Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Bilateral Teleoperation Experiments: Scattering Transformation and Passive Output Synchronization Revisited ⋆

Emmanuel Nuño * **Luis Basañez** * **Erick Rodríguez-Seda** ** **Mark W. Spong** **

* *Institute of Industrial and Control Engineering, Technical University of Catalonia. Barcelona 08028, Spain. (e-mail: emmanuel.nuno@upc.edu; luis.basanez@upc.edu).*
** *Coordinated Science Laboratory, University of Illinois at Urbana Champaign. 1308 W. Main St., Urbana, IL 61801, USA. (e-mail: spong@control.csl.uiuc.edu; erodrig4@uiuc.edu).*

Abstract: It is well known that the scattering variables transform the transmission delays into a passive *virtual* transmission line, hence, its interconnection with passive subsystems preserves passivity of the overall system. However, *wave reflections* may occur. Using a symmetric velocity controller, on the master and the slave, and by matching the impedances, the scattering transformation reduces to a passive output synchronization scheme. In this paper we revisit this relation and perform some experiments on this line.

Keywords: Telerobotics, Time-delay, Robot control, Lyapunov functions.

1. INTRODUCTION

Most bilateral teleoperators use a communication channel that imposes a time-delay between data transfers. It is well known that this time-delay affects the overall stability of the teleoperator. The control of these systems has become an highly active research field amongst engineering scientists. The ground–breaking work of Anderson and Spong [1989] has ever since dominated this field. They proposed to send the scattering signals to transform the transmission delays into a passive *virtual* transmission line. The transmission line is then interconnected with the master and slave robots, which define passive force to velocity operators, while the human operator and the contact environment constitute the terminations to the transmission line. Since power–preserving interconnection of passive systems is again passive $\mathcal{L}_2$–stability of the overall system is ensured under the reasonable assumption that the human operator and the environment define passive (force to velocity) maps. Since then, the use of scattering theory has been widely extended. The reader is referred to Hokayem and Spong [2006] and Arcara and Melchiorri [2002] for two detailed surveys regarding the control of teleoperators.

Recently, Chopra and Spong have presented an interesting control architecture, based on the passive output synchronization of n-agents. In their work they have achieved delay-independent output synchronization of the agents, for any constant time-delay (Chopra and Spong [2007], Chopra and Spong [2006]). They have shown that using a symmetric controller on the teleoperator and by *matching their impedances* with the *virtual* transmission line, the scheme reduces to the one used for the passive output synchronization. The stability for both schemes is analyzed using the passivity property of the transmission line, for the scattering transformation, and with a Liapunov-Krasovskiĭ functional, respectively. In this work we revisit this analysis and present some teleoperation simulations and experiments that show the stable behavior of the overall system.

The paper is arranged as follows: modeling the n-DOF teleoperator is shown in Section 2; Section 3 and Section 4 analyze the stability of the scattering transformation and the passive output synchronization schemes, respectively; some simulations are shown in Section 5 with the experiments in Section 6; finally, the conclusions and future work are outlined in Section 7.

2. MODELING THE N–DOF TELEOPERATOR

Before continuing, let us introduce the following notation. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}^+ := (0, \infty)$, $\mathbb{R}_0^+ := [0, \infty)$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ represent the minimum and maximum eigenvalue of matrix $\mathbf{A}$, respectively. $|\cdot|$ stands for the Euclidean norm and $\|\cdot\|_2$ for the $\mathcal{L}_2$ norm. In order to keep equations as clear as possible, the argument of all time dependent signals will be omitted (e.g. $\dot{\mathbf{q}}(t) \equiv \dot{\mathbf{q}}$), except for those which are time-delayed (e.g. $\dot{\mathbf{q}}(t-T)$). Following this reasoning, the argument of the signals inside the integrals will be omitted, and it is supposed that is equal to the variable on the differential, unless otherwise noted (e.g. $\int_0^t \mathbf{x}(\sigma)d\sigma \equiv \int_0^t \mathbf{x}d\sigma$).

⋆ This work has been partially supported by the spanish CICYT projects: DPI2005-00112 and DPI2007-63665, the FPI program with reference BES-2006-13393, and also by the mexican CONACyT grant-169003.

The master and the slave are modeled as a pair of n–degree of freedom (DOF) serial links with revolute joints. Their corresponding nonlinear dynamics are described by

$$\begin{aligned}\mathbf{M}_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + \mathbf{C}_m(\mathbf{q}_m, \dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) &= \boldsymbol{\tau}_m^* - \boldsymbol{\tau}_h \\ \mathbf{M}_s(\mathbf{q}_s)\ddot{\mathbf{q}}_s + \mathbf{C}_s(\mathbf{q}_s, \dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s + \mathbf{g}_s(\mathbf{q}_s) &= \boldsymbol{\tau}_e - \boldsymbol{\tau}_s^*, \end{aligned} \quad (1)$$

where $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathbb{R}^n$ are the acceleration, velocity and joint position, respectively. $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ the coriolis and centrifugal effects, $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ represent the vectors of gravitational forces, $\boldsymbol{\tau}_i^* \in \mathbb{R}^n$ are the control signals and $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the forces exerted by the human operator and the environment interaction, respectively. $i = m$ represents the master and $i = s$ the slave.

These robot dynamic models have some important properties [1]:

P1. Due to the fact that all joints are revolute then $\mathbf{M}_i(\mathbf{q}_i)$ is lower and upper bounded. i.e.

$$0 < \lambda_m(\mathbf{M}_i(\mathbf{q}_i))\mathbf{I} \le \mathbf{M}_i(\mathbf{q}_i) \le \lambda_M(\mathbf{M}_i(\mathbf{q}_i))\mathbf{I} < \infty$$

P2. The Coriolis matrix $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is given by

$$C_i^{jk}(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \sum_{l=1}^{n} \underbrace{\frac{1}{2}\left[\frac{\partial M_i^{jk}}{\partial q_i^l} + \frac{\partial M_i^{jl}}{\partial q_i^k} - \frac{\partial M_i^{kl}}{\partial q_i^j}\right]}_{{}^i\Gamma_{kl}^j(q_i)} \dot{q}_i^l$$

where ${}^i\Gamma_{kl}^j(q_i)$ are the Christoffel symbols of the first kind with the symmetric property that ${}^i\Gamma_{kl}^j(q_i) = {}^i\Gamma_{lk}^j(q_i)$, hence, the matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric. i.e.

$$\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{C}_i^T(\mathbf{q}_i, \dot{\mathbf{q}}_i) \quad (2)$$

2.1 General Assumptions

A1. Following standard considerations, we assume the human operator and the environment define passive (force to velocity) maps, that is, there exists $\kappa_i \in \mathbb{R}_0^+$ s.t.

$$\int_0^t \dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h d\sigma \ge -\kappa_m, \quad -\int_0^t \dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e d\sigma \ge -\kappa_s, \quad (3)$$

for all $t \ge 0$.

A2. In order to simplify some calculations and to focus on the main idea of this article, we will assume that the gravitational forces are precompensated by the controllers $\boldsymbol{\tau}_m^*, \boldsymbol{\tau}_s^*$ (i.e. $\boldsymbol{\tau}_m^* = \boldsymbol{\tau}_m + \mathbf{g}_m(\mathbf{q}_m)$ and $\boldsymbol{\tau}_s^* = \boldsymbol{\tau}_s - \mathbf{g}_s(\mathbf{q}_s)$). Hence, the dynamical models (1) are reduced to

$$\begin{aligned}\mathbf{M}_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + \mathbf{C}_m(\mathbf{q}_m, \dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m &= \boldsymbol{\tau}_m - \boldsymbol{\tau}_h \\ \mathbf{M}_s(\mathbf{q}_s)\ddot{\mathbf{q}}_s + \mathbf{C}_s(\mathbf{q}_s, \dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s &= \boldsymbol{\tau}_e - \boldsymbol{\tau}_s\end{aligned} \quad (4)$$

A3. We assume that the time-delay imposed by the communication channel is constant on each direction, but it may differ from one to another. The total round trip time-delay is equal to $T_m + T_s \ge 0$.

3. SCATTERING TRANSFORMATION

The scattering transformation is given by

$$\begin{aligned}\mathbf{u}_m &= \frac{1}{\sqrt{2b}}[\boldsymbol{\tau}_{md} - b\dot{\mathbf{q}}_{md}] \quad & \mathbf{u}_s &= \frac{1}{\sqrt{2b}}[\boldsymbol{\tau}_{sd} - b\dot{\mathbf{q}}_{sd}] \\ \mathbf{v}_m &= \frac{1}{\sqrt{2b}}[\boldsymbol{\tau}_{md} + b\dot{\mathbf{q}}_{md}] \quad & \mathbf{v}_s &= \frac{1}{\sqrt{2b}}[\boldsymbol{\tau}_{sd} + b\dot{\mathbf{q}}_{sd}]\end{aligned} \quad (5)$$

where b is the *virtual* impedance of the transmission line. The master and the slave are interconnected, for constant time-delays in the forward and the backward paths (T_m and T_s, respectively), as

$$\mathbf{u}_s = \mathbf{u}_m(t - T_m) \quad \mathbf{v}_m = \mathbf{v}_s(t - T_s) \quad (6)$$

Proposition 1. Consider the teleoperator given by (4) controlled by

$$\begin{aligned}\boldsymbol{\tau}_m &= \boldsymbol{\tau}_{md} - B_m\dot{\mathbf{q}}_m \\ \boldsymbol{\tau}_s &= \boldsymbol{\tau}_{sd} + B_s\dot{\mathbf{q}}_s\end{aligned} \quad (7)$$

where

$$\begin{aligned}\boldsymbol{\tau}_{md} &= -K_{dm}[\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_{md}] \\ \boldsymbol{\tau}_{sd} &= K_{ds}[\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_{sd}],\end{aligned} \quad (8)$$

with the communications governed by (5) and (6). The control gains [2] $b, B_i, K_{di} \in \mathbb{R}^+$. Under the assumptions A1 and A3

i. The system velocities asymptotically converge to the origin for any time-delay $T_i \ge 0$.

ii. By matching the *impedances* (i.e. $K_{dm} = K_{ds} = b$), *wave reflections* do not occur, and the system velocity error, defined by $\mathbf{e} = \dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s(t - T_s)$, asymptotically converges to the origin for any time-delay and any $b > 0$.

Proof. Let us propose the following Liapunov function $V(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ given by

$$\begin{aligned}V = &\frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \frac{1}{2}\dot{\mathbf{q}}_s^\top \mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s + \kappa_m + \kappa_s + \\ &+ \int_0^t [\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e] d\sigma + \int_0^t [\dot{\mathbf{q}}_{sd}^\top \boldsymbol{\tau}_{sd} - \dot{\mathbf{q}}_{md}^\top \boldsymbol{\tau}_{md}] d\sigma,\end{aligned} \quad (9)$$

relying on the assumption A1, the property P1 of robot manipulators, and the well known fact that

$$\begin{aligned}\int_0^t [\dot{\mathbf{q}}_{sd}^\top \boldsymbol{\tau}_{sd} - \dot{\mathbf{q}}_{md}^\top \boldsymbol{\tau}_{md}] d\sigma = &\frac{1}{2}\int_{t-T_m}^t |\mathbf{u}_m|^2 d\sigma + \\ &+ \frac{1}{2}\int_{t-T_s}^t |\mathbf{v}_s|^2 d\sigma \\ \ge &\ 0\end{aligned}$$

which is the main contribution of the scattering transformation, we show that the Lyapunov candidate (9) is positive definite and radially unbounded.

Using the property P2, the resulting time derivative of V is given by

$$\dot{V} = \dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_m - \dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_s + \dot{\mathbf{q}}_{sd}^\top \boldsymbol{\tau}_{sd} - \dot{\mathbf{q}}_{md}^\top \boldsymbol{\tau}_{md} \quad (10)$$

Substituting the expressions (7) and (8) we get

$$\begin{aligned}\dot{V} = &- B_m|\dot{\mathbf{q}}_m|^2 - B_s|\dot{\mathbf{q}}_s|^2 - K_{dm}|\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_{md}|^2 - \\ &- K_{ds}|\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_{sd}|^2 \\ \le &\ 0\end{aligned} \quad (11)$$

[1] The reader may refer to Kelly et al. [2005] and Spong et al. [2005] for a complete guide on advanced robot modeling.

[2] The use of scalar gains is made for simplicity. When these gains are positive definite diagonal matrices can be treated with slight modifications to the proof.

From which can conclude that the system velocities $\dot{\mathbf{q}}_m$ and $\dot{\mathbf{q}}_s$ asymptotically converge to the origin (part *i*, of the proposition). Moreover, invoking LaSalle Theorem for time-delayed systems (Hale [2006]), all solutions of (4), (7) and (6) converge to M, the largest invariant set in S. Where $S = \{\dot{q}_i \in \mathbb{R}^n,\ i = \{m, s\} : \dot{V} = 0\}$ s.t. $\{\dot{\mathbf{q}}_m = \dot{\mathbf{q}}_{md},\ \dot{\mathbf{q}}_s = \dot{\mathbf{q}}_{sd}\}$.

In order to prove part *ii*, from (7) and (8) we can find that

$$
\begin{aligned}
\dot{\mathbf{q}}_{md} = &\ \frac{K_{ds}}{b + K_{dm}}\dot{\mathbf{q}}_s(t - T_s) + \frac{K_{dm}}{b + K_{dm}}\dot{\mathbf{q}}_m + \\
&+ \frac{b - K_{ds}}{b + K_{dm}}\dot{\mathbf{q}}_{sd}(t - T_s) \\
\dot{\mathbf{q}}_{sd} = &\ \frac{K_{dm}}{b + K_{ds}}\dot{\mathbf{q}}_m(t - T_m) + \frac{K_{ds}}{b + K_{ds}}\dot{\mathbf{q}}_s + \\
&+ \frac{b - K_{dm}}{b + K_{ds}}\dot{\mathbf{q}}_{md}(t - T_m)
\end{aligned}
\tag{12}
$$

note that, by matching the impedances, the terms $\dot{\mathbf{q}}_{id}(t - T_i)$ on (12) disappear, and so do wave reflections. Using these new expressions the controllers become

$$
\begin{aligned}
\boldsymbol{\tau}_m &= \frac{b}{2}[\dot{\mathbf{q}}_s(t - T_s) - \dot{\mathbf{q}}_m] - B_m\dot{\mathbf{q}}_m \\
\boldsymbol{\tau}_s &= \frac{b}{2}[\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_m(t - T_m)] + B_s\dot{\mathbf{q}}_s
\end{aligned}
\tag{13}
$$

and the time derivative of V given by (11) transforms into

$$
\begin{aligned}
\dot{V} = &- B_m|\dot{\mathbf{q}}_m|^2 - B_s|\dot{\mathbf{q}}_s|^2 - \frac{b}{4}|\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s(t - T_s)|^2 - \\
&- \frac{b}{4}|\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_m(t - T_m)|^2
\end{aligned}
\tag{14}
$$

Similarly by LaSalle, $\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s(t - T_s) \to 0$ as $t \to \infty$. □

4. OUTPUT SYNCHRONIZATION

The control laws for the teleoperator are given by

$$
\begin{aligned}
\boldsymbol{\tau}_m &= K[\dot{\mathbf{q}}_s(t - T_s) - \dot{\mathbf{q}}_m] - B_m\dot{\mathbf{q}}_m \\
\boldsymbol{\tau}_s &= K[\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_m(t - T_m)] + B_s\dot{\mathbf{q}}_s
\end{aligned}
\tag{15}
$$

where $K > 0$ is the controller gain. $T_i \geq 0$, $i \in \{m, s\}$, is the time delay in the forward and backward paths, respectively. It is assumed that the initial conditions $\dot{\mathbf{q}}_i(\theta) \in \mathcal{C}^{\varepsilon}_{n,r}$ for $\theta \in [-T_i, 0]$.

Proposition 2. (Chopra and Spong [2006]). Consider the teleoperator system (4) controlled by (15). Assume that $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_e$ verify (3). Then all signals are bounded and the system velocity error, defined by $\mathbf{e} = \dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s(t - T_s)$, asymptotically converges to the origin, for any $\mathbf{K} > 0$ and all $T_i \geq 0$.

Proof. Consider the following Lyapunov-Krasovskiĭ functional, $V(t, \dot{\mathbf{q}}_i, \dot{\mathbf{q}}_i(t - T_i))$, given by [3]

$$
\begin{aligned}
V = &\ \dot{\mathbf{q}}_m^\top\mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \dot{\mathbf{q}}_s^\top\mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s + \\
&+ \int_0^t [\dot{\mathbf{q}}_m^\top\boldsymbol{\tau}_h - \dot{\mathbf{q}}_s^\top\boldsymbol{\tau}_e]d\sigma + \kappa_m + \kappa_s + \\
&+ K\int_{t-T_m}^t |\dot{\mathbf{q}}_m(\theta)|^2 d\theta + K\int_{t-T_s}^t |\dot{\mathbf{q}}_s(\theta)|^2 d\theta
\end{aligned}
\tag{16}
$$

[3] For a complete description of Liapunov-based analysis for time-delay systems, the reader should refer to Niculescu [2001].

Taking the time derivative of V and using the property P2 of robot manipulators, we obtain

$$
\begin{aligned}
\dot{V} = &\ 2\dot{\mathbf{q}}_m^\top\boldsymbol{\tau}_m - 2\dot{\mathbf{q}}_s^\top\boldsymbol{\tau}_s + K|\dot{\mathbf{q}}_m|^2 - K|\dot{\mathbf{q}}_m(t - T_m)|^2 + \\
&+ K|\dot{\mathbf{q}}_s|^2 - K|\dot{\mathbf{q}}_s(t - T_s)|^2
\end{aligned}
\tag{17}
$$

The last four terms come from the Krasovskiĭ-functional, and can be written as:

$$
\begin{aligned}
&K[\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s(t - T_s)]^\top[\dot{\mathbf{q}}_m + \dot{\mathbf{q}}_s(t - T_s)] + \\
&+ K[\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_m(t - T_m)]^\top[\dot{\mathbf{q}}_s + \dot{\mathbf{q}}_m(t - T_m)]
\end{aligned}
\tag{18}
$$

using the expressions (15) and some algebraic manipulations, it is easily seen that (17) becomes

$$
\begin{aligned}
\dot{V} = &- 2B_m|\dot{\mathbf{q}}_m|^2 - 2B_s|\dot{\mathbf{q}}_s|^2 - K|\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s(t - T_s)|^2 - \\
&- K|\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_m(t - T_m)|^2 \qquad (19) \\
\leq &\ 0
\end{aligned}
$$

Towards this end, the Liapunov-Krasovskiĭ functional only ensures uniform stability, but, using an extension of LaSalle's Invariance principle for FDE, we can obtain uniform asymptotic stability of the $\dot{V}$ signals.

Note that, the function $V(t, \dot{\mathbf{q}}_i, \dot{\mathbf{q}}_i(t - T_i))$ is positive definite, and $\dot{V} \leq 0$ any arbitrary initial level set is positively invariant, hence, all signals in (4) are bounded. Now, consider the set $S = \{\dot{q}_i \in \mathbb{R}^n,\ i = \{m, s\} : \dot{V} = 0\}$ characterized by all trajectories s.t. $\{\dot{\mathbf{q}}_s(t - T_s) = \dot{\mathbf{q}}_m,\ \dot{\mathbf{q}}_m(t - T_m) = \dot{\mathbf{q}}_s\}$. Let $M \subset S$ be the largest invariant set in S. Using LaSalle Theorem (Hale [2006]), all solutions of (4) and (15) converge to M as $t \to \infty$. This implies that the velocity error $\mathbf{e}$ asymptotically converges to the origin for all $T_i \geq 0$ and all positive definite gain K. □

5. SIMULATIONS

This section presents a simulation of the teleoperator system. The master and slave are two link manipulators with revolute joints. Their corresponding nonlinear dynamics follow (1). The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is given by

$$
\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i\cos(q_{2_i}) & \delta_i + \beta_i\cos(q_{2_i}) \\ \delta_i + \beta_i\cos(q_{2_i}) & \delta_i \end{bmatrix}
$$

$q_{k_i}, k \in \{1, 2\}$ is the articular position of each link, $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i}l_{2_i}m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$. The lengths for both links l_{1_i} and l_{2_i}, in each manipulator, are $0.38m$. The mass of each link correspond to $m_{1_m} = 3.9473$kg, $m_{2_m} = 0.6232$kg, $m_{1_s} = 3.2409$kg and $m_{2_s} = 0.3185$kg, respectively. These values are the same of those used by in Lee and Spong [2006]. Coriolis and centrifugal forces are modeled as the vector $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i$ which are

$$
\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i = \begin{bmatrix} -\beta_i\sin(q_{2_i})\dot{q}_{2_i}^2 - \beta_i\sin(q_{2_i})\dot{q}_{1_i}\dot{q}_{2_i} \\ \beta_i\sin(q_{2_i})\dot{q}_{1_i}^2 \end{bmatrix}
$$

$\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. The gravity effects $(\mathbf{g}_i(\mathbf{q}_i))$ for each manipulator are represented by

$$
\mathbf{g}_i(\mathbf{q}_i) = \begin{bmatrix} \frac{1}{l_{2_i}}g\delta_i\cos(q_{1_i} + q_{2_i}) + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)\cos(q_{1_i}) \\ \frac{1}{l_{2_i}}g\delta_i\cos(q_{1_i} + q_{2_i}) \end{bmatrix}
$$

Note that this vector is precompensated following Assumption 2. $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_e$ are the operator and environmental torques. At this point, it should be addressed that the human exerts a force $\mathbf{f}_h$ on the master manipulator's tip, and the slave interaction force with the environment $\mathbf{f}_e$ is also measured at the manipulator's tip. Hence, for the simulations the following expressions are used $\boldsymbol{\tau}_h = \mathbf{J}_m^\top(\mathbf{q}_m)\mathbf{f}_h$ and $\boldsymbol{\tau}_e = \mathbf{J}_s^\top(\mathbf{q}_s)\mathbf{f}_e$, where $\mathbf{J}_i^\top(\mathbf{q}_i)$ is the Jacobian transposed of the robot manipulator. The controllers for these simulations are given by (15), with $K = 5$ and $B_i = 0.5$. The time delay is $T_m = 2.5$s and $T_s = 3.5$s. The simulation has been carried out using MatLab SimuLink $^{\text{TM}}$.

Two simulations were carried out: on the first, the human exerts a force on the master and the slave moves freely on the environment (Fig. 1); on the second, the human moves the master and the slave touches a high-stiffness virtual wall on the environment (Fig. 2). This virtual wall has been located in cartesian coordinates at $y = 0.25m$. The stiffness of the wall was set to 20000 $\frac{N}{m}$ with 10 $\frac{Ns}{m}$ of damping. We assume that the system is frictionless. On Fig. 1 the system moves freely, velocity error converges to the origin and it is clearly seen that the teleoperator is stable. Moreover, if touching a high-stiff wall, around 6s on Fig. 2, the system remains stable, but, position drift arises.

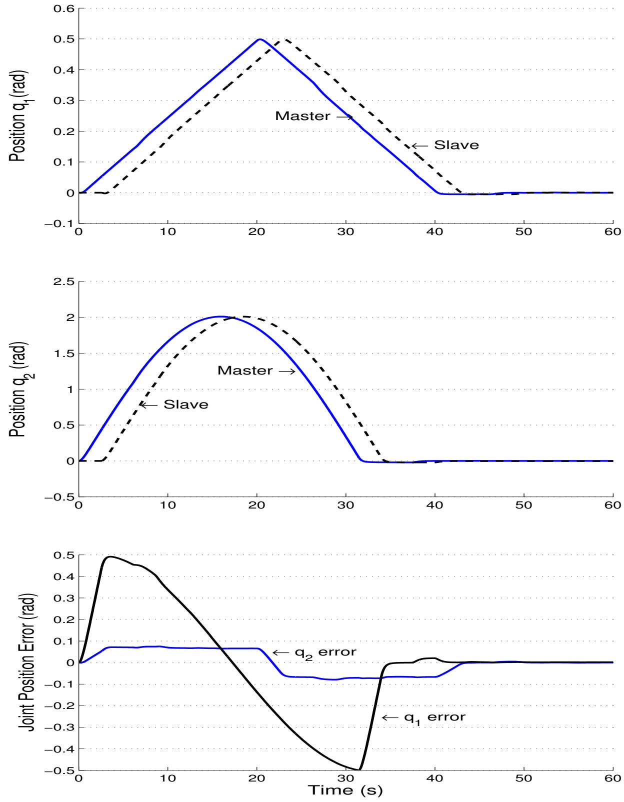
6. EXPERIMENTS

The experimental test-bed mainly consists of two direct-drive two DOF nonlinear manipulators. These manipulators are made of aluminium and are actuated by two pairs of Compumotor DM1015-B brushless DC motors. Optical encoders are used to measure the joint position, the joint velocity is digitally estimated and filtered. Two JR3 force-torque sensors, located at the manipulators end-effectors, are used to measure the force interaction with the human operator and environment, respectively. The controllers are implemented using WinCom 3.3 that enables Simulink$^{\text{TM}}$ models to interact with external hardware in real time. The sampling time is set to 4ms. An aluminium wall is located at one side of the slave in order to test the stability while interacting with an stiff environment. This setup is located at the CSL, UIUC, and is depicted in figure 3.

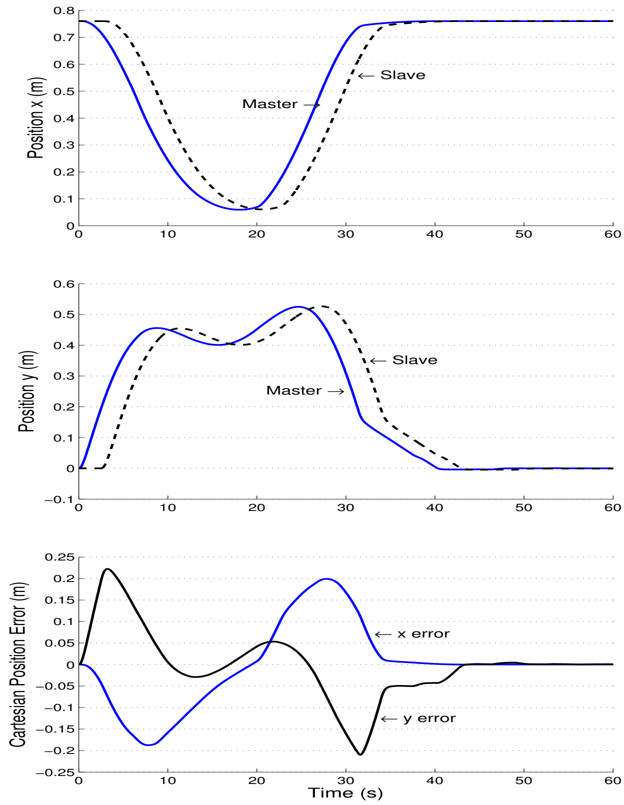
There are two experiments, the teleoperator moves freely on Fig. 4 and touches an aluminium wall around 12s on Fig. 5. The static friction has caused the teleoperator to exhibit a position error, although moving freely (q_1, q_2 on Fig. 4). However, the system is stable despite touching the aluminium wall. In order to set the control gain K it can be used a model-based tuning method such as the one on pp. 213 of Kelly et al. [2005]. The master and slave controllers follow (15) and the gains were set to $K = 10$, $B_i = 2$, for both experiments. The two 2 DOF manipulators move on a parallel plane tangential to the earth surface, hence, the gravity vector is zero.

7. CONCLUSIONS AND FUTURE WORK

Chopra and Spong [2006] introduced the passive output synchronization, which has been the base platform for this paper. Using a symmetric controller with the scattering



a) Joint space



b) Cartesian space

Fig. 1. Simulation of the teleoperator system with $T_m = 2.5s$ and $T_s = 3.5s$.

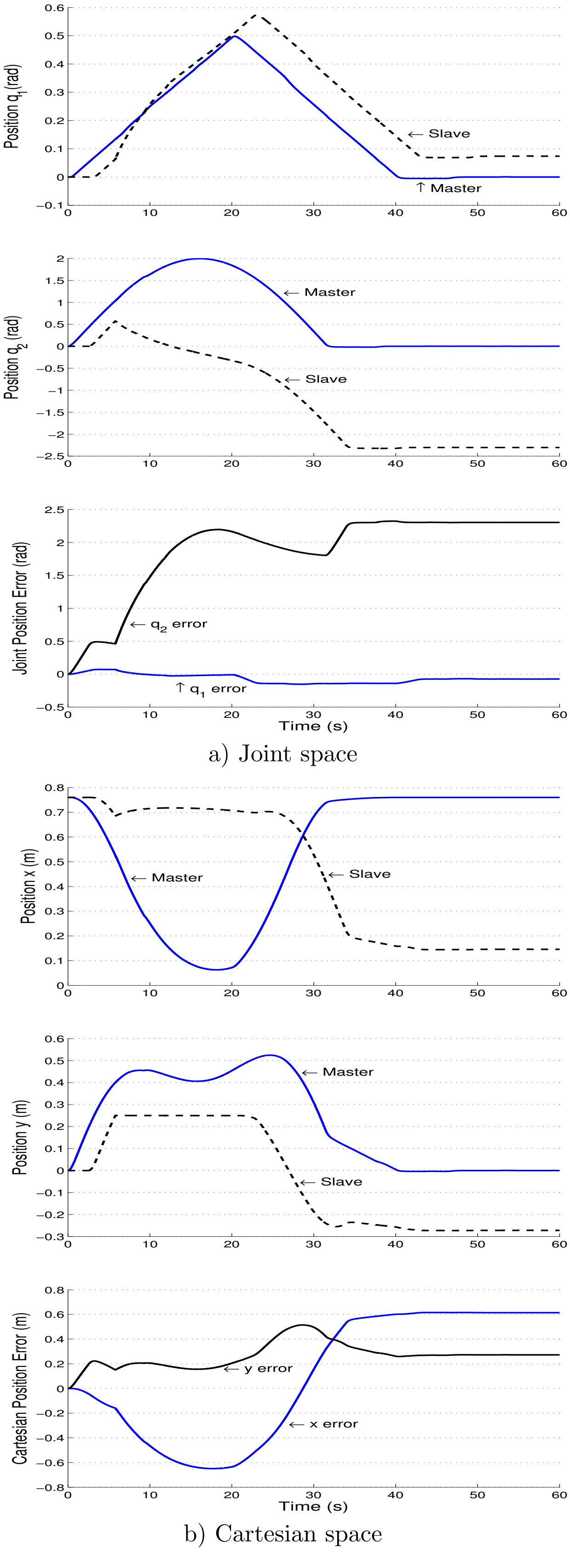


Fig. 2. Simulation of the teleoperator system touching a stiff wall with $T_m = 2.5s$ and $T_s = 3.5s$.

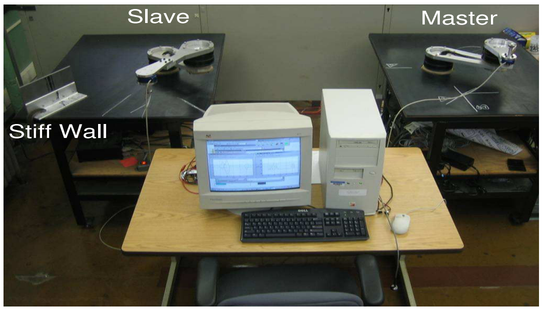


Fig. 3. Experimental teleoperator at CSL, UIUC.

transformation and matching the impedances, one can reduce the system to a passive output synchronization scheme. Both schemes ensure stability for any arbitrary constant time-delay. However, under velocity control, both schemes do not provide position synchronization. Future steps on this line are the study of the effects of variable time-delays and the analysis of position drift free schemes.

ACKNOWLEDGEMENTS

The first author gratefully acknowledges the hospitality of the people at the CSL at the UIUC, especially to Prof. Mark W. Spong, he also thanks the comments of the anonymous reviewers, that helped to improve the contents of this work.

REFERENCES

R.J. Anderson and M.W. Spong. Bilateral control of teleoperators with time delay. *IEEE Transactions on Automatic Control*, 34(5):494–501, May 1989.

P. Arcara and C. Melchiorri. Control schemes for teleoperation with time delay: A comparative study. *Robotics and Autonomous Systems*, 38:49–64, 2002.

N. Chopra and M.W. Spong. *Passivity-Based Control of Multi-Agent Systems*, pages 107–134. Advances in Robot Control From Everyday Physics to Human-Like Movements. Springer, 2006.

N. Chopra and M.W. Spong. *Adaptive Synchronization of Bilateral Teleoperators with Time Delay*, pages 257–270. Advances in Telerrobotics. Springer, 2007.

J.K. Hale. *History of Delay Equations*, chapter 1, pages 1–28. Number 205 in Delay Differential Equations and Applications. Springer-Verlag, 2006.

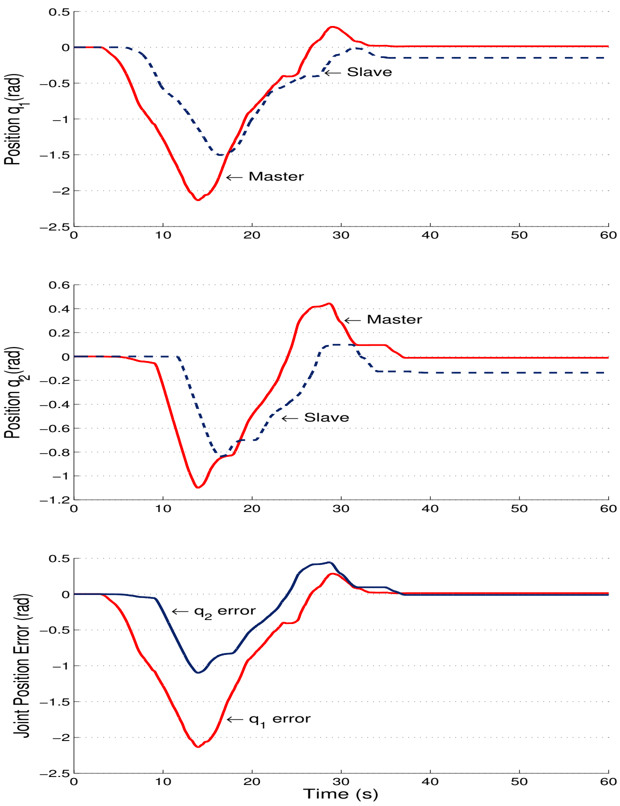
P.F. Hokayem and M.W. Spong. Bilateral teleoperation: An historical survey. *Automatica*, 42:2035–2057, 2006.

R. Kelly, V.Santibáñez, and A. Loría. *Control of robot manipulators in joint space*. Advanced textbooks in control and signal processing. Springer-Verlag, 2005.

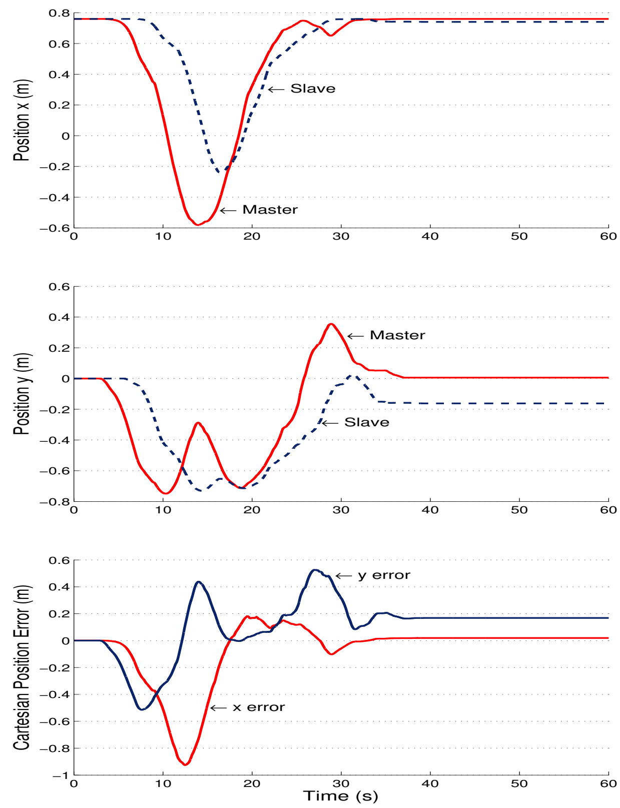
D. Lee and M.W. Spong. Passive bilateral teleoperation with constant time delay. *IEEE Transactions on Robotics*, 22(2):269–281, April 2006.

S.I. Niculescu. *Delay Effects on Stability: A Robust Control Approach*. Number 269 in Lecture Notes in Control and Information Sciences. Springer-Verlag, 2001.

M.W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley, 2005.

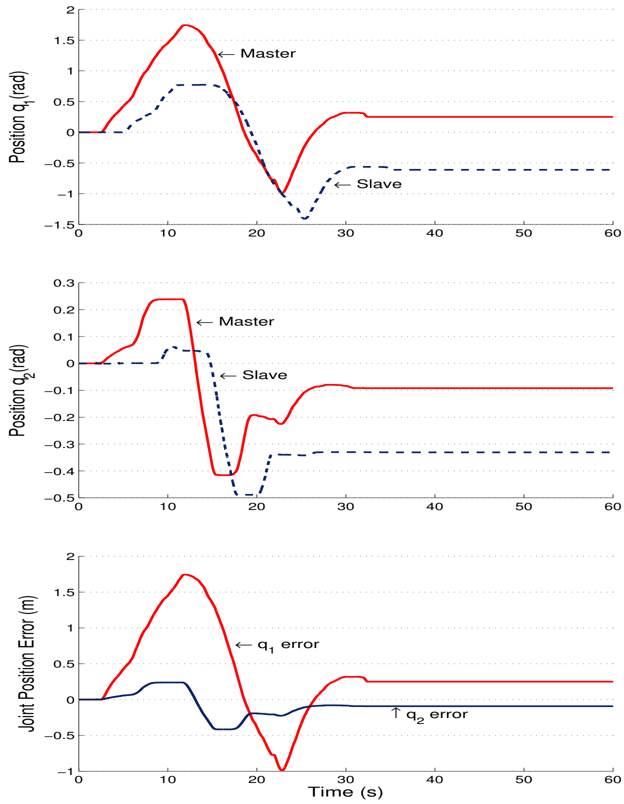


a) Joint space

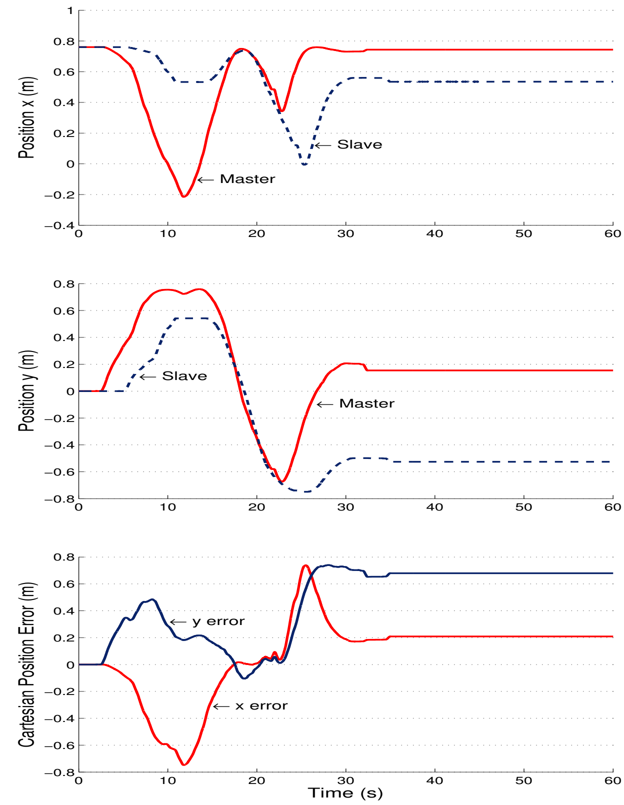


b) Cartesian space

Fig. 4. Experiments with $T_m = 2.5s$ and $T_s = 3.5s$.



a) Joint space



b) Cartesian space

Fig. 5. Experiments touching the aluminium wall, with $T_m = 2.5s$ and $T_s = 3.5s$.