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Delay Independent Stabilization and ASC

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Abstract: On the quadratic stabilization of uncertain linear time varying systems by means of linear state variable feedback, Wei introduced the concept of antisymmetric stepwise configuration (ASC) and proved that having this configuration is a necessary and sufficient condition for uncertain linear systems to be quadratically stabilizable by means of a linear state variable feedback. However, because his condition is constructed on the basis of quadratic lyapunov functions, his method is not applicable if the state variables contains delays.
In this report the conditions for the delay independent stabilization so far obtained on the basis of delay differential inequalities is further developed and it is proved that generally to have ASC is also a sufficient condition for the delay independent stabilizability of linear uncertain delayed systems by means of linear state variable feedback.

1. INTRODUCTION

Delay independent stabilization, of the delayed systems provides a fairly simple and useful method to obtain a stabilizing control for uncertain delayed systems. Delay independent stabilization is of course a kind of robust stabilization for systems with delays. However, it is so named because it provides a condition, whose results contains no terms of delays. This condition is applied to delayed systems however large the contained delayed may be, so long as they are bounded. Boundedness is necessary only because we cannot consider the infinite delays. Contrarily, the other conditions which explicitly depends on delayed terms are called delay dependent stabilization. The authors presented several results [1,2,3] for delay independent stabilization of delayed systems. In the first report all state variables were assumed to be known. This condition was developed to the case that state variables are partly observed in [2]. In [3] necessary and sufficient conditions for the stabilizabiity of the problem were investigated. Recently, several results on the delay dependent stability analysis have been presented often. with the help of LMI. However, these conditions are usually quite complicated and difficult to see the efficiency of them. Comparison with these results was shown in [3].

On the other hand, on uncertain time varying linear systems, Wei proved that to have certain special form called anti-symmetric stepwise configuration (ASC) is the necessary and sufficient for the quadratic stabilizability via state variable feedback for systems containing uncertainties in system parameters. So far we noticed [3] that the condition we have proved is equivalent to the most basic form of this ASC. However, it does not satisfy the general form of ASC. Here it is considered whether our conditions can be improved, showing that ASC constitutes a sufficient condition for the delay independent stabilization of uncertain linear delayed systems.

One of the refelee suggested to refer the results of [6]. However, the stabilizing method in [6] uses Lyapunov function and therefore is not applicable to our systems with time-varying delays.

The paper is organized as follows. First, in the next section, some notations and terminologies are presented. In Section 3, the description of the considered system with some basic assumptions are given. Then, the conditions for stabilization of uncertain systems so far obtained are presented. In Section 5, Wei's ASC and his results are introduced. In Section 6 the basic mathematical background for our results is presented. In the succeeding section our main theorem is derived on the basis of these theorems. Since the proof of this theorem is very complicated and there are various cases to be considered, they are given in Appendices. Section 7 is devoted to an illustrative example, which is given for the help of understanding. Finally, conclusions are given in Section 8.

2. NOTATIONS

For $A = (a_{ij}), B = (b_{ij}) \in R^{n\times m}$, every inequality such as $A > B$ indicates that it is satisfied componentwisely as $a_{ij} > b_{ij}$. For $A = (a_{ij}) \in R^{m\times m}$, matrices $B = (b_{ij}) \in R^{m\times m}$ and $C = (c_{ij}) \in R^{m\times m}$ defined as $b_{ij} = |a_{ij}|$ or $c_{ij} = |a_{ij}|$, $(j \neq i)$, $c_{ij} = a_{ij}$, $(j = i)$, are called the absolute companion matrix of A or quasiabsolute companion matrix of A, respectively. Here, notations $|A|$ or $|A|_q$ are used to denote absolute companion matrices or quasiabsolute companion matrix of A, respectively. For $A \in R^{m\times m}$, the inequalty $A \geq 0$ indicates A is a nonnegative matrix.

A real nonsingular matrix $D = (d_{ij}) \in R^{n\times n}$ is called an $M-$matrix, if it satisfies all the following conditions;
(i) all off-diagonal elements satisfy $d_{ij} \leq 0, (i \neq j)$,
(ii) the inverse of D satisfies $D^{-1} \geq 0$.
The set of all $M-$matrices is denoted as $\mathcal{M}$.

Let $[a, b]$ be an interval in R. The sets of all R^n continuous or piecewise continuous functions with domain $[a, b]$ are denoted by $C^n[a, b]$ or $D^n[a, b]$, respectively.

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3. SYSTEM DESCRIPTION

The system considered in this paper is given as follows,

$$\dot{x}(t) = A^0x(t) + \Delta A^1(t)x(t) + \sum_{i=1}^{m} \Delta A^{2i}(t)\tilde{x}(t-\tau_i(t)) + (b+\Delta b)u(t). \quad (1)$$

where $x \in R^n$ and $t \in [t_0, \infty)$. The solution of (1) with initial curve $\phi \in D^n[t_0 - \tau_0, t_0]$ is denoted as $x(t, \phi)$. $u \in R$ is a control variable and $A \in R^{n\times n}$, $b \in R^n$ are constant. $\tau_i : R \to R$ is a piecewise continuous function and is assumed to be bounded, i.e. for a constant $\tau_0 \in R$ it satisfies,

$$0 \le \tau_i(t) \le \tau_0,\ i = 1, ..., n,\ \text{for } t \in R. \quad (2)$$

Since the upper bound τ_0 in (2) does not affect the stability condition, given below, it is not necessarily assumed to be known and may be arbitrarily large. That is why this condition is called delay independent. The concept of 'delay independent' can also be found in [5].

$\Delta A^1, \Delta A^{2i} \in R^{n\times n}$, $i = 1, \cdots, m$, Δb denote uncertain parts of system parameters. All elements of them are piecewise continuous functions of t. These matrices satisfy for constant $n\times n$ matrices ΔA^{10}, ΔA^{2i0}, and $\Delta b^0 \in R^n$,

$$|\Delta A^1| \le \Delta A^{10},\ |\Delta A^{2i}| \le \Delta A^{2i0} \text{ for } t \ge t_0. \quad (3.1)$$

$$|\Delta b| \le \Delta b^0 \quad (3.2)$$

On Δb^0 it is assumed that if some element $\Delta b_i \equiv 0$ for $t \ge 0$ then $\Delta b_i^0 = 0$.

On the system parameters $A + \Delta A^*$ and input coefficients $b + \Delta b \in R^n$, the following assumption is introduced.

Assumption 1 If delays are all zero and all uncertain parameters are constant, the system is controllable, whatever values these uncertain parameters may take, satisfying the restrictions.

Definition 1. The system (1) is called robustly stabilizable if it can be made asymptotically stable independently of uncertain coefficients, satisfying the restrictions, by constructing certain control u. Specially, if u can be constructed as linear memoryless state variable feedback,

$$u = c'x \quad (4)$$

by choosing proper constant vector $c \in R^n$, the system is called stabilizable via linear state variable feedback.

It is also assumed that all state variables are directly accessible. Here the system is called globally asymptotically stable if every solution of it converges asymptotically to $x = 0$ whatever initial curve $\phi \in D^n[t_0 - \tau_0, t_0]$ may it start from.

Definition 2. (Delay independent stabilizability) If the system (1) is robustly stabilizable by the condition, which does not depend explicitly on the delayed term, the system is called delay independently stabilizable,(DIS).

The following is the problem considered in this paper.

Problem What conditions must system parameters satisfy for the system to be DIS, via linear state variable feedback (4), however large the upperbound of uncertainties may be, provided upper bounds are known, exept τ_0.

As the most basic system the following assumptions are introduced.

Assumption 2 The pair (A^0, b) of the nominal system is a controllable pair and is in the controllable canonical form and A^0 is given as

$$A^0 = \begin{pmatrix} 0 & 1 & 0 & . & . \\ . & . & . & . & . \\ . & . & . & . & 1 \\ 0 & 0 & . & . & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ . \\ . \\ 1 \end{pmatrix}.$$

4. UNCERTAIN COEFFICIENTS AND ASC

Here the configuration of the uncertain coefficients to be considered is defined. For this purpose a set of matrices with regards to the uncertain parameters ΔA^1 , ΔA^{2i} is introduced.

Definition 3. For an integer k satisfying $0 \le k \le n$, let $\Omega(k) = \{G = (g_{ij}) \in R^{n\times n}\}$ be a set of all matrices with the following properties:
(i) If $1 \le i \le n - k$ then $g_{ij} = 0$, for $j \le i + 1$ and $j \ge 2n - 2k - i + 1$.
(ii) If $n-k+1 \le i \le n$ then $g_{ij} = 0$, for $j \le 2n-2k-i+1$ and $j \ge i + 1$.

For the delay independent stabilization of this uncertain system, the following theorem has been obtained.

Theorem 1 [1] *In case $\Delta A^1 \in \Omega(k)$, $\Delta A^{2i} \in \Omega(k), i = 1, \cdots, m$ for certain common k, the system system (1) is DIS by a constant linear state variable feedback (4).*

Here the condition by Wei[3] and the ASC are presented for comparison sake.

Consider the system with no delays.

$$\dot{x}(t) = \tilde{A}^0x(t) + \Delta A^1(t)x(t) + \tilde{b}u(t). \quad (5)$$

where $\tilde{A}^0 \in R^{n\times n}$ and $\tilde{b} \in R^n$ are defined as

$$\tilde{A}^0 = \begin{pmatrix} 0 & \theta & 0 & . & . \\ . & . & . & . & . \\ . & . & . & . & \theta \\ 0 & 0 & . & . & 0 \end{pmatrix}, \tilde{b} = \begin{pmatrix} 0 \\ 0 \\ . \\ \theta \end{pmatrix}$$

Here $\theta's$ are all sign fixed uncertainties.

Definition 4. An uncertain system is called quadratically stable if there exists a positive definite lyapunov function of quadratic form $V = x'Px$ such that the time derivative of this lyapunov function is negative definite or semidefinite along the solution of the equation however the uncertainties may change.

The following assumption is introduced.

Assumption 3 All $(i, i + 1)$ elements of ΔA^1 are 0.

Wei called such form of matrix $(\tilde{A}^0 + \Delta A^1, b + \Delta b)$ as standard form.

Definition 5. A system is called quadratically stabilizable via linear state variable feedback if there exists a linear feedback (4) such that the resulting system is quadratically stable.

Definition 6. An $(n + 1) \times (n + 1)$ real matrix is called to have an antisymmetric stepwise configuration if it has the following properties:

$$
\begin{array}{c}
\hat{k} \\
\begin{pmatrix}
0\;0\;*\;*\;*\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;*\;*\;*\;*\;*\;*\;*\;0 \\
0\;0\;0\;*\;*\;*\;\cdot\;\cdot\;\cdot\;\cdot\;*\;0\;0\;0\;0\;0\;0\;0 \\
\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot \\
0\;0\;\cdot\;\cdot\;\cdot\;0\;0\;*\;*\;*\;*\;0\;0\;0\;0\;0\;0\;0 \\
0\;0\;\cdot\;\cdot\;\cdot\;\cdot\;0\;0\;*\;*\;*\;0\;0\;0\;\cdot\;0\;0\;0 \\
0\;0\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;0\;0\;*\;*\;0\;0\;\cdot\;\cdot\;0\;0\;0 \\
0\;0\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;0\;0\;0\;0\;\cdot\;\cdot\;\cdot\;0\;0\;0 \\
0\;0\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;0\;0\;*\;0\;0\;\cdot\;\cdot\;\cdot\;0\;0\;0 \\
0\;0\;0\;*\;*\;\cdot\;\cdot\;\cdot\;*\;*\;*\;0\;0\;\cdot\;\cdot\;0\;0\;0 \\
0\;0\;0\;*\;*\;\cdot\;\cdot\;\cdot\;*\;*\;*\;*\;0\;0\;0\;0\;0\;0 \\
\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot \\
0\;0\;0\;*\;*\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;*\;*\;*\;0\;0 \\
0\;0\;*\;*\;*\;*\;*\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;*\;*\;*\;*\;0
\end{pmatrix}
\end{array}
\leftarrow \hat{k}
$$

Fig. 1. Sample of a matrix $G \in \Omega^0_{ASC}$

(i) It is in the standard form.
(ii) Let m_{kp} and m_{uv} be (k, p) and (u, v) element of matrix M respectively. In this case, if $p \geq k + 2$ and $m_{kp} \neq 0$ for all q, then $m_{uv} = 0$ for all $u \geq v$, $u \leq p - 1$ and $v \leq k + 1$ and for any q.

Definition 7. Here we introduce a little arranged ASC. The set of all matrices which have ASC and have 0 as $(i, i+1)$-th elements $i = 1, \cdots n$ is denoted as Ω^0_{ASC}. We call them as basic ASC. Especially, the set $\Omega^0_{ASC}(k)$ indicates that the element matrix has $(n-k, n-k)$ diagonal uncertain element as highest one. A schematic view of a sample matrix $G \in \Omega^0_{ASC}$ is given in Fig.1.

Wei proved the following theorem.

Theorem 2([3]) *Denote $\tilde{A} = \tilde{A}^0 + \Delta A^1$ and construct an $(n+1) \times (n+1)$ matrix X as*

$$X = \begin{pmatrix} \tilde{A} & \tilde{b} \\ 0 & 0 \end{pmatrix}$$

Then, if and only if this X has an ASC, the system (5) is quadratically stabilizable via linear state variable feedback.

5. MATHEMATICAL BACKGROUND

First a condition for the uniform convergence of the solutions of a delayed differential equation is presented. Let $x \in R^n$ and consider,

$$\dot{x}(t) = Cx(t) + C^1(t)x(t) + \sum_{i=1}^{m} C^{2i}(t)x(t - \tau_i(t)) \qquad (6)$$

defined for $t \geq t_0$, with initial curve $\phi \in D^n[t_0 - \tau_0, t_0]$. Here C, C^1, C^{2i} are all $n \times n$ real matrices. C^1, C^{2i} may be time varying. All elments of them are assumed to be piecewise continuous and bounded, i.e. for given constant nonnegative matrices $C^{10}, C^{2i0} \in R^{n\times n}$, $i = 1, \cdots, m$ they satisfy $|C^1(t)| \leq C^{10}$, $|C^{2i}(t)| \leq C^{2i0}$, for $t > t_0$. Each delay τ_i is assumed to satisfy (2).

The following theorem has been proven.

Theorem 3 ([1]) *Assume*

$$(-|C|_q - C^{10} - \sum_{i=1}^{m} C^{2i0}) \in \mathcal{M}. \qquad (7)$$

Then, every solution of (10) converges uniformly and exponentially in the large and also independently of delays, to the equilibrium point x = 0.

Note that the above condition does not depend on the bound τ_0 of delays, this is why we call it the delay independent stability. On the evaluation of (7), the following propositions [4] is utilized in the subsequent. Let A, $B \in R^{n\times n}$ be constant matrices, satisfying $A \geq B$.

Proposition 1 *For any $K \in R^{n\times n}$, if $(K-A) \in \mathcal{M}$, and if all off-diagonal elements of $(K - B)$ are nonpositive, then $(K - B) \in \mathcal{M}$.*

Substituting (4) into (1) and then introducing the variable transformation

$$v = T^{-1}x \qquad (8)$$

then the equation (1) can be transformed into,

$$\begin{aligned} \dot{v}(t) &= T^{-1}(A^0 + bc')Tv(t) + T^{-1}\Delta A^1 Tv(t) \qquad (9) \\ &+ T^{-1}\Delta bc'Tv(t) + \sum_{i=1}^{n} T^{-1}\Delta A^{2i}Tv(t - \tau_i(t)). \end{aligned}$$

Owing to the controllability assumption, it is possible to choose $c \in R^n$ so that all the eigenvalues of $(A^0 + bc')$, are real, negative and distinct. Let c be defined as such. And let $\lambda_1, \lambda_2, \cdots, \lambda n$ be the eigen values of $(A^0 + bc')$ respectively. Define T as

$$T = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{n-1} & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_{n-1}^2 & \lambda_n^2 \\ & \cdot & \cdots & \cdot & \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_{n-1}^{n-1} & \lambda_n^{n-1} \end{pmatrix}, \qquad (10)$$

Then we obtain

$$T^{-1}(A^0 + bc')T = \Lambda = \text{diag.}(\lambda_1, \lambda_2, \cdots, \lambda_n)$$

Define $\hat{C}$,$\hat{C}^{11}$,$\hat{C}^{12}$,$\hat{C}^{2i}$ as

$$\begin{aligned} \hat{C} &= \Lambda \\ \hat{C}^{11} &= T^{-1}(\Delta A^1)T \\ \hat{C}^{12} &= T^{-1}(\Delta bc')T \\ \hat{C}^{2i} &= T^{-1}\Delta A^{2i}T \end{aligned} \qquad (11)$$

Now the system (10) can be written as

$$\dot{v}(t) = \hat{C}v(t) + \hat{C}^{11}v(t) + \hat{C}^{12}v(t) + \sum_{i=1}^{n} \hat{C}^{2i}v(t - \tau_i(t)).(12)$$

Then we obtain

$$|\hat{C}|_q = |\Lambda|_q \qquad (13.1)$$

$$|\hat{C}^{11}| \leq |T^{-1}|(\Delta A^{10})|T| \qquad (13.2)$$

$$|\hat{C}^{12}| \leq |T^{-1}|(|\Delta bc'|)|T| \qquad (13.3)$$

$$\sum_{i=1}^{n} |\hat{C}^{2i}| \leq |T^{-1}|\Delta A^{20}|T| \qquad (13.4)$$

ΔA^{20} in (13.4) is given by $\sum_{i=1}^{n} \Delta A^{2i0} = \Delta A^{20}$. Let ΔA^{30} be defined as

$$\Delta A^{30} = \Delta A^{10} + \Delta b^0|c'| + \Delta A^{20}, \qquad (14)$$

Define P^1, P^2 and P^0 as

$$\begin{cases} P^1 = -|\hat{C}|_q \\ P^2 = -|T^{-1}|\Delta A^{30}|T| \end{cases} \quad (15)$$

$$P^0 = P^1 + P^2. \quad (16)$$

Then, owing to Theorem 5, we obtain,

Proposition 2 *If there exist T which assure*

$$P^0 \in \mathcal{M}, \quad (17)$$

then, every solution of the system (12) converges to the equilibrium point.

To show the existence of T to assure the equation (17), a notation for a class of functions is again introduced here, following the previous paper[1, 2], Let $\xi(\sigma) \in C^1$ and let $m \in R$ be a constant. If ξ satisfies the conditions,

$$|\frac{\xi(\sigma)}{\sigma^m}| < \infty, \quad |\frac{\xi(\sigma)}{\sigma^{m-a}}| \to \infty \quad \text{as } |\sigma| \to \infty,$$

for any constant $a > 0 \in R$, then ξ is called a function of order m and is written as $\mathrm{Ord}(\xi) = m$. The set of all C^1 functions of order m is denoted as $O(m)$. It should be noted that m can be a negative number.

6. MAIN RESULTS

In this section it is shown that our previously obtained condition can be developed to the general form of ASC, on the basis of the mathematical background just given above. It is stated in the following theorem.

Following Wei, we here introduce the extended matrix Q, constructed by space coefficients such that,

$$Q = \begin{pmatrix} \Delta A^\star & \Delta b \\ 0' & 0 \end{pmatrix}. \quad (18)$$

Theorem 4 (Main Theorem) *Assume thus obtained Q satisfy $Q \in \Omega^0_{ASC}(k)$, for fixed k, then the uncertain systems (1) is DIS via linear state vairable feedback (4), where $\star$ in (18) indicates 1 or $2i$.*

The proof of the above theorem depends crucially on the way to choose eigen values of the nominal system. Here it is shown as rules for choosing them. Before presenting the rules, it must be made sure that all eigen values are assumed to be real, negative and distinct. It is possible by the assumption of the controllability of the nominal system.

Let σ be a negative number. The theorem is proved by assigning the order of each eigen values as a function of σ.

Whereas for the proof of Theorem so far proved on the case $\Delta A^* \in \Omega(k)$, these eigen values were to be of order only 1 or -1, just two kinds of eigen values have been needed, a more sophisticated method must be chosen to improve the previous results.

6.1 Edge Points and Rules for choosing eingen values

Let $(p_1, q_1), \cdots, (p_i, q_i)$, $p_i > p_{i+1}, q_i < q_{i+1}$, $p_i + 1 < q_i$, $q_k \le n + 1$ be uncertain elements in ASC, consisting the corner positions of ASC. That means all $(p_j, q_j + s)$ elements are zero for all $s > 0$ and all $(p_j + r, q_j)$ elements are zero for all $q_i - p_i - 1 > r > 0$. These points are called edge points in this paper. For the proof, eigen values of the nominal systems must be chosen depending on these points. Since the rules for choosing eigen values are slightly different on the shape of ASC, to describe this property, define q_0 and h as,

$$q_0 = (p_1 + 2), \quad h = q_1 - q_0 = q_1 - p_1 - 2.$$

The way to choose eigen values and therefore the proof should be separated into two cases according to this h. They are cases of $h > 1$ and $h = 1$.

I. Case $h > 1$

(*Step 1*) Choose $q_1 - q_0$ eigen values of order 1.

(*Step 2*) Choose $p_1 - p_2$ eigen values of order $-(q_1 - q_0)$

(*Step 3*) Assume in the previous steps r_i eigenvalues of order s_i have been chosen, $i = 1, \cdots, k-1$, then the order of the next eigen valeus are given as $s_k = -\sum_i (r_i s_i)$ if k is even, that means s_k is negaticve, and $q_k \ne n+1$. If k is odd $s_k = -\sum_i (r_i s_i) + 1$. In case of $q_k = n+1$, and k is even, s_k should be $s_k = -\sum_i (r_i s_i) - 1$. In the process if the calculated value s_k satisfies $|s_k| < |s_{k-2}|$, then s_k must selected as $s_k = s_{k-2}$.

As for the numbers of these eigen values, they should be chosen as $r_{2k} = p_{k-1} - p_k$, for negative order ones or $r_{2k+1} = q_{k+1} - q_k$ for positive order ones. If k is the last one and $q_k \le n$ then q_{k+1} should be given as $q_{k+1} = n+1$.

(*Step 4*) Go to Step 3 until there exist no edge points. The last eigen value to be chosen is positive order one unless $q_k = n + 1$.

U. Case $h = 1$

(*Step 1*) Choose $p_1 - p_2$ eigen values of order -1.

(*Step 2*) Choose $q_2 - q_1$ eigen values of order $(q_1 - (p_1))$

(*Step 3*) Assume in the previous steps r_i eigenvalues of order s_i have been chosen, $i = 1, \cdots, k-1$, then the order of the next eigen valeus are given as $s_k = -\sum_i (r_i s_i + 1)$ for odd k and $s_k = -\sum_i (r_i s_i)$ for even k. If odd s_{2k-1} becomes the last eigen value, that is $q_k = n+1$ s_{2k-1} should be moreover replaced by $s_{2k-1} - 1$ If the calculated value s_k satisfies $|s_k| < |s_{k-2}|$, then s_k must selected as $s_k = s_{k-2}$.

Whereas the numbers of these eigen values should be $r_{2k} = p_{k-1} - p_k$, for negative order eigen values or $r_{2k+1} = q_{k+1} - q_k$ for positive order eigen values. If k is the last one q_{k+1} should be given as $q_{k+1} = n+1$. However, if $q_k = n+1$, the the eigen value which should be chosen last must have negative ordero and the number of this eigen value r_{2k-1} must be incresed by one to the ordinary value and r_{2k-3} must be replaced by $r_{2k-3} - 1$.

(*Step 4*) Go to Step 3 until there exist no edge points. The last eigen value to be chosen is positive order one unless $q_k = n + 1$.

7. EXAMPLES

7.1 Example 1

To help the understanding, the following illustrative example is considered. Let $n = 8$ and the uncertain matrix ΔA^* is given as in Fig.3, In this case (p_i, q_i) are given as,

$$(p_1, q_1) = (3, 6), \quad (p_2, q_2) = (1, 8),$$

For this system the previous method is not applicable. By using the above given decision rule, eigenvalues are given as follows.

$$\left(\begin{array}{cccccccc|c} 0 & 0 & * & * & \cdot & * & * & * & 0 \\ 0 & 0 & 0 & * & \cdot & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \leftarrow k$$

Fig. 2. Matrix $\Delta A^* \in \Omega(k)^0_{ASC}$, $s = 2$ $\hat{k} = 5$

$$\left(\begin{array}{cccccccc|c} 0 & 0 & * & * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \leftarrow \hat{k}$$

Fig. 3. Matrix $\Delta A^* \in \Omega(k)^0_{ASC}$, $s = 1$, $\hat{k} = 5$

(*Step 1*) 1 eigen value of order 1
(*Step 2*) 2 eigen value of order -1
(*Step 3-1*) 2 eigen values of order 2
(*Step 3-2*) 2 eigen values of order -3
(*Step 3-3*) 1 eigen values of order 4

By using the above given rules, the following P^0 is obtained. Note that every number in the matrix indicates the order of the functions of σ and necessary restrictions to be an M-matrix on the signs of all diagonal or off diagonal elements are assured to be satisfied. It is clear that this matrix becomes an M-matrix for sufficient large σ.

$$P^0 \simeq \begin{pmatrix} -3 & -4 & 0 & 0 & 9 & 14 & 14 & 28 \\ -4 & -3 & 0 & 0 & 9 & 14 & 14 & 28 \\ -6 & -6 & -1 & -2 & 7 & 12 & 12 & 24 \\ -6 & -6 & -2 & -1 & 7 & 12 & 12 & 24 \\ -12 & -12 & -8 & -8 & 1 & 4 & 4 & 16 \\ -16 & -16 & -12 & -12 & -5 & 2 & 0 & 12 \\ -16 & -16 & -12 & -12 & -5 & 0 & 2 & 12 \\ -28 & -28 & -26 & -26 & -19 & -14 & -14 & 4 \end{pmatrix}$$

7.2 Example 2

Let $n = 8$ and assume

$$(p_1, q_1) = (4, 6), \quad (p_2, q_2) = (2, 8),$$

This is the example for the case U. The uncertain matrix of this case ΔA^* is given as in Fig.3. For this system the following eigen values should be selected.

(*Step 1*) 2 eigen value of order -1
(*Step 2*) 2 eigen values of order 2
(*Step 3*) 2 eigen values of order -3
(*Step 3-3*) 2 eigen values of order 7

In this case P^0 can be shown as

$$P^0 \simeq \begin{pmatrix} -3 & -6 & 0 & 0 & 17 & 17 & 52 & 52 \\ -6 & -3 & 0 & 0 & 17 & 17 & 52 & 52 \\ -10 & -10 & -1 & -2 & 13 & 13 & 48 & 48 \\ -10 & -10 & -2 & -1 & 13 & 13 & 48 & 48 \\ -19 & -19 & -13 & -13 & 2 & 1 & 36 & 36 \\ -19 & -19 & -13 & -13 & 1 & 2 & 36 & 36 \\ -49 & -49 & -43 & -43 & -29 & -29 & 7 & 6 \\ -49 & -49 & -43 & -43 & -29 & -29 & 6 & 7 \end{pmatrix}$$

It is clear that this matrix is again an M-matrix, provided that the additional conditions on the signs of elements are sutisfied.

8. CONCLUDING REMARKS

Here it is proved that the Wei's ASC is also a sufficient condition for the delay independent stabilizability of uncertain time-varying delayed linear systems by means of linear state variable feedback. However, here it is assumed that all state variables are directly measurable and can be utilized for feedback control. It is rather a restrictive condition. It should be developed to the cases that only a part of state variables are directly accessible. In such case some observing mechanisms must be introduced. These are expected to be the future works on this subject. As for the proof, only a part of it is shown in the appendix. However, it should be noted that all cases of $n < 10$ were ckecked to be true.

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Appendix A. OUTLINE OF THE PROOF OF THE MAIN THEOREM

As the proof is quite complicated it is difficult to show it all in this restrictive spaces. Here only a brief outline of the proof of the main theorem is presented.

First define h as

$$h = q_1 - q_0 = q_1 - p_1 - 2.$$

The proof should be separated into two cases according to this h. First consider,

I Case 1 , $h > 1$

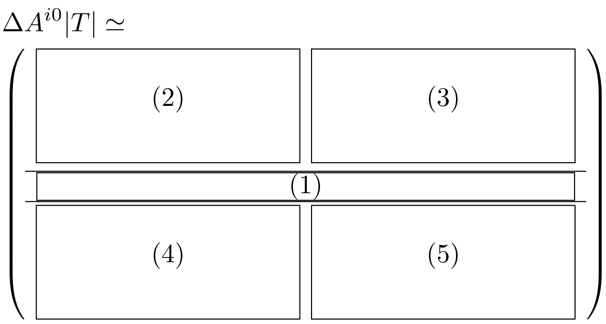

Fig. A.1. Schematic view of a matrix $\Delta A^{i0}T$

Let the number of edge points be s. In this case the number of different kinds of eigen values are $2s+1$. Let it be denoted by μ. That is $\mu = 2s+1$. The transformation matrix T of (10) and inverse of it T^{-1} can be shown to have the following structure, in which the selected eigen values should be listed from the smallest to the largest. Let, $\bar{r}_1, \cdots, \bar{r}_\mu$ be the order of eigen values as functin of σ orderd from the smallest to the largest. Thus $\bar{r}_1$ shows the lowest order of r_i, and $\bar{r}_\mu$ indicates the largest. Let $\bar{s_k}$ be the number of the corresponding eigen values. Then T can be shown as

$$T = \left(T^1, T^2, \cdots, T^{\mu-1}, T^\mu\right)$$

where T^q be a $n \times \bar{s_q}$ matrix constructed from the $q-$th eigen value from the smallest as

$$T^q = \begin{pmatrix} \boxed{0} \\ \boxed{\bar{r}_q} \\ \vdots \\ \boxed{(n-2)\bar{r}_q} \\ \boxed{(n-1)\bar{r}_q} \end{pmatrix}, q = 1, 2, \cdots, \mu.$$

In the above figure the box $\boxed{\lambda}$ shows that they are all row vectors $\in R^{\bar{s_q}}$, whose elements are all functions of σ of order λ . If $\lambda = 0$ then the all elements of this matrix are constants.

As for the inverse matrix of T, it has the following structure as,

$$T^{-1} = \begin{pmatrix} \hat{T}^1 \\ \hat{T}^2 \\ \cdots \\ \hat{T}^{\mu-1} \\ \hat{T}^\mu \end{pmatrix}$$

where $\hat{T}^i$, are all $\bar{s}_i \times n$, matrices, respectively, and are found to have similar characteristics, which are rather complicated as shown below. Each $\hat{T}^i$ consists of μ parts and all elements of each column vector of $\hat{T}^i$ are of the same order function of σ. The order of each column vector in these parts of $\hat{T}^i$ monotonically decrase or increase as column nuber increase. Because of the limitation of the space, the precise description of the structure of $\hat{T}^i$ must be omitted.

Next $|T^{-1}|\Delta A^{30}|T|$ is considered. First, $\Delta A^{30}T$ is found to be devided into 5 blocks as shown in Fig.A.1. In this figure, block (B1) $\sim$ block (B5) are defined as follows.

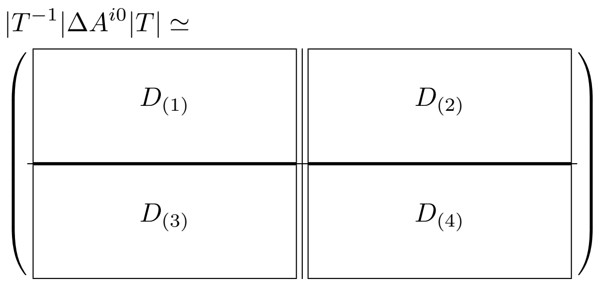

Fig. A.2. Schematic View of $|T^{-1}|\Delta A^{30}|T|$

(B1), This region consists of p-th row of the matrix of $\Delta A^{30}T$ and all elements belonging to this block are 0.
(B2,B3) These blocks consist of 1 to p-1 th row of $\Delta A^{30}T$ and 1 to p+1 and, p+2 to nth column in B2 and B3.respectively.
(B4,B5) These blocks consist of p+1-th row to the n-th row of $\Delta A^{30}T$. 1st to p+1-th column in B4, and p+2-th to n-th column in B5.
By carefuly multiplying $|T^{-1}|$ from left to the above matrix, the following results are obrtained.

Property 1 The obtained matrix is a $\mu \times \mu$ blocks matrix. Each (u, v) element blcok is a $s_u \times s_v$.
Property 2 The elements of each block $B_{u,v}$ are functions of σ of the same order.
Property 3 $|T^{-1}|\Delta A^{30}|T|$ can be devided into 4 major blocks as shown in Fig.A.2.

In case that there exists only one edge point (p, q), $q < n+1$ then $|T^{-1}|\Delta A^{03}|T|$ is shown as follows. Note that in this case only 3 kind of eigen values are needed. They are $p+1$ eigenvalue of order $-h$, h of order 1 and $n-h-p-1$ of order $hp+1$. Therefore in this case matrix $|T^{-1}|\Delta A^{03}|T|$ consists of 3×3 blocks as shown below. That means $D_{(4)}$ can be devided into 2×2 blocks.

$$|T^{-1}|\Delta A^{03}|T| \simeq \quad \text{(A.1)}$$
$$\begin{pmatrix} -h-1 & ph+p+1 & \begin{matrix} p^2h+ph^2+2ph \\ +p+1 \end{matrix} \\ -ph-p-h-1 & 0 & \begin{matrix} p^2h+ph^2+ \\ ph-h \end{matrix} \\ \begin{matrix} -p^2h-ph^2-ph \\ -p-h-1 \end{matrix} & -p^2h-ph^2-ph & 0 \end{pmatrix}$$

Each block means a submatrix and the number in its indicates the order of element functions of these matrices, all of which have same orders. All diagonal blocks of the RHS of (A.1) are $(p+1) \times (p+1)$, $h \times h$, $(n-p-h-1) \times (n-p-h-1)$ matricews, respectively. Number of rows and columns of other matrices follow from them.

Considereing $\Lambda \simeq \text{diag}(-h, \cdots, 1, \cdots, hp+1, \cdots,)$ it can easily be proven that in such case a proper σ which make the condition (17) is satisfied can be found.