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High-gain observer-based parameter identification with application in a gas turbine engine ⋆

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Abstract: In this paper, a novel identification technique, that is high-gain observer-based identification approach, is proposed for systems with bounded process and measurement noises. For system parameters with abnormal changes, an adaptive change detection and parameter identification algorithm is next presented. The presented technique and algorithm is finally applied to the parameter identification of the gas turbine engine by using the recorded input data from the engine test-bed. The identified parameters and the response curves are desired. The simulations have proved the effectiveness of the proposed procedure compared with the previous identification approach.

Keywords: Parameter identification, bounded noise environment, gas turbine engine

1. INTRODUCTION

System identification is essentially important for establishing system model, and it is a starting point for system analysis and control synthesis. During the past decades, there were huge results reported in this field (see Ljung [1987]). Basically, there are two kinds of identification techniques: polynomial model identification and state-space model identification. The polynomial identification approach (e.g. see Villadsen et al. [1978], Evans et al. [2000], Sakai et al. [2005], Diversi et al. [2007]) is particularly suitable for a single-input-single-output (SISO) model. However, for multiple-input-multiple-output (MIMO) systems, the polynomial model identification approach may give rise to numerically ill-conditioned mathematical problems. So far, state-space model identification is still a hot research direction for MIMO systems. Specifically, in the literature by Larimore [1990], Verhaegen [1994] and Overschee et al. [1994], subspace identification technique was adopted, and the resulting identification toolbox *n4sid* was developed. In the meanwhile, on the basis of different observers/filters, several parameter estimation algorithms were developed by Friedland [1997], Liu et al. [1998] and Rajaraman et al. [2005]. However, it is not difficult to find that all the results in Larimore [1990], Verhaegen [1994], Overschee et al. [1994], Friedland [1997], Liu et al. [1998] and Rajaraman et al. [2005] are based on the noise-free or white noise assumption. Obviously, this kind of noise assumption cannot meet some practical situations. Recently in the reference by Baev et al. [2006], based on a high-order sliding-mode observer, a parameter identification approach was proposed under the bounded noise assumption rather than the white noise assumption. The bounded noise assumption is obviously popular in many control issues. Unfortunately, only the measurement noise was taken into account by Baev et al. [2006]. Actually, a process noise always exists in any practical processes. Therefore, this motivates us to develop a novel parameter identification technique for systems with both bounded process and measurement noises.

Very recently, an interesting high-gain observer was developed by Gao et al. [2007] with application in fault estimation and fault-tolerant control design. In this study, by using the high-gain observer technique with slight modifications, the system state, the bounded output noise and the process uncertainty, composed of the parameter perturbations, are estimated simultaneously. Using the estimated state, estimated process uncertainty and the mean method, the parameters are then identified. The dynamic response curves of the identified parameters are also given. When the parameters to be identified have obvious changes in different time intervals, it is indispensable to detect the abnormal changing points and identify the parameters in the resulting changing intervals. A new adaptive identification technique is addressed to simultaneously detect the abrupt changing points and identify the parameters over the resulting time intervals.

Gas turbine engines are widely used in many fields such as aerospace, marine and power generating etc (see Tim et al. [2005]). It is a fundamental and key task to identify the model of the gas turbine engine accurately. In the paper, by using the proposed high-gain observer-based identification technique and the recorded input data from the engine test-bed, simulation study is investigated in detail. The identified parameters are desired for the gas turbine engine with unexpected bounded process and output noises. The abnormal change detection and adaptive parameter identification has also proved effective in this simulated study.

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2. HIGH-GAIN OBSERVER DESIGN

Consider the following dynamic system

$$\begin{cases} \dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)u(t) + \omega_i(t) \\ y(t) = x(t) + \omega_o(t) \end{cases} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is a state vector, $u(t) \in \mathcal{R}^m$ represents a control input vector, $y(t) \in \mathcal{R}^n$ is a measurement output vector, $\omega_i(t) \in \mathcal{R}^n$ and $\omega_o(t) \in \mathcal{R}^n$ are input and output noise vectors, respectively; A and B are known constant matrices, and ΔA and ΔB are unknown matrices to be determined. Denote

$$\begin{aligned} & d(t) = \Delta A x(t) + \Delta B u(t), \\ & \bar{x}(t) = \begin{bmatrix} x(t) \\ d(t) \\ \omega_o(t) \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0_{n\times m} \\ 0_{n\times m} \end{bmatrix}, \\ & \bar{G} = \begin{bmatrix} I_n \\ 0 \\ 0 \end{bmatrix}, \ \bar{H} = \begin{bmatrix} 0 \\ I_n \\ 0 \end{bmatrix}, \ \bar{N} = \begin{bmatrix} 0 \\ 0 \\ I_n \end{bmatrix}, \\ & \bar{E} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0_{n\times n} \end{bmatrix}, \\ & \bar{A} = \begin{bmatrix} A & I_n & 0 \\ 0 & 0_{n\times n} & 0 \\ 0 & 0 & -I_n \end{bmatrix}, \\ & \bar{C} = [\, I_n \ 0 \ I_n \,]. \end{aligned} \quad (2)$$

As a result, an augmented descriptor system can be obtained from (1) and (2) to give

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{G}\omega_i(t) + \bar{H}\dot{d}(t) + \bar{N}\omega_o(t) \\ y(t) = \bar{C}\bar{x}(t). \end{cases} \quad (3)$$

In this study, $\dot{d}$ (t), $\omega_i(t)$ and $\omega_o(t)$ are all assumed to bounded. In this context, the following observer can be constructed

$$\begin{cases} \bar{S}\dot{\xi}(t) = (\bar{A} - \bar{K}\bar{C})\xi(t) + \bar{B}u(t) - \bar{N}y(t) \\ \hat{\bar{x}}(t) = \xi(t) + \bar{S}^{-1}\bar{L}y(t) \end{cases} \quad (4)$$

where $\xi(t) \in \mathcal{R}^{3n}$ is the state vector of the dynamic system above, $\hat{\bar{x}}(t) \in \mathcal{R}^{3n}$ is the estimate of $\bar{x}(t) \in \mathcal{R}^{3n}$, $\bar{S} = \bar{E} + \bar{L}\bar{C}$, and $\bar{K}, \bar{L} \in \mathcal{R}^{3n\times p}$ are the gain matrices to be designed.

Here we choose

$$\bar{L} = \begin{bmatrix} 0 \\ 0 \\ M \end{bmatrix} \quad (5)$$

where $M \in \mathcal{R}^{n\times n}$ is a non-singular matrix. One thus can calculate:

$$\bar{S} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ M & 0 & M \end{bmatrix}, \quad \bar{S}^{-1} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ -I_n & 0 & M^{-1} \end{bmatrix}. \quad (6)$$

In terms of (2) and (6), it is further derived that

$$\bar{C}\bar{S}^{-1}\bar{L} = I_n, \ \bar{A}\bar{S}^{-1}\bar{L} = -\bar{N}. \quad (7)$$

Using (7), the estimator (4) can be expressed as

$$\bar{S}\dot{\hat{\bar{x}}}(t) = \bar{A}\hat{\bar{x}}(t) + \bar{B}u(t) + \bar{K}(y(t) - \bar{C}\hat{\bar{x}}(t)) + \bar{L}\dot{y}(t) \quad (8)$$

The dynamic equation of the plant (3) can be expressed as

$$\begin{aligned} \bar{S}\dot{\bar{x}}(t) = & \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{G}\omega_i(t) \\ & + \bar{H}\dot{d}(t) + \bar{N}\omega_o(t) + \bar{L}\dot{y}(t) \end{aligned} \quad (9)$$

Letting $\bar{e}(t) = \bar{x}(t) - \hat{\bar{x}}(t)$ and subtracting (8) from (9), one has

$$\begin{aligned} \dot{\bar{e}}(t) & = \bar{S}^{-1}[(\bar{A} - \bar{K}\bar{C})\bar{e}(t) + \bar{G}\omega_i(t) + \bar{H}\dot{d}(t) + \bar{N}\omega_o(t)] \\ & = \bar{S}^{-1}(\bar{A} - \bar{K}\bar{C})\bar{e}(t) + \bar{N}M^{-1}\omega_o(t) \\ & \quad + \bar{H}\dot{d}(t) + (\bar{G} - \bar{N})\omega_i(t). \end{aligned} \quad (10)$$

From (10), one can choose a high-gain M to reduce the effect from $\omega_o(t)$ on the estimation error dynamics.

According to the work by Gao et al. [2007], the high-gain matrix $\bar{K}$ can be computed as

$$\bar{K} = \bar{S}\bar{P}^{-1}\bar{C}^T, \quad (11)$$

where $\bar{P}$ is solved from the following Lyapunov equation

$$-(\mu I + \bar{S}^{-1}\bar{A})^T\bar{P} - \bar{P}(\mu I + \bar{S}^{-1}\bar{A}) = -\bar{C}^T\bar{C}, \quad (12)$$

with $\mu > 0$ satisfying $\Re[\lambda_i(\bar{S}^{-1}\bar{A})] > -\mu$, $\forall i \in \{1, 2, \ldots, 3n\}$.

Remark 1.
By letting $d(t) = \Delta A x(t) + \Delta B u(t)$, the augmented system (3) is constructed. Therefore, the observer (4), stemmed form the design technique proposed by Gao et al. [2007], can be used to estimate the state x, the input uncertainty d and the output noise ω_o simultaneously:

$$\hat{x}(t) = [\, I_n \ 0_{n\times 2n} \,]\,\hat{\bar{x}}(t), \quad (13)$$

$$\hat{d}(t) = [\, 0_n \ I_n \ 0_n \,]\,\hat{\bar{x}}(t), \quad (14)$$

$$\hat{\omega}_o(t) = [\, 0_{n\times 2n} \ I_n \,]\,\hat{\bar{x}}(t). \quad (15)$$

It is noted that, the observer (4) does exist without any constraints since $C = I$. It it worthy to point out that the estimates of $x(t)$ and $d(t)$ pave the way for the parameter identification in the succeeding section.

3. NOVEL PARAMETER IDENTIFICATION TECHNIQUES

3.1 Observer-based parameter identification

The estimates of $x(t)$ and $d(t)$ will be used to identify the unknown parameters ΔA and ΔB. The $d(t)$ and its estimate $\hat{d}(t)$ can be expressed as

$$d(t) = [\, \Delta A \ \Delta B \,] \begin{bmatrix} \hat{x}(t) \\ u(t) \end{bmatrix} + \Delta A e_x(t) \quad (16)$$

$$\hat{d}(t) = [\, \Delta\hat{A} \ \Delta\hat{B} \,] \begin{bmatrix} \hat{x}(t) \\ u(t) \end{bmatrix} \quad (17)$$

where $e_x(t) = x(t) - \hat{x}(t) = [\, I_n \ 0_{n\times 2n} \,]\,\bar{e}(t)$; $\Delta\hat{A}$ and $\Delta\hat{B}$ are the estimates of ΔA and ΔB, respectively.

Subtracting (17) from (16), one can derive that

$$d(t) - \hat{d}(t) = \begin{bmatrix} \Delta A - \Delta\hat{A} & \Delta B - \Delta\hat{A} \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ u(t) \end{bmatrix} + \Delta A e_x(t) \quad (18)$$

Since $e_x(t)$ and $e_d(t) = d(t) - \hat{d}(t)$ both tend to be desired small as time tends to infinity, one can conclude that $\Delta\hat{A} \to \Delta A$ and $\Delta\hat{B} \to \Delta B$ as time tends to infinity.

From (17), one has

$$\Gamma(t)\theta(t) = \hat{d}(t) \quad (19)$$

where

$$\Gamma(t) = \begin{bmatrix} (\hat{x}^T \;\; u^T) & 0 & \cdots & 0 \\ 0 & (\hat{x}^T \;\; u^T) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\hat{x}^T \;\; u^T) \end{bmatrix} \in \mathcal{R}^{n\times n(n+m)},$$

and

$$\theta(t) = \begin{bmatrix} \Delta\hat{A}_1^T \\ \Delta\hat{B}_1^T \\ \vdots \\ \Delta\hat{A}_n^T \\ \Delta\hat{B}_n^T \end{bmatrix} \in \mathcal{R}^{n(n+m)}$$

where $\begin{bmatrix} \Delta\hat{A}_i & \Delta\hat{B}_i \end{bmatrix} \in \mathcal{R}^{1\times(n+m)}$ represents the ith row of the matrix $\begin{bmatrix} \Delta\hat{A} & \Delta\hat{B} \end{bmatrix}$, $\theta(t)$ is the parameter to be identified.

An important formula (19) has been given. However, there are $n(n + m)$ identified parameters in the n equations described by (19). Therefore, the equation (19) can not be solved uniquely. Motivated by the work (Baev et al. [2006]) for continuous-system parameter identification and with slight modifications, the following equations can be constructed in terms of (19):

$$\begin{aligned} &\Gamma(t-\delta_1)\theta(t) = \hat{d}(t-\delta_1) \\ &\Gamma(t-\delta_2)\theta(t) = \hat{d}(t-\delta_2) \\ &\vdots \\ &\Gamma(t-\delta_{n+m-1})\theta(t) = \hat{d}(t-\delta_{n+m-1}) \end{aligned} \quad (20)$$

where $\delta_i = i\delta$, $i = 1, 2, \cdots n + m - 1$, δ is some constant time interval, and $t > (n + m)\delta$.

Equations (19) and (20) can be grouped into the one $(n + m)n$-order linear algebraic system:

$$\underbrace{\begin{bmatrix} \Gamma(t) \\ \Gamma(t-\delta_1) \\ \vdots \\ \Gamma(t-\delta_{n+m-1}) \end{bmatrix}}_{\Gamma_H} \theta(t) = \underbrace{\begin{bmatrix} \hat{d}(t) \\ \hat{d}(t-\delta_1) \\ \vdots \\ \hat{d}(t-\delta_{n+m-1}) \end{bmatrix}}_{\hat{d}_H} \quad (21)$$

The solution to (21) is

$$\begin{aligned} \theta(t) &= (\Gamma_H)^{-1}\hat{d}_H \\ &= \frac{Adj(\Gamma_H)}{\det(\Gamma_H)}\hat{d}_H \end{aligned} \quad (22)$$

By using the well-known Cramer's rule (e.g. see Lay [2003]), one has

$$\theta_j(t) = \frac{\det(\Gamma_{Hj})}{\det(\Gamma_H)} \quad (23)$$

where $\theta_j(t)$ is the jth component of the vector $\theta(t)$, and Γ_{Hj} is the matrix obtained by replacing the entries in the jth column of Γ_H by the entries in the matrix $\hat{d}_H$.

In order to avoid possible jumps at some points due to the numerical computation, a way is to take the integral to give the following formula:

$$\theta_j(t) = \frac{\int_{t-T_L}^{t} \det(\Gamma_{Hj})\det(\Gamma_H)dt}{\int_{t-T_L}^{t}[\det(\Gamma_H)]^2 dt}, \quad t > (n+m)\delta + T_L \quad (24)$$

where t is the end time of the current integral window, and T_L is the selected integral length. Therefore, the response curve of $\theta_j(t)$ can be given as t increases along the time axis. In (24), the least-square idea proposed by Smith et al. [2002] is used actually.

In order to further smooth the response curve, we can take the mean calculation over the time interval as follows:

$$\bar{\theta}_j(t) = \frac{1}{N}\int_{t-N}^{t} \theta_j(t)dt, \quad t \geq N \quad (25)$$

where $\theta_j(t)$ is the identified parameter obtained from (24), and N is the length to take the mean calculation.

The identified parameter $\theta_j(t)$ is assumed to stay constantly (not to change abnormally) over the whole simulation interval, the identified parameter can be thus calculated as

$$\bar{\theta}_j(T_2) = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \theta_j(t)dt \quad (26)$$

where $T_1 > (n + m)\delta$ is the starting time, and T_2 is the end time of the simulation.

Now it is ready to give the following identification algorithm.

Algorithm 1. High-gain observer-based identification (HOI) algorithm.

1) Construct the augmented matrices as in (2).
2) Select the observer gains $\bar{L}$ as in (5) and calculate $\bar{S}$ and $\bar{S}^{-1}$ in terms of (6).
3) Solve the Lyapunov equation (12) and compute the high-gain matrix $\bar{K}$ according to (11).
4) Implement the real-time estimation using (4), and obtain the estimates of $\hat{x}$, $\hat{d}$ and $\hat{\omega}_o$ in the forms of (13)-(15).
5) Construct the equation (19).
6) Select a time interval δ and construct the equation (20).
7) Calculate the identified parameters $\theta_j(t)$, $j = 1, 2, \cdots, n(n+m)$, according to the integral (24), and plot the parameter curves.
8) From (25), further calculate the refined identified parameter $\bar{\theta}_j(t)$, $j = 1, 2, \cdots, n(n + m)$, and plot the response curves of the parameters.

9) Finally, a set of constant identified parameters is given by (26).

3.2 Adaptive parameter identification

In the last subsection, the concerned case is that the identified system parameters do not have obvious changes or the changes are tolerable under the whole identification interval. However, if some parameter changes in ΔA and ΔB are obvious in different time intervals, the obtained identified parameters in terms of (26) will be inaccurate. Moreover, in practical systems, the system parameters may change abnormally due to the age or the accidents of the components. Therefore, it is very important to detect the changes and identify the changed parameters simultaneously for improving identification accuracy and system reliability. In this study, the simultaneous change detection and parameter identification is called *adaptive parameter identification*.

For a system parameter with a single abnormal changing point only, the constant threshold method can work well when the system runs at some known steady state. If a series of abrupt changes happen, it is more likely that such an approach can only detect the first change, but fail to detect the succeeding changes, due to the constant threshold.

In this study, we assume that the considered system parameters may have multiple intolerable abrupt change points. In order to solve this issue, adaptive threshold is adopted for detecting a sequence of changes. The basic concept is to re-calculate the threshold after detecting each change. The adaptive threshold is expressed as the tolerable maximal relative varying ratio with respect to the nominal value of the identified parameter. Actually, the nominal value is defined as the mean value of the identified parameter under some selected steady period. Firstly, we should calculate the nominal value over a selected initial period, and determine the first threshold. If any a real-time parameter calculated by (25) is beyond the threshold, and keeps the overflow over a confirmed time interval, a change alarm is given. By calculating the new nominal value after the change, and determine a new threshold, the change detection process can keep going. The adaptive change detection and parameter identification technique can be described by the following algorithm.

Algorithm 2. Adaptive parameter identification.

1) Choose T_0 such that
$$T_0 > (n+m)\delta + T_L + T_D$$
where T_D is the selected time length for the nominal parameter identification; n, m, δ and T_L are defined as before.
2) Calculate the nominal value of the identified parameter as
$$\theta_j^0 = \frac{1}{T_D}\int_{T_0-T_D}^{T_0} \theta_j(t)dt \tag{27}$$
where $\theta_j(t)$ is given by (24).
3) Set the adaptive thresholds for the changes detection:
$$\theta_j^+ = \begin{cases} (1+\rho)\theta_j^0 & \text{if } \theta_j^0 > 0 \\ (1-\rho)\theta_j^0 & \text{if } \theta_j^0 < 0 \end{cases} \tag{28}$$
and
$$\theta_j^- = \begin{cases} (1-\rho)\theta_j^0 & \text{if } \theta_j^0 > 0 \\ (1+\rho)\theta_j^0 & \text{if } \theta_j^0 < 0 \end{cases} \tag{29}$$
where $0 \le \rho < 1$; ρ is the tolerable maximal relative varying ratio with resect to the nominal value θ_j^0; θ_j^+ and θ_j^- are the up and low boundaries, respectively.
4) Select T_c as the so-called *confirm window* for the change. If the current time t satisfies
$$\exists j, \bar{\theta}_j(\tau) < \theta_j^- \text{ or } \bar{\theta}_j(\tau) > \theta_j^+, \forall \tau \in \{\tau | t - T_c \leqslant \tau \leqslant t\} \tag{30}$$
where the real-time parameter $\bar{\theta}_j(\tau)$ is given by (25), then a change is detected at the time t.
5) If any change is detected, calculate the following mean value:
$$\hat{\theta}_j = \frac{1}{t-T_0}\int_{T_0}^{t} \theta_j(t)dt \tag{31}$$
Set $T_0 = t + T_D$, and go back to step 2.
6) Otherwise, continue Step 3 until the simulation ends. Then go to next step.
7) Calculate the following mean value:
$$\hat{\theta}_j = \frac{1}{T_{end}-T_0}\int_{T_0}^{T_{end}} \theta_j(t)dt \tag{32}$$
where T_{end} is the end time of the simulation.

4. PARAMETER IDENTIFICATION FOR A GAS TURBINE SYSTEM

A gas turbine model is characterized by
$$\begin{cases} \dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)u(t) + \omega_i(t) \\ y(t) = x(t) + \omega_o(t) \end{cases} \tag{33}$$
where the input $u(t)$ is the mass flow rate to the combustion chamber, the components of the output $y(t)$ are the low-pressure and high-pressure shaft speeds, respectively. The coefficient matrices
$$A = \begin{bmatrix} -0.9426 & 0.1601 \\ 3.9439 & -3.2348 \end{bmatrix}, \ B = \begin{bmatrix} 86.7941 \\ 154.6907 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{34}$$
are known. In the meanwhile, the matrices ΔA and ΔB, due to the modeling accuracy, the aging of physical apparatus and the noise effect, are the parameters to be estimated. In this simulation, we assume
$$\Delta A = \begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix} = \begin{bmatrix} 0.3000 & 0.1000 \\ 0.8000 & -0.8000 \end{bmatrix},$$
$$\Delta B = \begin{bmatrix} \Delta b_1 \\ \Delta b_2 \end{bmatrix} = \begin{bmatrix} 1.2000 \\ -1.2000 \end{bmatrix}. \tag{35}$$

4.1 Parameter identification: standard n4sid algorithm

For the gas turbine system (33), the input data are gathered from the engine test-bed at the normal engine operating point Tim et al. [2005]. Using the standard system identification toolbox *n4sid* with the input and output data, one can obtain the perturbed parameters ΔA and ΔB as shown in third row (see Identification 1) of Table 1. Comparing the identified parameters in the third row with the true parameters in the second row, one can

Table 1. Identified parameters via standard *n4sid* technique

Parameters	Δa_{11}	Δa_{12}	Δb_1	Δa_{21}	Δa_{22}	Δb_2
True	0.3	0.1	1.2	0.8	-0.8	-1.2
Identification 1	0.3000	0.1000	1.1864	0.8000	-0.8000	-1.1038
Identification 2	-9.1684	7.0839	-55.6453	-8.2551	5.8790	-10.9569

Table 2. Identified parameters via the proposed technique

Parameters	Δa_{11}	Δa_{12}	Δb_1	Δa_{21}	Δa_{22}	Δb_2
True	0.3	0.1	1.2	0.8	-0.8	-1.2
Identification HOI 1	0.290257	0.107670	1.126614	0.815732	-0.812296	-1.110485
Identification HOI 2	0.288044	0.109498	1.120677	0.813510	-0.810460	-1.116432

see that the standard subspace identification approach is obviously effective for systems without noises.

Let

$$\omega_i = 0.2sin(20t) \tag{36}$$

which is corrupted by a random signal with the variance 0.0001; and

$$\omega_o = \begin{cases} 0, & t < 5, \\ r(t), & t \geq 5 \end{cases}$$

where $r(t) = 0.025sin(120t)$ corrupted by a random signal with the variance 0.00005. The noises considered are not white noises, but quasi-stationary noises.

Using the standard system identification toolbox *n4sid*, one can obtain the identified parameters as shown in the fourth row (see Identification 2) of Table 1. One can see that the identified parameters are not acceptable compared with the real parameters as shown in the second row. It is not strange, because the standard subspace identification approach is only valid for systems under white noise environment.

4.2 Novel parameter identification approach

(i) High-gain observer-based identification (HOI)

The augmented plant in the form of (3) can be constructed. Choose

$$\bar{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}^T.$$

Select $\mu = 1000$, the high gain $\bar{K}$ is computed as

$$\bar{K} = \begin{bmatrix} 0.01199014349023 & 0.00001675489315 \\ 0.00002430658075 & 0.01196732057822 \\ 7.99579705740000 & 0.01579720213789 \\ 0.00066280862061 & 7.98669223765000 \\ 0.00000599790547 & 0.00000000411038 \\ 0.00000000411038 & 0.00000599333828 \end{bmatrix} \times 10^{10}$$

The identified parameters are given as shown in Table 2. Compared with the true values (see the second row of Table 2), the identified parameters for systems without noises (see HOI 1, the third row of Table 2) and subjected to noises (see HOI 2, the fourth row of Table 2) are both desired.

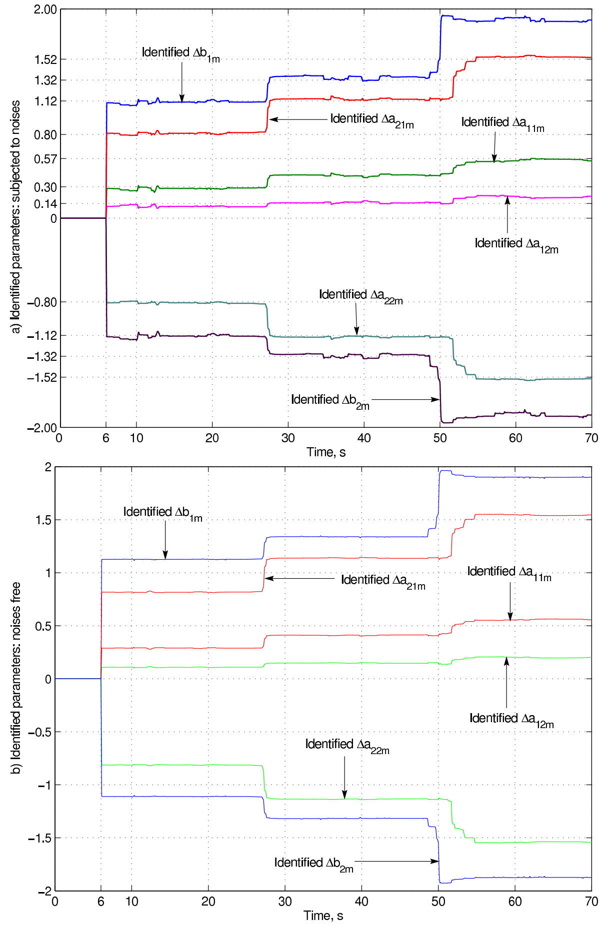

Fig. 1. Responses of the identified parameters with changing intervals

(ii) Adaptive parameter identification

In this case, we assume the perturbed parameters with obvious changes at different time intervals:

$$\Delta A_m = \begin{cases} \Delta A, & 0 \leq t < 25 \\ 1.4\Delta A, & 25 \leq t < 50 \\ 1.9\Delta A, & t \geq 50 \end{cases}$$
$$\Delta B_m = \begin{cases} \Delta B, & 0 \leq t < 25 \\ 1.2\Delta B, & 25 \leq t < 45 \\ 1.7\Delta B, & t \geq 45 \end{cases} \tag{37}$$

where ΔA and ΔB are defined in (35).

From (4.2), there are three abnormal changing points in ΔA and ΔB, which occur at $25s$, $45s$ and $50s$, respectively.

Table 3. Identified parameter changes: subjected to noises

Interval	Parameters	Δa_{11m}	Δa_{12m}	Δb_{1m}	Δa_{21m}	Δa_{22m}	Δb_{2m}
1^a	True	0.3	0.1	1.2	0.8	-0.8	-1.2
	Identified	0.28355	0.11304	1.1083	0.81108	-0.80857	-1.1283
2^b	True	0.42	0.14	1.44	1.12	-1.12	-1.44
	Identified	0.41006	0.1481	1.3485	1.1373	-1.1341	-1.3153
3^c	True	0.42	0.14	2.04	1.12	-1.12	-2.04
	Identified	0.48237	0.17378	1.9078	1.3316	-1.3291	-1.9257
4^d	True	0.57	0.19	2.04	1.52	-1.52	-2.04
	Identified	0.55374	0.20412	1.895	1.5411	-1.5385	-1.8791

$a : \Delta A$ & ΔB; $b : 1.4\Delta A$ & $1.2\Delta B$; $c : 1.4\Delta A$ & $1.7\Delta B$; $d : 1.9\Delta A$ & $1.7\Delta B$.

Table 4. Identified parameter changes: noises free

Interval	Parameters	Δa_{11m}	Δa_{12m}	Δb_{1m}	Δa_{21m}	Δa_{22m}	Δb_{2m}
1^a	True	0.3	0.1	1.2	0.8	-0.8	-1.2
	Identified	0.28941	0.10839	1.1264	0.81696	-0.81325	-1.1103
2^b	True	0.42	0.14	1.44	1.12	-1.12	-1.44
	Identified	0.4102	0.14792	1.3433	1.1375	-1.1344	-1.3215
3^c	True	0.42	0.14	2.04	1.12	-1.12	-2.04
	Identified	0.48855	0.16964	1.9326	1.3395	-1.3362	-1.9008
4^d	True	0.57	0.19	2.04	1.52	-1.52	-2.04
	Identified	0.55733	0.201	1.9003	1.5447	-1.5416	-1.8736

$a : \Delta A$ & ΔB; $b : 1.4\Delta A$ & $1.2\Delta B$; $c : 1.4\Delta A$ & $1.7\Delta B$; $d : 1.9\Delta A$ & $1.7\Delta B$.

Using the proposed algorithm 2, the three changing points are detected respectively at $27.247s$, $49.97s$ and $54.708s$ for systems subjected to noises. The adaptive identified parameters are shown in Table 3. One can see that the parameters under four time intervals are identified adaptively and satisfactorily. For systems without noises, the three changing points are detected at $27.27s$, $50.024s$ and $54.709s$, respectively. The desired identified parameters are given by Table 4. Moreover, the dynamic responses of the identified parameters are given by Figure 1. In consequence, the performance of the adaptive changes detection and parameters identification is desired.

5. CONCLUSIONS

On the basis of the high-gain observer technique, two novel parameters identification algorithms have been proposed. The presented algorithms are effective for systems under bounded process and measurement noises environment, which is more realistic in practical cases. The proposed procedures have been applied to the simulated study of the gas turbine engine. The simulated curves and the identified parameters are both desired.

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