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Non-Fragile Observer-Based Control of Vehicle Dynamics Using T-S Fuzzy Approach ⋆

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Abstract: This paper deals with the problem of non-fragile observer-based vehicle control. Vehicle dynamics are described by a 10-DOF (Degree Of Freedom) model which include lateral, longitudinal, yaw and roll dynamics. Plant dynamics uncertainties as well as the vehicle longitudinal velocity variation are taken into account in the controller synthesis. The controller to be designed is assumed to be subject to gain variations, due to additive unknown noise and environmental influence. The nonlinear vehicle model is approximated by a Takagi-Sugeno fuzzy model with structured parametric uncertainties. Combined pole placement and H_∞ algorithm is used to satisfy performance specifications. Closed-loop stability conditions are given in the form of LMI (Linear Matrix Inequalities).

Keywords: Fuzzy observer; LMI; Non-fragile control; Pole placement; Takagi-Sugeno Fuzzy models, Vehicle dynamics; Uncertainty.

Nomenclature :

V_x, V_y	Longitudinal velocity, lateral velocity
r, p, β	Yaw rate, roll rate, side slip angle
φ, ϕ	Yaw angle, roll angle
ω_1, ω_2	Angular velocity of front left, front right wheel
ω_3, ω_4	Angular velocity of rear left, rear right wheel
m_v, m_s	Vehicle's total mass, vehicle's sprung mass
$d_{f,r}$	Distance from (front, rear) axle to vehicle's gravity center
$C_{f,r}$	Cornering stiffness coefficients (front,rear)
C_{σ_f,σ_r}	Longitudinal stiffness coefficients (front,rear)
R_t	Effective wheel rolling radius
h_s	Height of sprung mass center gravity above roll axis
J_{xx}	Sprung mass roll moment of inertia
J_{zz}	Principal yaw inertia moment
J_{xz}	Sprung mass inertia moment about yaw and roll axes
J_t	Effective rotational inertia moment
$k_{sf,sr}$	Rotary compliances of steering actuators (front, rear)
csf, sr	Rotary viscous damping coefficients (front, rear)
$\delta_{f,r}$	Steering angle (front, rear)
$\delta_{sf,sr}$	Actuators command (front, rear)
c_{ϕ_f,ϕ_r}	Rotational damping coefficients (front,rear)
k_{ϕ_f,ϕ_r}	Rotational stiffness coefficients (front,rear)
τ_{b1}, τ_{b2}	Brake torque of front wheels (left,right)
τ_{b3}, τ_{b4}	Brake torque of rear wheels (left,right)
g	Gravity acceleration

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1. INTRODUCTION

Many safety systems have been developed and installed in vehicles like ABS and ESP. However, in some critical driving situations (variation of road state, emergency braking, skid in cornering), these systems are still not optimal and can be improved using advanced control methods: Liaw et al. [2005], B. A. Güvenç et al. [2004], Benton et al. [2005], Ono et al. [1999],Catino et al. [2003], Ono et al. [1993] and You et al. [1999]. However, most of these works are based on simplified vehicle models or assume that the vehicle longitudinal velocity is constant. Recent research works, which take into account the variation of the longitudinal vehicle velocity, have been based on simplified vehicle models, see Palladino et al. [2006] and Leith et al. [2005]. Moreover many specific information are necessary to achieve control. However, some sensors are still very expensive (like the side slip angle sensor). Thus, observer-based pole placement method has been proposed to satisfy desired control performances, see El Messoussi et al. [2006]. Moreover, although several research works have cope up with different control methods that take into consideration uncertainties in the plant dynamics, they always assume that the designed controller is precise and exactly implemented. However, in practical applications, controller parameters can have some variations due to additive unknown noise and environmental influence, see Dorato [1998] and Jadbabaie et al. [1998]. In the recent years, non-fragile control has been proposed to design a feedback control that will be insensitive to some errors or variations in gains of feedback control. In Yang et al. [2001] and Zhang et al. [2007] the authors present a the non-fragile control but without observer. Recent results in Lien [2007] have brought attention to observer-based non-fragile control, but with no consideration to uncertainty in the plant dynamics. Thus, in this paper, we extend the results of El Messoussi et al. [2007], in which we have developed a vehicle control method to deal with uncertainty in the plant dynamics. In this work, controller gains which include uncertainties are considered. The paper is organized as follows: in section 2, we present the vehicle dynamics mathematical model. The controller design strategy is given in section 3 whereas, in section 4, simulation results of the developed controller, applied to the nonlinear vehicle model, are given to show the effectiveness of our approach.

2. VEHICLE DYNAMICS DESCRIPTION

In this section, we describe a four-wheel steering (4WS) vehicle by a 10-DOF model including lateral, longitudinal, yaw and roll dynamics.We suppose that front and rear cornering forces are given as follows:

$$F_{yfi} = -C_f \alpha_f, \quad F_{yri} = -C_r \alpha_r, \quad i = l, r$$

Where α_f,α_r are front and rear tire slip angles given by:

$$\alpha_f = -\beta - \frac{d_f}{V_x} r + \delta_f + R_f \phi, \;\; \alpha_r = -\beta + \frac{d_f}{V_x} r + \delta_r + R_r \phi.$$

And front and rear longitudinal forces are given by:

$$F_{xfi} = C_{\sigma f} \sigma_{xfj}, F_{xri} = C_{\sigma r} \sigma_{xrj}, \;\; i = l, r.$$

Where σ_f, σ_r are front and rear longitudinal slip ratio given by:

$$\sigma_{xfj} = \frac{R_t \omega_j - V_x}{V_x}, \, j = 1, 2. \quad \sigma_{xrj} = \frac{R_t \omega_j - V_x}{V_x}, \, j = 3, 4.$$

During braking maneuver, and by considering small angles, the following dynamic equations are obtained:

- Longitudinal dynamics

$$\begin{aligned} m_v \dot{V}_x \; = \; & m_v V_y r - 2C_{\sigma f} - 2C_{\sigma r} \\ & + \frac{C_{\sigma f}}{V_x} R_t \left(\omega_1 + \omega_2 \right) \\ & + \frac{C_{\sigma r}}{V_x} R_t \left(\omega_3 + \omega_4 \right) \end{aligned} \tag{1}$$

- Lateral dynamics

$$\begin{aligned} m_v \dot{V}_y \; = \; & -m_v V_x r - m_s h_s \dot{p} + (2C_f + 2C_r) \frac{V_y}{V_x} \\ & + \left(2C_f \frac{d_f}{V_x} - 2C_r \frac{d_r}{V_x} \right) r \\ & -2C_f \delta_f - 2C_r \delta_r - (2C_f R_f + 2C_r R_r) \phi \end{aligned} \tag{2}$$

- Yaw dynamics

$$\begin{aligned} J_{zz} \dot{r} - J_{xz} \dot{p} \; = \; & (2d_f C_f - 2d_r C_r) \frac{V_y}{V_x} \\ & + \left(2C_f \frac{d_f^2}{V_x} + 2C_r \frac{d_r^2}{V_x} \right) r \\ & -2d_f C_f \delta_f + 2d_r C_r \delta_r \\ & +2 \left(d_r C_r R_r - d_f C_f R_f \right) \phi \\ & + \frac{d_t C_{\sigma f} r_{eff}}{V_x} \left(\omega_1 - \omega_2 \right) \\ & + \frac{d_t C_{\sigma r} r_{eff}}{V_x} \left(\omega_3 - \omega_4 \right). \end{aligned} \tag{3}$$

- Roll dynamics

$$\begin{aligned} & J_{xx} \dot{p} - J_{xz} \dot{r} = -m_s h_s \dot{V}_y - m_s h_s V_x r \\ & \qquad -C_\phi p - (k_\phi - m_s g h_s) \phi \\ & \dot{\phi} = p. \end{aligned} \tag{4}$$

- Wheel dynamics

$$J_t \dot{\omega}_i = -\tau_{bi} + R_t C_{\sigma f} - \frac{C_{\sigma f} R_t^2}{V_x} \omega_i, \, i = 1, 2, 3, 4. \tag{5}$$

- Actuator dynamics

$$\begin{aligned} \dot{\delta}_f &= \frac{-k_{sf}}{c_{sf}} \delta_f + \frac{k_{sf}}{c_{sf}} \delta_{sf} - \frac{k_{sf}}{c_{sf}} \delta_c \\ \dot{\delta}_r &= \frac{-k_{sr}}{c_{sr}} \delta_r + \frac{k_{sr}}{c_{sr}} \delta_{sr}. \end{aligned} \tag{6}$$

Where $C_\phi = c_{\phi f} + c_{\phi r}$, $k_\phi = k_{\phi f} + k_{\phi r}$ and δ_c is the driver action. Let us consider the state vector:

$$x = \left[\, V_y \;\; r \;\; p \;\; \phi \;\; \delta_f \;\; \delta_r \;\; \omega_1 \;\; \omega_2 \;\; \omega_3 \;\; \omega_4 \, \right]^t$$

And the control input:

$$u = \left[\, \delta_{sf} \;\; \delta_{sr} \;\; \tau_{b1} \;\; \tau_{b2} \;\; \tau_{b3} \;\; \tau_{b4} \, \right]^t$$

Then, from (1), (2), (3), (4), (5) and (6), we can describe the nonlinear vehicle model as follows:

$$\dot{x}(t) = A(V_x(t)) x(t) + Bu(t) + B_c \delta_c(t) + B_d u_d \tag{7}$$

Where

$$A(V_x(t)) = \left(\begin{array}{c|c|cc|cc} \Re & \aleph & \begin{matrix} a_{17} \\ a_{27} \\ a_{37} \end{matrix} & \begin{matrix} a_{18} \\ a_{28} \\ a_{38} \end{matrix} & \begin{matrix} a_{19} \\ a_{29} \\ a_{39} \end{matrix} & \begin{matrix} a_{110} \\ a_{210} \\ a_{310} \end{matrix} \\ \hline 0_{2\times3} & \Im & \multicolumn{2}{c|}{0_{3\times2}} & \multicolumn{2}{c}{0_{3\times2}} \\ \hline 0_{2\times3} & 0_{2\times3} & \multicolumn{2}{c|}{I_f} & \multicolumn{2}{c}{0_{2\times2}} \\ \hline 0_{2\times3} & 0_{2\times3} & \multicolumn{2}{c|}{0_{2\times2}} & \multicolumn{2}{c}{I_r} \end{array} \right),$$

$$\Re = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \; \aleph = \begin{pmatrix} a_{14} & a_{15} & a_{16} \\ a_{24} & a_{25} & a_{26} \\ a_{34} & a_{35} & a_{36} \end{pmatrix}$$

$$\Im = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{k_{sf}}{c_{sf}} & 0 \\ 0 & 0 & -\frac{k_{sr}}{c_{sr}} \end{pmatrix}, \; B_c = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{k_{sf}}{c_{sf}} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^t,$$

$$B = \left(\begin{array}{cc|c} \multicolumn{2}{c|}{0_{4\times2}} & 0_{4\times4} \\ \hline \frac{k_{sf}}{c_{sf}} & 0 & 0_{1\times4} \\ 0 & \frac{k_{sr}}{c_{sr}} & 0_{1\times4} \\ \hline \multicolumn{2}{c|}{0_{4\times2}} & \frac{-1}{J_t} \times I_{4\times4} \end{array} \right), \; I_{f,r} = \frac{-C_{\sigma f, \sigma r} R_t^2}{V_x J_t} \times I_{2\times2},$$

$$B_d = \begin{pmatrix} 0_{6\times4} \\ \hline \frac{1}{J_t} \times I_{4\times4} \end{pmatrix}, \; u_d = \begin{pmatrix} R_t C_{sf} & R_t C_{sf} & R_t C_{sr} & R_t C_{sr} \end{pmatrix}^t$$

With I the identity matrix of appropiate dimension.

Remark 1. Note that the nonlinear vehicle model given by (7) depends on the longitudinal vehicle velocity. During braking maneuver, the latter parameter greatly influences the vehicle dynamics. In the following, the idea is to describe the nonlinear vehicle model by a T-S fuzzy model.

Let as define $V_x(t)$ as the premise variable of the T-S fuzzy model, where $V_x(t) \in [V_{\min}, V_{\max}]$, and M_i, $i = 1, 2$ the linguistic variables of $V_x(t)$. By applying the least-square method, the membership functions $h_i(V_x(t))$ are given as follows:

$$\begin{pmatrix} h_1(V_x(t)) \\ h_2(V_x(t)) \end{pmatrix} = \left(\left(\Omega^t \times \Omega\right)^{-1} \times \Omega^t \right) \times \begin{pmatrix} \frac{1}{V_x(t)} \\ V_x(t) \\ 1 \end{pmatrix},$$

$$\Omega = \begin{pmatrix} \frac{1}{V_{\min}} & \frac{1}{V_{\max}} \\ V_{\max} & V_{\min} \\ 1 & 1 \end{pmatrix}, \; h_i(V_x(t)) \geq 0, \; i = 1, 2.$$

Then, the T-S fuzzy model can be given by the following rules:

Model rule i: *If* $V_x(t)$ *Is* M_i *Then*

$$\begin{cases} \dot{x}(t) = A_i x(t) + Bu(t) + B_c \delta_c(t) + B_d u_d \\ y = Cx(t) \end{cases} \tag{8}$$

Thus, the vehicle model can be approximated by the following T-S fuzzy model :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(V_x(t)) \{A_i x(t) + Bu(t) + B_c \delta_c(t) \\ \qquad + B_d u_d\} \\ y = Cx(t) \end{cases} \tag{9}$$

See El Messoussi et al. [2007] for more details. On the other hand, we know that the road adhesion greatly influences the vehicle dynamics. The cornering stiffness coefficients C_f, C_r and also the longitudinal stiffness coefficients $C_{\sigma f}, C_{\sigma r}$ vary according to the road type. Thus, to take this fact into account, we consider that this coefficients are uncertain and can be given by $C_k = C_{k0}(1 + e\Delta), \quad k = f, r, \sigma f, \sigma r$ where $C_{k0}, \quad k = f, r, \sigma f, \sigma r$ are the nominal coefficients values, $|\Delta| \leq 1$ and e is the magnitude deviation of the different coefficients from their nominal values. After development, we can write the equation (9) as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(V_x(t)) \{(A_{i0} + \Delta A_{i0}) x(t) + Bu(t) \\ \qquad + B_c \delta_c(t) + B_d u_d\} \\ y = Cx(t) \end{cases} \tag{10}$$

Where A_{i0} is the nominal state space matrix and $\Delta A_{i0} = H_{ai} \Delta_{ai}(t) E_{ai}$. $\Delta_{ai}(t)$ is an unknown function satisfying: $\Delta_{ai}^T(t) \Delta_{ai}(t) \leq I$. H_{ai} are constant matrices given as follows: $H_{ai} = e_i \times I_{10\times10}$.

Remark 2. Due to the lack of space, the coefficient a_{ij}, $i, j = 1, ..., 10$, the matrices A_i, A_{i0} and E_{ai}, $i = 1, 2$ are not given in this paper.

Remark 3. The model given by (10) is non linear with respect to the speed and the uncertainties take into account the different road types.

3. NON-FRAGILE VEHICLE CONTROL STRATEGY

In this work, we assume that the lateral vehicle velocity is not measurable. Then, an observer is used to estimate this parameter. The structure of the considered vehicle control system is given in (Fig. 1). Fuzzy state observer is formulated as follows:

Observer rule i: *If* $V_x(t)$ *Is* M_i *Then*

$$\begin{cases} \hat{\dot{x}} = A_{i0}\hat{x}(t) + Bu(t) \\ \qquad - G_i(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t), \quad i = 1, 2 \end{cases} \tag{11}$$

The fuzzy observer design is to determine the local gains G_i, $i = 1, 2$. The output of (11) is given as follows:

$$\begin{cases} \hat{\dot{x}} = \sum_{i=1}^{2} h_i(z(t)) \{A_{i0}\hat{x}(t) + Bu(t) \\ \qquad - G_i(y(t) - \hat{y}(t))\} \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{12}$$

To stabilize this class of systems given by (10), we use the Parallel Distributed Compensation (PDC) observer-based controller, see Tanaka et al. [1998], defined as follows:

Controller rule i : *If* $z(t)$ *Is* M_i *Then*

$$u(t) = K_i \hat{x}(t), i = 1, 2 \tag{13}$$

Where K_i, $i = 1, 2$ are the controller gains to be determined. The overall observer-based controller is given by:

$$u(t) = \sum_{i=1}^{2} h_i(V_x(t)) K_i \hat{x}(t) \tag{14}$$

The controller in the form of (14) does not involve uncertainties. However, uncertainties always appear in control systems for many reasons such as imprecision inherent in analog systems and the need for additional tuning of the controller parameters in the final implementation. Thus, in the following, we study the design of fuzzy controller with respect to parametric controller gains perturbations. By considering that $K_i = K_{i0}(1 + e_k\Delta), i = 1, 2$ where $K_{i0}, i = 1, 2$ are the nominal controller gains, $|\Delta| \leq 1$ and e_k is the magnitude deviation from the nominal values, An observer-based non-fragile controller can be given by:

$$u(t) = \sum_{i=1}^{2} h_i(V_x(t)) (K_{i0} + \Delta K_{i0}) \hat{x}(t) \tag{15}$$

Where $\Delta K_{i0} = H_{ki0} \Delta_{ki0}(t) E_{ki0}$. $\Delta_{ki0}(t)$ is an unknown function satisfying: $\Delta_{ki0}^T(t) \Delta_{ki0}(t) \leq I$. $E_{ki0} = K_{i0}$, $i = 1, 2$ and H_{ki0} are constant matrices given as follows: $H_{ki0} = e_k \times I_{6\times6}$. Let us denote the estimation error by: $e(t) = x(t) - \hat{x}(t)$. The augmented system containing both the controller and the observer is represented as follows:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{e}(t) \end{pmatrix} = \bar{A}_i(V_x(t)) \begin{pmatrix} x(t) \\ e(t) \end{pmatrix} + \begin{pmatrix} B_c \\ B_c \end{pmatrix} \delta_c(t) + \begin{pmatrix} B_d \\ 0 \end{pmatrix} u_d \tag{16}$$

Where

$$\bar{A}_i(V_x(t)) = \sum_{i=1}^{2} h_i(V_x(t)) A_i^{as},$$

$$A_i^{as} = \begin{bmatrix} A_i + BK_{i0} + \Delta A_i + B\Delta K_{i0} & -BK_{i0} - B\Delta K_{i0} \\ \Delta A_i & A_i + G_i C \end{bmatrix}$$

The main goal is to find the sets of matrices $K_{i0}, i = 1, 2$ and $G_i, i = 1, 2$ in order to guarantee the global asymptotic stability of the equilibrium point of (16) with performance specifications using a combined H_∞ and pole placement approach in order to guarantee that the error between the state and its estimation converges faster to zero even if the plant dynamics is subject to uncertainties, see El Messoussi et al. [2006]. The control system has to be able to tolerate some uncertainty in the controller as well as in the plant dynamics . In lemma 1, we give sufficient conditions for the global asymptotic stability of the closed-loop augmented system given by (16):

Lemma 1. The equilibrium point of the augmented system described by (16) is globally asymptotically stable if there exist common positive definite matrices P and Q, matrices W_i, matrices V_i and positive scalars $\epsilon_i \succ 0$ such as

$$\begin{bmatrix} D_i & B & BH_{ki0} & PE_{ai}^t & V_i^t \\ B^t & -\varepsilon_i I & 0 & 0 & 0 \\ H_{ki0}^t B^t & 0 & -\varepsilon_i I & 0 & 0 \\ E_{ai}P & 0 & 0 & -0.5\varepsilon_i I & 0 \\ V_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \leq 0, \quad i = 1, 2, \tag{17}$$

And

$$\begin{aligned} QA_i + W_iC + A_i^tQ + C^tW_i^t + \varepsilon_i H_{ai}H_{ai}^t \\ +2\varepsilon_i K_{i0}^t K_{i0} \leq 0, \;\; i = 1, 2, \end{aligned} \tag{18}$$

Where

$$\begin{aligned} D_i = A_iP + BV_i + PA_i^t + V_i^tB^t + \varepsilon_i BH_{ki0}H_{ki0}^t B^t \\ +\varepsilon_i H_{ai}H_{ai}^t, \\ V_i = K_{i0}P, \; W_i = QG_i. \end{aligned}$$

Proof. Using theorem 7 in Tanaka et al. [1998], the separation lemma, see Shi et al. [1992], and the Schur's complement, see Boyd et al. [1994], conditions (17) and (18) hold with some changes of variables.

Remark 4. Note that the controller and the observer design is a two-step procedure. First, we solve (17) for decision variables $(P, K_{i0}, \varepsilon_i)$ and secondly, we solve (18) for decision variables (Q, G_i) by using the results of the first step.

Remark 5. The location of the poles associated with the state dynamics and with the estimation error dynamics is unknown. However, since the design algorithm is a two-step procedure, we can impose two pole placements separately, the first one for the state and the second one for the estimation error. To ensure control performances, in the following, we focus on robust pole placement, see El Messoussi et al. [2005]. From (16), Let us define:

$$\begin{cases} T_{ci} = \sum_{i=1}^{2} h_i(V_x(t)) \left(A_i + BK_{i0} + \Delta A_i + B\Delta K_{i0}\right) \\ T_{oi} = \sum_{i=1}^{2} h_i(V_x(t)) \left(A_i + G_iC\right) \end{cases}$$

Lemma 2. Matrix T_{ci} is D-stable if and only if there exist a symmetric matrix $P > 0$, matrices $V_i, i = 1, 2$ and positive scalars $\tau_i, \eta_i i = 1, 2$ such that

$$\begin{pmatrix} Z_i & \left(I \otimes XE_{ai}^t\right) & \left(I \otimes V_i^t\right) \\ \left(I \otimes E_{ai}X\right) & -\tau_i I & 0 \\ \left(I \otimes V_i\right) & 0 & -\eta_i I \end{pmatrix} \leq 0, \quad i = 1, 2, \tag{19}$$

Where

$$\begin{aligned} Z_i = \alpha \otimes X + \beta \otimes A_iX + \beta^t \otimes XA_i^t + \beta \otimes BV_i \\ +\beta^t \otimes V_i^tB^t + \tau_i \left(\beta \otimes H_{ai}\right)\left(\beta^t \otimes H_{ai}^t\right) \\ +\eta_i \left(\beta \otimes BH_{ki}\right)\left(\beta^t \otimes H_{ki}^tB^t\right) \end{aligned}$$

And $\otimes$ denotes the Kronecker product.

Proof. The same method as in lemma 2 in El Messoussi et al. [2006] can be used to prove this lemma.

Lemma 3. Matrix T_{oi} is D-stable if and only if there exist a symmetric matrix $Q > 0$, matrices $W_i, i = 1, 2$ such that

$$\begin{aligned} \alpha \otimes Q + \beta \otimes QA_i + \beta^t \otimes A_i^tQ \\ +\beta \otimes W_iC + \beta^t \otimes C^tW_i^t \leq 0, \quad i = 1, 2 \end{aligned} \tag{20}$$

Where $\otimes$ denotes the Kronecker product and α, β determine the desired LMI region.

Proof. The same method as in lemma 3 in El Messoussi et al. [2006] can be used to prove this lemma.

Remark 6. From (16), the estimation error dynamics depend on the state. However, if the state dynamics are slow, we will have a slow convergence of the estimation error to the equilibrium point in spite of its own fast dynamics. So in this paper, we add an algorithm using the H_∞ approach to ensure that the estimation error converges faster to the equilibrium point, see El Messoussi et al. [2006]. We know from (16) that:

$$\begin{aligned} \dot{e}(t) = \sum_{i=1}^{2} h_i(z(t)) \{(A_i + G_iC)\, e(t) + \Delta A_i x(t) \\ +B_c\delta_c(t)\} \end{aligned} \tag{21}$$

Let us denote $B_{wi} = (\, \Delta A_i \;\; B_c \,)$ and $w(t) = (\, x(t) \;\; \delta_c(t) \,)^t$. The following system can be obtained:

$$\begin{pmatrix} \dot{e}(t) \\ e(t) \end{pmatrix} = \sum_{i=1}^{2} h_i(z(t)) \begin{pmatrix} A_i + G_iC & B_{wi} \\ I & 0 \end{pmatrix} \begin{pmatrix} e(t) \\ w(t) \end{pmatrix} \tag{22}$$

The objective is to minimize the L_2 gain from $w(t)$ to $e(t)$. Thus, we define the following H_∞ performance criterion under zero initial conditions:

$$\int_0^\infty \left\{e^t(t)\, e(t) - \gamma^2 w^t(t)\, w(t)\right\} dt \prec 0 \tag{23}$$

Where γ has to be minimized. We give the following lemma to satisfy the H_∞ performance.

Lemma 4. If there exist symmetric positive definite matrix $Q > 0$, matrices $W_i, i = 1, 2$ and positive scalars ς_i,γ such as:

$$\begin{pmatrix} F_i & QH_{ai} & 0 & QB_c \\ H_{ai}^TQ & -\varsigma_i & 0 & 0 \\ 0 & 0 & -\gamma^2 I + \varsigma_i E_{ai}^T E_{ai} & 0 \\ B_c^tQ & 0 & 0 & -\gamma^2 I \end{pmatrix} \leq 0, \; i = 1, 2 \tag{24}$$

Where

$$F_i = QA_i + A_i^tQ + W_iC + C^tW_i^t + I$$

Then, the system given by (22) satisfies the H_∞ performance with a L_2 gain equal or less than γ.

Proof. The same method as in lemma 4 in El Messoussi et al. [2006] can be used to prove this lemma.

In order to improve the estimation error convergence, we obtain the following convex optimization problem:

minimization γ under the LMI constraints (24). Finally, from lemma 1, 2, 3 and 4 yields the following theorem:

Theorem 1. The closed-loop uncertain system given by (16) is robustly stabilizable via the observer-based non-fragile controller given by (15) with control performances defined by a pole placement constraint for the state dynamics, as well as for the estimation error dynamics in an LMI region and a gain performance (23) as small as possible if first, LMI conditions (17) and (19) are solvable for the decision variables $(P, K_{i0}, \varepsilon_i, \tau_i, \eta_i)$ and secondly, LMI conditions (18), (20) and (24) are solvable for the decision variables $(Q, G_i, \varsigma_i, \gamma)$. Furthermore, the controller and observer gains are $K_{i0} = V_i P^{-1}, i = 1, 2$ and $G_i = Q^{-1} W_i, i = 1, 2$ respectively.

Remark 7. The effectiveness of the combined pole placement and H_∞ algorithm, used in this work, has been demonstrated in El Messoussi et al. [2006].

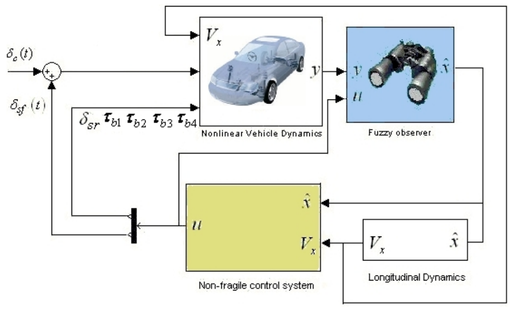

Fig. 1. Vehicle control system structure

4. NUMERICAL EXAMPLE

The control design purpose of this example is to design a robust observer-based non-fragile controller in order to improve the vehicle stability and manoeuvrability when this latter is subject to change lane manoeuvre. The control system can tolerate presence of parametric variations (variation of the road type) as well as controller gains variation. To ensure good performances of the controlled system, we place both the poles linked to the state dynamics and the ones linked to the estimation error dynamics in the LMI region defined by the half-left complex plane given by $\alpha = -1$ and $\beta = 1$ to ensure minimum settling time less then $4s$. The evolution of the control signal (15) is given in (Fig. 3) (the controller and observer gains are obtained from LMIs of theorem 1). A comparison between the measured and the observed lateral vehicle velocity is shown in (Fig. 4). Although the variation of the vehicle longitudinal speed and the front steering angle, see (Fig. 2), we can see that the vehicle is still stable. (Fig. 5) shows the robustness of the designed control system with respect to plant dynamics and controller uncertainty (variation until $\pm 25\%$ of the plant dynamics and $\pm 20\%$ of the controller gains). On the other hand, to show our control method effectiveness, a comparison of the controlled and uncontrolled system outputs is given in figure (Fig. 6). Note that the developed vehicle control system has been applied to the nonlinear vehicle model given by (1), (2), (3), (4), (5) and (6).

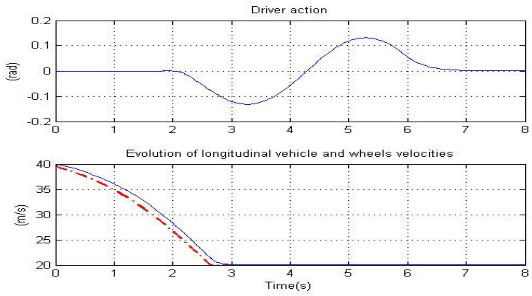

Fig. 2. a)Driver action, b)Longitudinal vehicle (solid line) and wheels velocities (dotted line)evolution

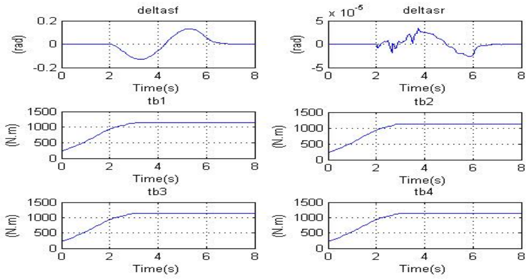

Fig. 3. Control Signal evolution

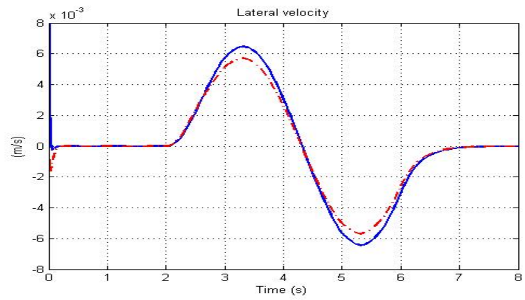

Fig. 4. Estimation of the lateral vehicle velocity

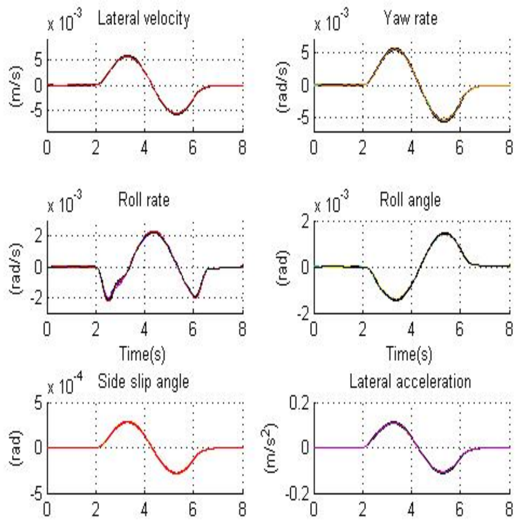

Fig. 5. Controller robustness

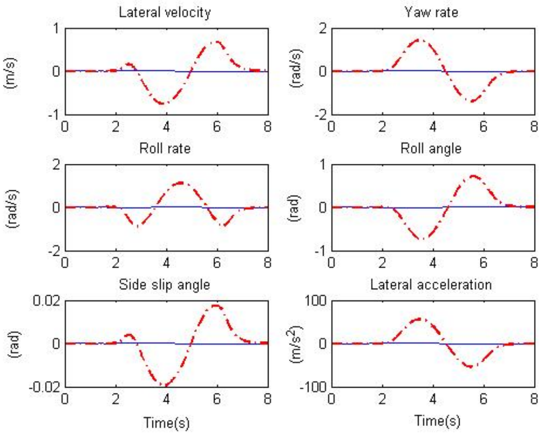

Fig. 6. Comparison of controlled (solid line) and uncontrolled (dotted line) outputs

5. CONCLUSION

In this paper, a robust observer-based non-fragile control system have been proposed in order to improve vehicle stability although the presence of plant dynamics and controller uncertainty. A 10-DOF mathematical model has been used to describe the vehicle dynamics. Variation of the longitudinal vehicle velocity as well as variation of the road adhesion was considered in the controller design. LMI conditions have been developed using T-S multi-model representation, H_∞ approach and robust pole placement, in order to guarantee global stability of the closed loop system with desired performances. The effectiveness of our approach has been demonstrated through simulations on the nonlinear vehicle model.

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