observer-based LPV control for lateral driving assistance, *5th IFAC Symposium on Robust Control Design, Toulouse, France.*

Scherer C. W., (2001), LPV control and full block multipliers, *Automatica (37), pp. 361-375.*

Stilwell D. J., Rugh W. J., (1999), Interpolation of observer state feedback controllers for gain scheduling, *IEEE Transaction on Automatic and Control 44, N°6, pp. 1225-1229.*

Stilwell D. J., Rugh W. J., (2000), Stability preserving interpolation methods for the synthesis of gain scheduled controllers, *Automatica, (36), pp. 665-671.*