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Adaptive Control of Unknown Dynamic Hysteretic Systems

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Abstract: This paper first discusses the adaptive control for the hysteresis described by Prandtl-Ishlinskii model. Then, the adaptive control for the continuous-time linear dynamical systems preceded with hysteresis described by Prandtl-Ishlinskii model is considered. The relative degree and the upper bound of the order of the linear dynamical system are assumed to be known. The contribution of the paper is the fusion of the hysteresis model with the adaptive control techniques. Only the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the controller are adaptively estimated online. The output tracking error can be controlled to approach to zero. Simulation results show the effectiveness of the proposed algorithm.

1. INTRODUCTION

The hysteresis phenomenon can be found in diverse disciplines ranging from, e.g., smart materials (Banks and Smith, 2000; Moheimani and Goodwin, 2001; Webb, et al, 1998), to ferromagnetism and superconductivity (Mayergoyz, 1991), to economics (Cross, et al, 2001), to geosciences (Guyer, et al, 1994). When a plant is preceded by the hysteresis, the system usually exhibits undesirable inaccuracies or oscillations and even instability (Tao and Kokotovic, 1995). The development of control techniques to mitigate the effects of hysteresis has been studied for decades and has recently re-attracted significant attention, e.g. Moheimani and Goodwin (2001) and the references therein. Much of the interest is a direct consequence of the importance of hysteresis in numerous new applications. Interest in studying dynamic systems with actuator hysteresis is also motivated by the fact that they are nonlinear system with nonsmooth nonlinearities for which traditional control methods are insufficient and thus require development of alternate effective approaches (Tan and Baras, 2004; Tao and Lewis, 2001). Development of a general frame for control of an uncertain dynamical system in the presence of unknown hysteresis is quite a challenging task.

To deal with the control problem of a dynamical system preceded by hysteresis, the thorough characterization of the hysteresis nonlinearities forms the foremost task. Appropriate hysteresis models may then be applied to the formulation of control algorithms. Hysteresis model can be roughly classified into physics-based models and phenomenological models. Physics-based models are built on first principles of physics (Jiles and Atherton, 1986). Phenomenological models are used to produce behaviors similar to those of the physical systems without necessarily providing physical insight into the problems. The basic idea consists of the modeling of the real complex hysteresis nonlinearities by the weighted aggregate effect of all possible so-called elementary hysteresis operators. Elementary hysteresis operators are noncomplex hysteretic nonlinearities with a simple mathematical structure. The popular phenomenological models are Preisach model (Adly, et al, 1991; Croft, et al, 2001; Natale, et al, 2001; Mayergoyz, 1991), Prandtl-Ishlinskii model (Brokate and Sprekels, 1996; Visintin, 1994), and Krasnosel'skii-Pokrovskii model (Krasnosel'skii-Pokrovskii, 1989; Visintin, 1994). The Preisach model and Krasnosel'skii-Pokrovskii (KP) model are parameterized by a pair of threshold variables (Mayergoyz, 1991), whereas the Prandtl-Ishlinskii (PI) model is a superposition of elementary stop operators which are parameterized by a single threshold variable (Visintin, 1994).

With the developments in various hysteresis models, it is natural to seek means to fuse these hysteresis models with the available control techniques to mitigate the effects of hysteresis, especially when the hysteresis is unknown, which is a typical case in many practical applications. However, the results on the fusion of the available hysteresis models with the available control techniques is surprisingly spare in the literature (Chen, et al, 2006; Su, et al, 2000, 2005; Tao and Kokotovic, 1995; Zhou, et al, 2004). The most common approach in coping with hysteresis in the literature is to construct an inverse operator, which is pioneered by Tao and Kokotovic (1995), and the reader may refer to, for instance, Krejci and Kuhnen (1999) and Tan and Baras (2004) and the references therein. Essentially, the inversion problem depends on the phenomenological modeling methods. Due to multi-valued and non-smooth features of hysteresis, the inversion always generates certain errors and possesses strong sensitivity to the model parameters. These errors directly make the stability analysis of the closed-loop system very difficult except for certain special cases, e.g. Tao and Kokotovic (1995).

This paper first develops an adaptive control method for the output of hysteresis described in PI model. Then, a new adaptive control approach is proposed for uncertain linear continuous time dynamical systems preceded with hysteresis, where the adaptive control techniques and the PI hysteresis model are fused together. The considered uncertain linear continuous time dynamical system contains unknown parameters, where the relative degree and the upper bound of the order of the linear system are assumed to be known. The Prandtl-Ishlinskii model of hysteresis is adopted in this paper. Only the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the controller are adaptively estimated online. The adaptive controller is synthesized by using the estimated parameters. All the signals in the closed loop are bounded and the output tracking error can be asymptotically controlled to be zero.

The remainder of this paper is organized as follows. Section 2 describes the PI-type hysteresis model and the adaptive control for the output of the PI-type hysteresis. In Section 3, first, the control problem for the linear continuous system preceded by hysteresis is formulated. Then, the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the controller are adaptively estimated. Finally, the adaptive controller is formulated and the stability of the closed system is analyzed. Simulation results are presented to show the effectiveness of the proposed method. Section 4 concludes this paper.

2. MODEL AND CONTROL OF HYSTERESIS

2.1 Hysteresis Model

In this paper, the Prandtl-Ishlinskii (PI) model is adopted. The hysteresis is denoted by the operator $H[*](t)$

$$u(t) = H[v](t), \tag{1}$$

where $v(t)$ is the input, $u(t)$ is the output of the hysteresis. The basic element of the PI operator is the so-called stop operator. For arbitrary piece-wise monotone function $v(t)$, define $e_r : \bar{R} \to \bar{R}$ (where $\bar{R}$ denotes the space of real numbers.) as

$$e_r(v) = min(r,\ max(-r, v)) \tag{2}$$

For any initial value $w_{-1} \in \bar{R}$ and $r \geq 0$, the stop operator $E_r[*; w_{-1}](t)$ is defined as

$$E_r[v; w_{-1}](0) = e_r(v(0) - w_{-1}), \tag{3}$$

$$E_r[v; w_{-1}](t) = e_r(v(t) - v(t_i) + E_r[v](t_i)), \tag{4}$$

for $t_i \leq t \leq t_{i+1}$, where the function $v(t)$ is monotone for $t_i \leq t \leq t_{i+1}$ (Brokate and Sprekels, 1996). The stop operator is mainly characterized by the threshold parameter $r \geq 0$ which determines the height of the hysteresis region in the (v, u) plane. For simplicity, denote $E_r[v; w_{-1}](t)$ by $E_r[v](t)$ in the following of this paper. It should be noted that the stop operator $E_r[v](t)$ is rate-independent. The PI hysteresis model is defined by

$$u(t) = \int_0^\infty p(r)E_r[v](t)dr. \tag{5}$$

where $p(r)$ is the density function which is usually unknown, satisfying $p(r) \geq 0$ with $\int_0^\infty rp(r)dr < \infty$ (Su, et al, 2005; Visintin, 1994; Webb, et al, 1998). Since the density function $p(r)$ vanishes for large values of r, it is reasonable to assume that there exists a constant R such that $p(r) = 0$ for $r > R$ (Brokate and Sprekels, 1996; Visintin, 1994). Thus, model (5) gives

$$u(t) = \int_0^R p(r)E_r[v](t)dr. \tag{6}$$

Figure 1 shows the relation between $v(t)$ and $u(t)$ given by model (6) with $p(r) = e^{-0.067(r-1)^2}$, $R = 20$, $w_{-1} = 0$ and $v(t) = 7\frac{sin(3t)}{1+t}$. It can be seen that the PI model (6) indeed generates the hysteresis curves and can be considered to be well-suited to describe the rate-independent hysteretic behavior.

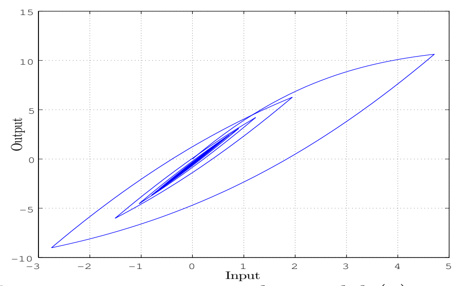

Fig. 1. Hysteresis curves given by model (6).

2.2 Implicit Inverse of Hysteresis

In this subsection, an input $v(t)$ is tried to calculate for a given output $u(t)$ for the operator described in (6). For this purpose, suppose the density function $p(r)$ is known in this subsection. Without loss of generality, suppose $u(t)$ is monotonically increasing on the interval $t_i \leq t \leq t_{i+1}$. For each $t \in [t_i, t_{i+1}]$, define a new variable $\bar{v}(t, \mu)$ with $\bar{v}(t, 0) = v(t_i)$ and another new variable $u_\mu(t)$, where μ is a parameter varying in the range $\mu \in [0, 2R]$

$$\bar{v}(t, \mu) = \bar{v}(t, 0) + \mu \tag{7}$$

$$u_\mu(t) = \int_0^R p(r)E_r[\bar{v}(t, \mu)]dr \tag{8}$$

Let $[v_{min}, v_{max}]$ be the input range, which is a subset of $[-R, R]$, to the hysteresis operator, and

$$\int_0^R p(r)E_r[v_{max}](t)dr = \bar{U} \tag{9}$$

$$\int_0^R p(r)E_r[v_{min}](t)dr = \underline{U} \tag{10}$$

If $u(t) > \bar{U}$, let $v(t) = v_{max}$
If $u(t) < \underline{U}$, let $v(t) = v_{min}$

If $\underline{U} \leq u(t) \leq \bar{U}$, the value of $v(t)$ is derived from the following algorithm.

Step 1: Let μ increase from 0.
Step 2: Calculate $\bar{v}(t,\mu)$ and $u_\mu(t)$. If $u_\mu(t) < u(t)$, then let μ increase continuously and go to Step 2;
Otherwise, go to Step 3.
Step 3: Stop the increasing of μ, memorize it as μ_0 and define $v(t) = \bar{v}(t,\mu_0)$

Remark 1. For $t = 0$, $\bar{v}(0,0)$ can be defined as $\bar{v}(0,0) = v_{min}$.

2.3 Implementation for the Implicit Inverse of Hysteresis

In this subsection, the implementation of the proposed implicit inverse algorithm for the hysteresis in Section 2.2 is considered. Suppose $\underline{U} \leq u(t) \leq \bar{U}$. For an assigned admissible error δ, we try to find the pseudo-inverse $v^*(t)$ such that

$$\Big|\int_0^R p(r)E_r[v^*](t)dr - u(t)\Big| \leq \delta \tag{11}$$

Now, select a small value $\Delta = \frac{R}{L}$, where L is a very large integer. The algorithm of determining $v^*(t)$ is as follows.

Step 1: $v^{(0)}(t) := v^*(t_i)$, $l := 0$.
Step 2: $u^{(l)}(t) := \int_0^R p(r)E_r[v^{(l)}](t)dr$.
If $|u^{(l)}(t) - u(t)| \leq \delta$, go to Step 6;
Else if $u^{(l)}(t) < u(t) - \delta$, let $v^{(l+1)}(t) := v^{(l)}(t) + \Delta$ and $l := l + 1$, then go to Step 3.
Else (i.e. $u^{(l)}(t) > u(t) + \delta$), let $v^{(l+1)}(t) := v^{(l)}(t) - \Delta$ and $l := l + 1$, then go to Step 4.
Step 3: $u^{(l)}(t) := \int_0^R p(r)E_r[v^{(l)}](t)dr$.
If $|u^{(l)}(t) - u(t)| \leq \delta$, go to Step 6;
Else if $u^{(l)}(t) < u(t) - \delta$, let $v^{(l+1)}(t) := v^{(l)}(t) + \Delta$ and $l := l + 1$, then go to Step 3.
Else (i.e. $u^{(l)}(t) > u(t) + \delta$), let $\underline{v}^{(l)}(t) := v^{l-1}(t)$ and $\bar{v}^{(l)}(t) := v^{(l)}(t)$, then go to Step 5.
Step 4: $u^{(l)}(t) := \int_0^R p(r)E_r[v^{(l)}](t)dr$.
If $|u^{(l)}(t) - u(t)| \leq \delta$, go to Step 6;
Else if $u^{(l)}(t) > u(t) + \delta$, let $v^{(l+1)}(t) := v^{(l)}(t) - \Delta$ and $l := l + 1$, then go to Step 4.
Else (i.e. $u^{(l)}(t) < u(t) - \delta$), let $\underline{v}^{(l)}(t) := v^l(t)$ and $\bar{v}^{(l)}(t) := v^{(l-1)}(t)$, then go to Step 5.
Step 5: $\underline{u}^{(l)}(t) := \int_0^R p(r)E_r[\underline{v}^{(l)}](t)dr$,
$\bar{u}^{(l)}(t) := \int_0^R p(r)E_r[\bar{v}^{(l)}](t)dr$,
$v^{(l+1)}(t) := \underline{v}^{(l)}(t) + (\bar{v}^{(l)}(t) - \underline{v}^{(l)}(t))\frac{u(t)-\underline{u}^{(l)}(t)}{\bar{u}^{(l)}(t)-\underline{u}^{(l)}(t)}$.
Let $l := l + 1$ and $u^{(l)}(t) := \int_0^R p(r)E_r[v^{(l)}](t)dr$.
If $|u^{(l)}(t) - u(t)| \leq \delta$, go to Step 6;
Else if $u^{(l)}(t) < u(t) - \delta$, let $\underline{v}^{(l)}(t) := v^{(l)}(t)$ and $\bar{v}^{(l)}(t) := \bar{v}^{(l-1)}(t)$, then go to Step 5;
Else (i.e. $u^{(l)}(t) > u(t) + \delta$), let $\underline{v}^{(l)}(t) := \underline{v}^{(l-1)}(t)$ and $\bar{v}^{(l)}(t) := v^{(l)}(t)$, then go to Step 5.
Step 6: $v^*(t) := v^{(l)}(t)$ and stop.

Remark 2. It is obvious that $v^*(t)$ can be found by finite steps of operations, i.e. l is finite when the operation is stopped. By discretizing $[0,R]$ uniformly into L subintervals, the integrals can be calculated by the well-known "Simpson Method".

2.4 Adptive Control for the Output of Hysteresis

In this subsection, the output $u(t)$ of the hysteresis is controlled to track a desired signal $u_d(t)$, which is differentiable and uniformly bounded. Since the density function $p(r)$ is unknown in the practical control. Let the estimate of $p(r)$ at instant t be $\hat{p}(r,t)$ for a fixed r. Define

$$\hat{u}(t) = \int_0^R \hat{p}(r,t)E_r[v](t)dr \tag{12}$$

and $e_1(t) = u(t) - \hat{u}(t)$.
The estimate $\hat{p}(r,t)$ is updated by the following algorithm with projection

$$\dot{\hat{p}}(r,t) = \begin{cases} \alpha e_1(t)E_r[v](t) & if\ \hat{p}(r,t) > 0 \\ 0 & if\ e_1(t)E_r[v](t) < 0 \\ & \ and\ \hat{p}(r,t) = 0 \end{cases} \tag{13}$$

where α is the adaptation gain satisfying $\alpha > 0$, $\hat{p}(r,0)$ should be chosen such that $\hat{p}(r,0) > 0$ and $\int_0^R r\hat{p}(r,0) < \infty$. Let $\tilde{p}(r,t) = \hat{p}(r,t) - p(r)$ and consider the function $L_1(t) = \int_0^R \tilde{p}^2(r,t)dr$. Taking the derivative of $L_1(t)$ yields

$$\frac{d}{dt}L_1(t) \leq -2\alpha e_1^2(t) \tag{14}$$

Lemma 1. For the estimated density function $\hat{p}(r,t)$, $\int_0^R \tilde{p}^2(r,t)dr$ is uniformly bounded and $e_1(t) \in L^2$.

For the input range $[v_{min}, v_{max}]$, suppose the saturation output of $\int_0^R \hat{p}(r,t)E_r[v](t)dr$ be $\underline{U}_{sat}$ and $\bar{U}_{sat}$. If $\underline{U}_{sat}(t) \leq u_d(t) \leq \bar{U}_{sat}(t)$, based on the method proposed in Section 2.2, a signal $v_1^*(t)$ can be derived such that

$$u_d(t) = \int_0^R \hat{p}(r,t)E_r[v_1^*](t)dr \tag{15}$$

The control input is chosen as

$$v(t) = v_1^*(t) \tag{16}$$

Theorem 1. If $\underline{U}_{sat}(t) \leq u_d(t) \leq \bar{U}_{sat}(t)$ for all $t \geq 0$, then the output tracking error of the hysteresis can be guaranteed to be zero by using the input (16).

Proof: By referring the derivation method of $v(t)$, it can be proved that $v(t)$ is uniformly continuous by some elementary operation. Then the uniform continuity of $u(t)$ can be proved. Thus $e_1(t)$ is uniformly continuous. The theorem is obvious by applying the famous Barbalat's Lemma to Lemma 1.

2.5 Simulation Results for the Output Control of Hysteresis

Consider the output control of the hysteresis described by

$$u(t) = \int_0^R p(r)E_r[v](t)dr., \tag{17}$$

with $p(r) = e^{-0.067(r-1)^2}$ and $w_{-1} = 0$. The control purpose is to drive the output of the above hysteresis to track the signal $u_d(t) = 5sin(2\pi t)$. In the simulation, the parameter R is chosen as $R = 20$, L is chosen as $L = 2000$, δ is set to $\delta = 1.0 \times 10^{-7}$, the sampling period is set to 0.001, the design parameter is chosen as $\alpha = 5.0$, the initial value is chosen as $\hat{p}(r,0) = 1.0$. The estimated parameter is shown in Figure 2. The control input is shown in Figure 3. The output tracking error is shown in Figure 4.

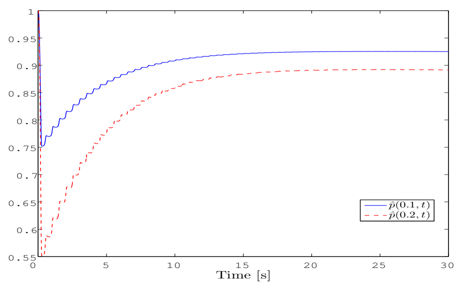

Fig. 2. The estimates of $\hat{p}(r,t)$ with $r = 0.1$ and $r = 0.2$.

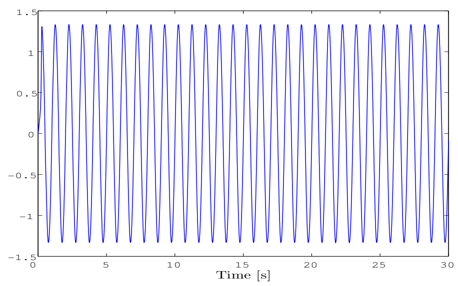

Fig. 3. The control input.

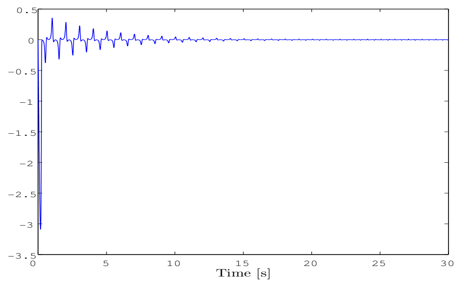

Fig. 4. The output tracking error.

3. MODEL AND CONTROL OF HYSTERESIS

3.1 Problem Statement

Consider the adaptive control for the continuous-time systems preceded by hysteresis described by

$$P(s)[y](t) = k_p Z(s)[u](t), \tag{18}$$
$$u(t) = H[v](t), \tag{19}$$

where $y(t) \in R$ is the output of the linear plant, $u(t) \in R$ is the input of the linear plant, k_p is the high frequency gain, $P(s), Z(s)$ are described by the following polynomials.

$$P(s) = s^{n_0} + p_{n_0-1}s^{n_0-1} + \cdots + p_1 s + p_0, \tag{20}$$
$$Z(s) = s^m + z_{m-1}s^{m-1} + \cdots + z_1 s + z_0,\ n_0 > m \tag{21}$$

The control purpose is to drive the output of the system to track the output $y_m(t)$ of the reference model described by

$$P_m(s)[y_m](t) = q(t), \tag{22}$$

where $P_m(s)$ is a monic polynomial with degree $n^* = n_0 - m$, $q(t)$ is the input of the reference model.
We make the following assumptions for the control system.

A1: $Z(s)$ is a stable polynomial.

A2: The upper bound for the degree n_0 of $P(s)$ is known as n.

A3: The sign of the plant high frequency gain k_p is known.

A4: The degree of n^* of $P_m(s)$ is known.

It is well known in the literature that the control input of the plant should have the following form (Tao, 2003)

$$u(t) = \theta_1^T\omega_1(t) + \theta_2^T\omega_2(t) + \theta_{20}y(t) + \theta_3 q(t), \tag{23}$$

where $\omega_1(t)$ and $\omega_2(t)$ are defined as

$$\omega_1(t) = \frac{a(s)}{\Lambda(s)}[u](t), \quad \omega_2(t) = \frac{a(s)}{\Lambda(s)}[y](t);$$

$\Lambda(s)$ is a $(n-1)th$ order monic stable polynomial; the parameters $\theta_1 \in R^{n-1}$, $\theta_2 \in R^{n-1}$, $\theta_{20} \in R$, $\theta_3 \in R$ should satisfy the following equation.

$$\begin{aligned}&\theta_1^T a(s)P(s) + (\theta_2^T a(s) + \theta_{20}\Lambda(s))k_p Z(s)\\ =&\Lambda(s)(P(s) - \theta_3 k_p Z(s)P_m(s)),\end{aligned} \tag{24}$$

Since the parameters in the P(s) and Z(s) are unknown, the parameters θ_1, θ_2, θ_{20}, θ_3 are all unknown.

3.2 Adaptive Control Algorithm

Multiplying the both sides of (24) with $y(t)$ and applying (18) yields

$$\begin{aligned}\int_0^R p(r)E_r[v](t)dr &= \theta_1^T\frac{a(s)}{\Lambda(s)}\int_0^R p(r)E_r[v](t)dr\\ &+\theta_2^T\frac{a(s)}{\Lambda(s)}[y](t) + \theta_{20}y(t) + \theta_3 P_m[y](t),\end{aligned} \tag{25}$$

In the following, without loss of generality, assume that $\theta_3 = k_p^{-1} = 1$. Otherwise, regard $k_p p(r)$ as $p(r)$, where $p(r)$ is the density function of the hysteresis operator.

Suppose that the estimates of $p(r)$, θ_2, θ_{20} are respectively $\hat{p}(r,t)$, $\hat{\theta}_2(t)$, $\hat{\theta}_{20}(t)$ at instant t, and suppose the estimate of the product θ_1 and $p(r)$ is $\hat{\theta}_1(r,t)$ at instant t.

Now, let us consider the variable $V(t)$ which should satisfy the next equation.

$$\begin{aligned}\int_0^R \hat{p}(r,t)E_r[V](t)dr &= \int_0^R \hat{\theta}_1^T(r,t)\frac{a(s)}{\Lambda(s)}E_r[V](t)dr\\ &+\hat{\theta}_2^T(t)\frac{a(s)}{\Lambda(s)}[y](t)+\hat{\theta}_{20}(t)y(t)+q(t).\end{aligned} \tag{26}$$

The variable $V(t)$ satisfying (26) can be derived by the method proposed in Section 2.2. The control input of the system preceded by hysteresis should be chosen as

$$v(t) = V(t) \tag{27}$$

Define

$$e(t) = y(t) - y_m(t) \tag{28}$$

Therefore, from (25) and (26), it yields

$$\begin{aligned}e(t) = \frac{1}{P_m(s)}\Big\{&-\int_0^R \tilde{p}(r,t)E_r[v](t)dr\\ &+\int_0^R \hat{\theta}_1^T(r,t)\frac{a(s)}{\Lambda(s)}E_r[v](t)\\ &-\theta_1^T\frac{a(s)}{\Lambda(s)}\int_0^R p(r)E_r[v](t)dr\\ &+\tilde{\theta}_2^T(t)\frac{a(s)}{\Lambda(s)}y(t) + \tilde{\theta}_{20}(t)y(t)\Big\},\end{aligned} \tag{29}$$

with $\tilde{p}(r,t) = \hat{p}(r,t) - p(r)$, $\tilde{\theta}_2(t) = \hat{\theta}_2(t) - \theta_2$, $\tilde{\theta}_{20}(t) = \hat{\theta}_{20}(t) - \theta_{20}$. Now, introduce a $(n^*-1)th$ order monic stable polynomial $L(s)$, and define a new error $\epsilon(t)$

$$\epsilon(t) = e(t) + \frac{L(s)}{P_m(s)}\Big\{\xi(t) - \kappa\epsilon(t)m_0^2(t)\Big\}, \qquad (30)$$

where $\kappa > 0$ is an arbitrary constant, and

$$\begin{aligned}\xi(t) = &\int_0^R \hat{\theta}_1^T(r,t)L^{-1}(s)\frac{a(s)}{\Lambda(s)}E_r[v](t)dr \\ &+ \hat{\theta}_2^T(t)L^{-1}(s)\frac{a(s)}{\Lambda(s)}y(t) + \hat{\theta}_{20}L^{-1}(s)y(t) \\ &- \int_0^R \hat{p}(r,t)L^{-1}(s)E_r[v](t)dr \\ &- L^{-1}(s)\zeta(t) + L^{-1}(s)\int_0^R \hat{p}(r,t)E_r[v](t)dr, \qquad (31)\end{aligned}$$

$$\begin{aligned}\zeta(t) = &\int_0^R \hat{\theta}_1^T(r,t)\frac{a(s)}{\Lambda(s)}E_r[v](t)dr \\ &+ \hat{\theta}_2^T(t)\frac{a(s)}{\Lambda(s)}[y](t) + \hat{\theta}_{20}y(t), \qquad (32)\end{aligned}$$

$$\begin{aligned}m_0(t) = \Big(&\int_0^R ||L^{-1}(s)\frac{a(s)}{\Lambda(s)}E_r[v](t)||^2 dr \\ &+ \int_0^R \big(L^{-1}(s)E_r[v](t)\big)^2 dr + \xi^2(t) \\ &+ ||L^{-1}(s)\frac{a(s)}{\Lambda(s)}y(t)||^2 + \big(L^{-1}(s)y(t)\big)^2\Big)^{\frac{1}{2}}, \qquad (33)\end{aligned}$$

The parameter adaptation law is chosen as

$$\dot{\hat{p}}(r,t) = \begin{cases}\gamma_0\epsilon(t)L^{-1}(s)E_r[v](t) & if\ \hat{p}(r,t) > 0 \\ 0 & if\ \epsilon(t)L^{-1}(s)E_r[v](t) < 0 \\ & and\ \hat{p}(r,t) = 0\end{cases} \qquad (34)$$

$$\dot{\hat{\theta}}_1(r,t) = -\Gamma\epsilon(t)L^{-1}(s)\frac{a(s)}{\Lambda(s)}E_r[v](t), \qquad (35)$$

$$\dot{\hat{\theta}}_2(t) = -B\epsilon(t)L^{-1}(s)\frac{a(s)}{\Lambda(s)}y(t), \qquad (36)$$

$$\dot{\hat{\theta}}_{20} = -\beta\epsilon(t)L^{-1}(s)y(t), \qquad (37)$$

with $\gamma_0 > 0$, $\Gamma = \Gamma^T > 0$, $B = B^T > 0$, $\beta > 0$. In the following, the stability of the system (1) controlled by (27) will be analyzed.

Lemma 2. The adaptive law (34)-(37) guarantees that $\epsilon(t) \in L^2 \cap L^\infty$, $\epsilon(t)m_0(t) \in L^2$, $\hat{p}(r,t) \in L^\infty$, $\hat{\theta}_1(r,t) \in L^\infty$, $\hat{\theta}_2(t) \in L^\infty$, $\hat{\theta}_{20} \in L^\infty$, $\dot{\hat{p}}(r,t) \in L^2$, $\dot{\hat{\theta}}_1(r,t) \in L^2$, $\dot{\hat{\theta}}_2(t) \in L^2$, $\dot{\hat{\theta}}_{20}(t) \in L^2$.

Theorem 2. All the signals in the closed-loop system consisting of the plant (18), reference model (22), controller (27), adaptive law (34)-(37) are bounded and the tracking error $e(t) = y(t) - y_m(t)$ belongs to $e(t) \in L^2$ and $\lim_{t\to\infty} e(t) \to 0$.

3.3 Simulation Results

In this subsection, consider the plant described by

$$(s+1)(s+2)[y](t) = (3s+5)[u](t), \qquad (38)$$

$$u(t) = \int_0^R p(r)E_r[v](t)dr, \qquad (39)$$

with $p(r) = e^{-0.067(r-1)^2}$ and $w_{-1} = 0$. The control purpose is to drive the output of the above system to track the output $y_m(t)$ of the reference model described by $(s+1)[y_m](t) = q(t)$ with $q(t) = 10sin(2\pi t)$. In the simulation, $\Lambda(s)$ is chosen as $\Lambda(s) = s+3$, the parameter R is chosen as $R = 20$, L is chosen as $L = 2000$, δ is set to $\delta = 0.00001$, the sampling period is set to 0.001, the design parameters are chosen as $\Gamma = 0.8I$, $\gamma_0 = 0.8$, $B = 0.8I$, $\beta = 0.8$, the initial values are chosen as $\hat{p}(r,0) = 2.0$, $\hat{\theta}_1(r,0) = 2.0$, $\hat{\theta}_2(0) = 1.0$, $\hat{\theta}_{20}(0) = -0.1$, $y(0) = 0.9$. The estimated parameters are shown in Figures 5-7. The control input is shown in Figure 8. The outputs of the controlled system and the reference model is shown in Figure 9. The tracking error is shown in Figure 10.

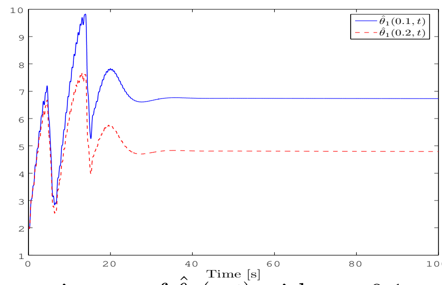

Fig. 5. The estimates of $\hat{\theta}_1(r,t)$ with $r = 0.1$ and $r = 0.2$.

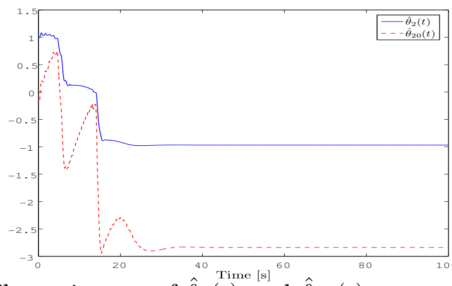

Fig. 6. The estimates of $\hat{\theta}_2(t)$ and $\hat{\theta}_{20}(t)$.

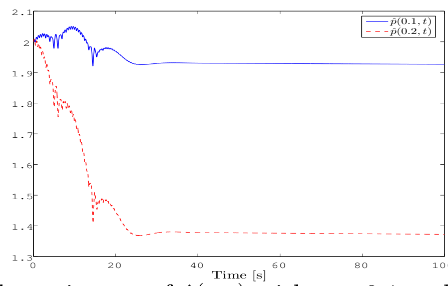

Fig. 7. The estimates of $\hat{p}(r,t)$ with $r = 0.1$ and $r = 0.2$.

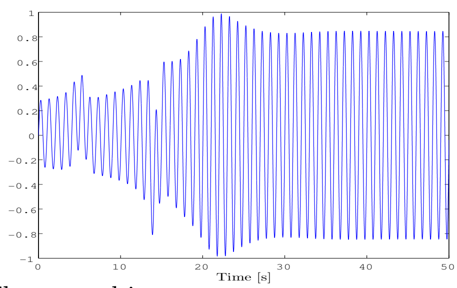

Fig. 8. The control input.

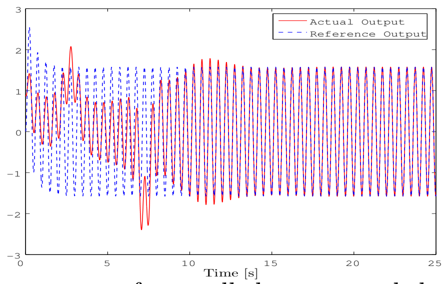

Fig. 9. The outputs of controlled system and the reference model.

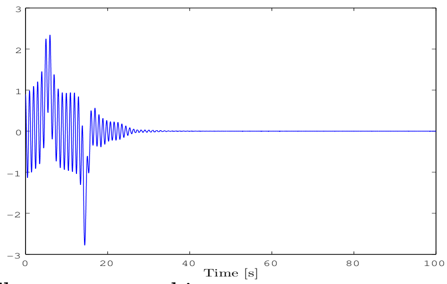

Fig. 10. The output tracking error .

4. CONCLUSIONS

This paper discussed the adaptive control for the hysteresis described by Prandtl-Ishlinskii model. Then, the adaptive control for the continuous-time linear dynamical systems preceded with hysteresis described by Prandtl-Ishlinskii model is considered. The contribution of the paper is the fusion of the hysteresis model with the adaptive control techniques. Only the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the controller are adaptively estimated online. The output tracking error can be controlled to approach to zero. Simulation results show the effectiveness of the proposed algorithm.

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