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A model predictive control approach for decentralized traffic signal control

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Abstract: In this paper, a decentralized control is utilized for the traffic signal control problem using model predictive control. A point-queue model is used with red-green signal transition times. A traffic signal controller is designed to minimize the queue lengths with information from the adjacent intersections. After the signal controller gathers the needed information from the adjacent intersections, it makes green time calculation of the next period.

1. INTRODUCTION

The Urban traffic control problem has received increasing attention in the last decades. Different control methods have been proposed considering various aspects of the problem, current, and future technologies. One may look at (Papageorgiou *et al*., 2003) for a detailed survey of the various proposed traffic control methods. These methods may be classified into two group; fixed time strategies in which historical traffic data is used to compute signal timing, and traffic responsive strategies where signal timing is determined considering current traffic status. Due to the dynamic nature of the problem and technological developments, the latter strategies are more promising. Implementation of fully traffic responsive system requires full update of legacy hardware in the present cities. However, constructing plans for futuristic cities, like the Korean government plans to build with the latest information technology infrastructure (Songdo City, 2007), will make traffic responsive strategies more attractive.

Traffic responsive strategies may also be classified depending on whether they deal with the full network with many intersections or mainly concentrated on isolated intersection. In this area, SCOOT (Hunt *et al*., 1982) and SCATS (Lowrie, 1982) are the two well-known and widely used coordinated traffic-responsive strategies. But, both SCOOT and SCATS decide on incremental change of splits, offsets, and cycles based on real time measurements which may not deal with traffic conditions changing rapidly. In another work, a centralized urban control strategy of consisting two steps is (Diakaki *et al*., 2002) presented. In the first step, Linear quadratic regulator theory has been used to calculate initial signal times in offline based on the simplified saturated intersection model proposed by Gazis (Gazis,1991). Later, these signal times and other systems constraints are combined in an optimization model to calculate the final green times of intersections in real time. Since the design of the regulator is based on the saturated traffic model, this approach can not achieve the best performance when some of the intersections are unsaturated. There are also some other approaches which use model based optimization methods for coordinated intersections. For example, PRODYN (Farges *et. al.*, 1982) and RHODES (Mirchandani and Head, 1998) employed dynamic programming, while OPAC (Gartner, 2000) employed exhaustive enumeration. Due to the computational complexity of these methods, the basic optimization kernel is not real-time feasible for more than one junction (Papageorgiou *et al*., 2003). The centralized strategies mentioned above consider the whole network during the signal time calculations, but they have clear limitations as the network size gets larger.

On the other hand, decentralized approaches mainly focus on isolated intersections and then integrate multiple intersections. Davison and Ozguner, (1983), apply a robust decentralized servomechanism method to balance the queue lengths when the intersections are saturated. Later, Khorrami and Ozguner (1984) developed a mathematical model including the effects of both signalization and routing at the same time in the traffic network. They showed the existence of decentralized controllers (state feedback and proportional integral feedback) to stabilize the traffic network. In other work, a decentralized real-time intersection control scheme based on the dynamic programming principle is proposed (Porche and Lafortune, 1996). Regarding future vehicle arrivals, it computes the optimal signal switching sequence in order to minimize the delay time. Recently, an optimal decentralized regulation approach that balances and decreases the queue lengths using a point-queuing model was reported

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(Lei and Ozguner, 2002). Hybrid models were used to handle saturated and unsaturated traffic conditions. Although some other works also exist (Schutter and Moor, 1998; Fouladvand and Nematollahi, 2001) for isolated intersections, the property of a completely decentralized operation (e.g., independent algorithm application at each intersection) needs further investigation.

In this study, model predictive control is used to calculate the signal times at each intersection considering the incoming flows of the upstream intersections and maximum queue length of the downstream intersections. After the analysis of the results for the single intersection, it will be generalized to the multi-intersection case.

The paper is organized as follows. The isolated intersection queue model and the proposed control method are given in Section 2 with simulation results. Then, Section 3 gives the decentralized multi-intersection control approach. Conclusion and future work is given in Section 4.

2. CONTROL OF SINGLE INTERSECTION

In this section, we first introduce the queue model for single intersection which considers the red-green switching times explicitly. Subsequently, the model predictive control model is developed to calculate the green signal times for an N step horizon.

2.1 Queue model of single intersection

Consider a four way intersection with lanes L_j where $j = 1,2,\ldots,8$ (Fig. 1). The green signal times t_i, $i = 1,2,3,4$ with corresponding directions are given in Table 1. There are four green time phases in this intersection. For example, in the first phase the vehicles on the lanes L_1 and L_5 should turn left. In the second phase, the vehicles on the lanes L_2 and L_6 should go directly or turn right. No car is allowed to turn right without a green signal for any given direction. The vehicles on the lanes L_3, L_7 and L_4, L_8 should move in the third and fourth phases respectively. The summation of the green signal times is equal to the cycle time C which may vary between a lower and upper bound depending on the traffic density.

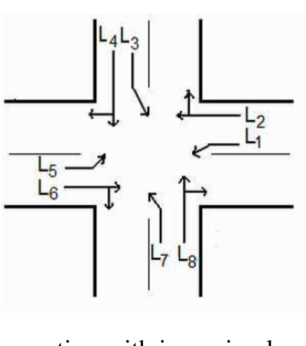

Fig. 1. Single intersection with incoming lanes

Table 1. Green signal times for the lanes

Signal times	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8
t_1	ON	-	-	-	ON	-	-	-
t_2	-	ON	-	-	-	ON	-	-
t_3	-	-	ON	-	-	-	ON	-
t_4	-	-	-	ON	-	-	-	ON

A single intersection can be modeled by a discrete time system in which the state variables $q_j(k)$ with $j = 1,2,\ldots,8$ represent the queue lengths at the beginning of the kth cycle (C). Individual queue lengths are mainly determined by incoming flows (f_j, $j = 1,2,\ldots,8$), outgoing flows (c_j, $j = 1,2,\ldots,8$) or lane capacities, and the duration of the green signal time. Assuming the average flow f_j^{td} to be known during any time interval (td), a generic queue model for the jth lane can be written as

$$q_j(k+1) = \max[q_j(k) + t_{bg} f_j^{bg} + t_g f_j^{g} - t_g c_j, 0] + t_{ag} f_j^{ag}$$

where t_{bg}, t_g, t_{ag} denote pre green-time duration, green time duration and post green time duration in a cycle respectively. The max term guarantees non-negative queue length in this model.

For the intersection given in Fig. 1, the queue length on each lane can be written as

$$
\begin{aligned}
q_1(k+1) &= \max[q_1(k) + t_1 f_1^{t_1} - t_1 c_1, 0] + t_2 f_1^{t_2} + t_3 f_1^{t_3} + t_4 f_1^{t_4}\\
q_2(k+1) &= \max[q_2(k) + t_1 f_2^{t_1} + t_2 f_2^{t_2} - t_2 c_2, 0] + t_3 f_2^{t_3} + t_4 f_2^{t_4}\\
q_3(k+1) &= \max[q_3(k) + t_1 f_3^{t_1} + t_2 f_3^{t_2} + t_3 f_3^{t_3} - t_3 c_3, 0] + t_4 f_3^{t_4}\\
q_4(k+1) &= \max[q_4(k) + t_1 f_4^{t_1} + t_2 f_4^{t_2} + t_3 f_4^{t_3} + t_4 f_4^{t_4} - t_4 c_4, 0]\\
q_5(k+1) &= \max[q_5(k) + t_1 f_5^{t_1} - t_1 c_5, 0] + t_2 f_5^{t_2} + t_3 f_5^{t_3} + t_4 f_5^{t_4}\\
q_6(k+1) &= \max[q_6(k) + t_1 f_6^{t_1} + t_2 f_6^{t_2} - t_2 c_6, 0] + t_3 f_6^{t_3} + t_4 f_6^{t_4}\\
q_7(k+1) &= \max[q_7(k) + t_1 f_7^{t_1} + t_2 f_7^{t_2} + t_3 f_7^{t_3} - t_3 c_7, 0] + t_4 f_7^{t_4}\\
q_8(k+1) &= \max[q_8(k) + t_1 f_8^{t_1} + t_2 f_8^{t_2} + t_3 f_8^{t_3} + t_4 f_8^{t_4} - t_4 c_8, 0]
\end{aligned}
$$

Although this model is designed to find the queue lengths over the period $[kC, (k+1)C]$, it has inherently two sub-states. One is from the start of the kth cycle to the end of the green time in which non-negative queue lengths are guarantied by the max terms. The other sub-state is between the end of the green time and the end of the kth cycle. So this model also helps to consider the incoming flow rates which cover unsaturated traffic conditions.

An intersection simulator is developed in MATLAB environment. This simulator uses queue lengths, green times, incoming flow values, and lane capacities of each lane to simulate the intersection.

2.2 Model Predictive control for determining signal times

Assuming average flow during one cycle helps using well known controller design tools of the control theory for the intersection signal control problem (Diakaki *et al.*, 2002). In this case, the resulting controller can do better for saturated traffic conditions. But if the network includes some unsaturated intersections, the designed controller may not well as a result of the average flow assumption during one cycle. If the average flow is known between red-green signal transitions rather than one cycle, then it will increase the performance of the controller for unsaturated intersections. But, in this case it will be almost impossible to apply standard control theory tools because of the resulting nonlinear model. On the other hand, some constraints related to the traffic signal control problem should also be handled. Model predictive control presents the capability of handling the nonlinear model as well as some constraints of the system. It is an increasing popular control approach because of its use of a possible nonlinear control model and ability to handle constraints on inputs, states and outputs (Rawlings, 2000).

In the following, the queue model and some constraints related to the traffic signal control problem are combined in the model predictive control formulation to calculate the green time signals at the beginning of the kth cycle.

$$\min_{\substack{t_i(k,N)\\ i=1,2,3,4}} \sum_{p=1}^{N}\sum_{j=1}^{8} w_j q_j(k+p)$$

s.t.

$$q_1(k+p+1)=\max[q_1(k+p)+t_1(k+p)f_1^{t_1(k+p)}-t_1(k+p)c_1,0] +t_2(k+p)f_1^{t_2(k+p)}+t_3(k+p)f_1^{t_3(k+p)}+t_4(k+p)f_1^{t_4(k+p)}$$

$$q_2(k+p+1)=\max[q_2(k+p)+t_1(k+p)f_2^{t_1(k+p)}+t_2(k+p)f_2^{t_2(k+p)} -t_2(k+p)c_2,0]+t_3(k+p)f_2^{t_3(k+p)}+t_4(k+p)f_2^{t_4(k+p)}$$

$$q_3(k+p+1)=\max[q_3(k+p)+t_1(k+p)f_3^{t_1(k+p)}+t_2(k+p)f_3^{t_2(k+p)} +t_3(k+p)f_3^{t_3(k+p)}-t_3(k+p)c_3,0]+t_4(k+p)f_3^{t_4(k+p)}$$

$$q_4(k+p+1)=\max[q_4(k+p)+t_1(k+p)f_4^{t_1(k+p)}+t_2(k+p)f_4^{t_2(k+p)} +t_3(k+p)f_4^{t_3(k+p)}+t_4(k+p)f_4^{t_4(k+p)}-t_4(k+p)c_4,0]$$

$$q_5(k+p+1)=\max[q_5(k+p)+t_1(k+p)f_5^{t_1(k+p)}-t_1(k+p)c_5,0] +t_2(k+p)f_5^{t_2(k+p)}+t_3(k+p)f_5^{t_3(k+p)}+t_4(k+p)f_5^{t_4(k+p)}$$

$$q_6(k+p+1)=\max[q_6(k+p)+t_1(k+p)f_6^{t_1(k+p)}+t_2(k+p)f_6^{t_2(k+p)} -t_2(k+p)c_6,0]+t_3(k+p)f_6^{t_3(k+p)}+t_4(k+p)f_6^{t_4(k+p)}$$

$$q_7(k+p+1)=\max(q_7(k+p)+t_1(k+p)f_7^{t_1(k+p)}+t_2(k+p)f_7^{t_2(k+p)} +t_3(k+p)f_7^{t_3(k+p)}-t_3(k+p)c_7,0)+t_4(k+p)f_7^{t_4(k+p)}$$

$$q_8(k+p+1)=\max(q_8(k+p)+t_1(k+p)f_8^{t_1(k+p)}+t_2(k+p)f_8^{t_2(k+p)} +t_3(k+p)f_8^{t_3(k+p)}+t_4(k+p)f_8^{t_4(k+p)}-t_4(k+p)c_8,0)$$

for $p=0,1,\ldots,N-1$

$$\sum_{i=1}^{4} t_i(k+p)=C\,,\ T_{\min}\le C\le T_{\max}\,,$$

$$t_{i\_\min}\le t_i(k+p)\le t_{i\_\max}\,, i=1,2,3,4$$

for $p=0,1,\ldots,N-1$,

$$q_j(k+p)\le\alpha_j\,,\ j=1,2,\ldots,8\,,\ p=1,2,\ldots,N$$

where C , $T_{\min}$ and , $T_{\max}$ represent cycle time, lower and upper bound for the cycle time respectively. The term $t_i(k,N)$ is the green signal time variable of each lane over the N-step horizon. It can be written as $t_i(k,N):=[t_i(k)\quad t_i(k+1)\quad \ldots\quad t_i(k+N)]^T$, $i=1,2,3,4$. This model calculates the green signal times for N-step horizon, $t_i(k,N)$, at the beginning of the kth cycle, but only apply the first value, $t_i(k)$. For each step of the N-step horizon, the summation of green times is equal to the cycle time, which may vary between a lower and upper value. Each green time value also has a minimum ($t_{i\_\min}$) and maximum value ($t_{i\_\max}$) which is fixed over the horizon. The sum of the minimum green time values are assumed to be equal or less than $T_{\max}$, also the sum of the maximum green time values are assumed to be equal or greater than $T_{\min}$ for feasibility concerns. The queue length of each lane is constrained by α_j $j=1,2,\ldots,8$ to avoid spillback for multi-intersection case.

The objective function includes weighting parameters w_j assigned to the each lane. They have a default value of $w_j=1$ in which the objective becomes minimizing the total queue length. The optimization objective can be changed by selecting different weighting parameters w_j regarding different criteria's; maximum or average delay of that lane, priority of the lane, emergency vehicle passing etc.

The resulting nonlinear programming problem described above is solved using the sequential quadratic programming algorithm, implemented by the MATLAB function *fmincon*.

2.3 Simulations for one intersection

In this sub section, the control method described above is applied to control of the single intersection. The initial queue lengths and incoming flows between 100-200 seconds are given as:

$q_1(0)=5$, $q_2(0)=65$, $q_3(0)=34$, $q_4(0)=80$, $q_5(0)=5$, $q_6(0)=20$, $q_7(0)=6$, $q_8(0)=12$,

$f_j=0.1\,veh/\sec$ for $j=1,3,5,7$

$f_j=0.2\,veh/\sec$ $j=2,4,6,8$

The capacity of lanes are assumed to be fixed and given

$c_1 = 0.25$ veh/sec, $c_2 = 0.5$ veh/sec, $c_3 = 0.25$ veh/sec, $c_4 = 0.5$ veh/sec, $c_5 = 0.25$ veh/sec, $c_6 = 0.5$ veh/sec, $c_7 = 0.25$ veh/sec, $c_8 = 0.5$ veh/sec,

The cycle time is fixed to 90 secs (equal lower and upper bound). The minimum green times are assumed as follows: $t_{i\_\min} = 10\,\text{sec}$ for $i=1,3,5,7$, $t_{i\_\min} = 15\,\text{sec}$ for $i=2,4,6,8$. Maximum queue lengths and the predicting future step are selected as $\alpha_j = 100$ and N=3 respectively.

The simulation results are given in Fig. 2.a-b. The queue lengths versus time results are shown in Fig. 2.a, and the duration of the green signal times at each cycle versus time are shown in Fig. 2.b.

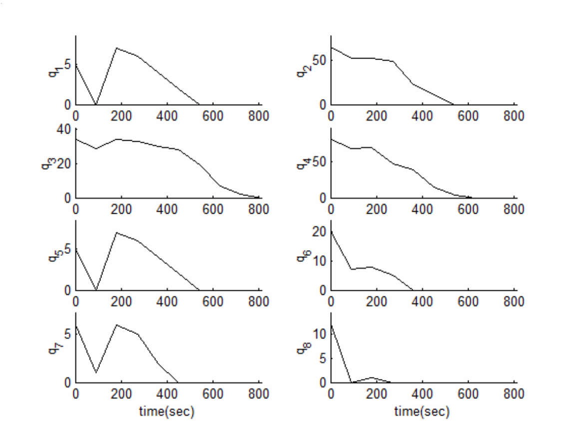

Fig. 2.a Queue lengths versus time at each lane

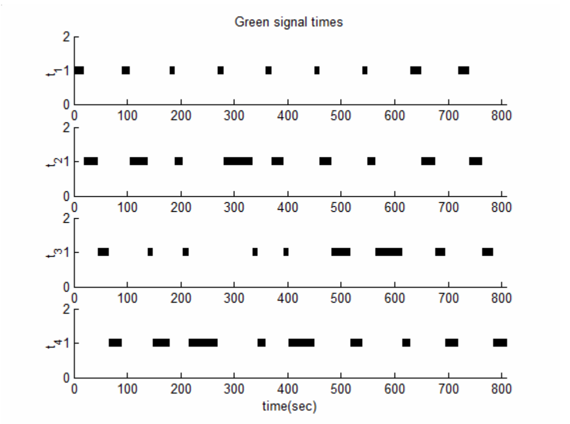

Fig. 2.b The Green signal times

In the second simulation, an emergency vehicle is assumed to pass from the lane 4 for the same conditions in Fig. 2. So we increase only the weighting factor of this lane to $w_4 = 20$. Fig. 3.a-b shows the queue lengths and the green signal times respectively. Notice that the queue length in the lane 4 gets zero faster (compare Fig. 2.a and Fig 3.a) and more green time is assigned to that lane in the starting cycles (compare fourth column of Fig. 2.b and Fig. 3.b).

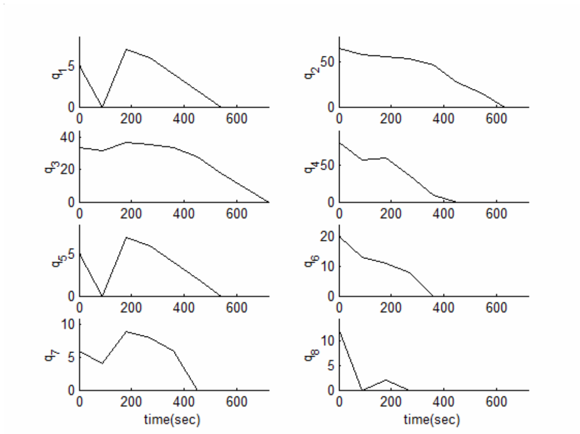

Fig. 3.a Queue lengths versus time at each lane

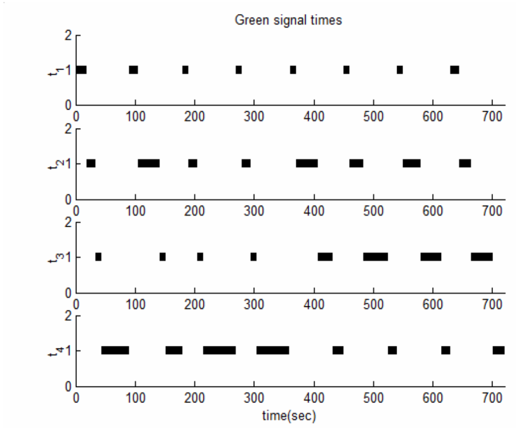

Fig. 3.b The Green signal times

The response of the proposed signal controller to the various test conditions have also been evaluated with the simulations. Due to page restrictions they are not given here. In the following section, the proposed controller is extended to multi-intersection case.

3. DECENTRALIZED MULTI-INTERSECTION TRAFFIC SIGNAL CONTROL

The single intersection traffic signal controller proposed in the previous section is able to calculate its signal times considering the adjacent intersections.

For the simplicity in presentation consider the arterial (Fig. 4). Let us redefine the variables for the multi intersection case using an upper index r which denote the intersection number. So, L_j^r and q_j^r represents the lane numbers and the queue length of rth intersection respectively. The weighting parameters, incoming flows, the lane capacities, and the maximum queue length (or spillback constraint) of the lane j of rth intersection becomes w_j^r, f_j^r, c_j^r, and α_j^r respectively.

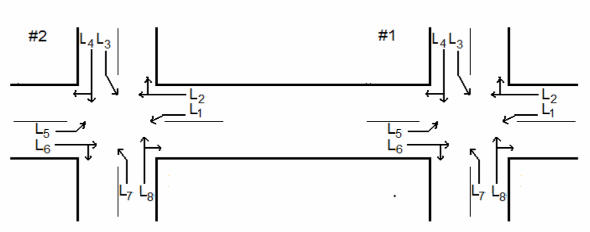

Fig. 4. Arterial road with two intersections

In the multi intersection case, the traffic signal controllers at each intersection consider the incoming flows of the upstream and congestion status of downstream intersections.

The incoming flow of each lane may be measured until a point near to adjacent intersection. For example, the incoming flow to lane one and two of intersection-two, $f^2_{1,2}$, may be measured by placing a device at the outgoing lane of the intersection-one directed to the intersection-two. This device may measure the flow value until an offset time which is related to distance between intersections. Providing the signal time values, the queue lengths, and the lane capacities of the intersection-one for next coming cycle also help predicting the incoming flow for some additional time. So, the incoming flow information may become a combination of measurement and prediction in the multi intersection case.

On the other hand, the single intersection signal controller may also look at the queue length of the downstream intersections to calculate the signal times. For example assume that the queue lengths of lanes $L^2_{1,2}$ (in intersection two) reach up to a threshold value but less than the critical value $\alpha^2_{1,2}$. Then, the signal controller in intersection-one may take care of this situation by increasing weight factors (w^1_j) of other directions that do not towards to intersection-two to avoid. Or, it may decrease the green signal time lower and upper bounds on this direction depending on downstream congestion status

Simulations for Multi intersection case

The simulation conditions other than flows are kept similar to the single intersection simulation case. External incoming flows values of 0.1 veh/sec and 0.5 veh/sec for the lane 1 and the lane 2 of intersection-one are assumed respectively between 0-120 seconds.

The offset time between intersections is assumed to be 60 seconds. Considering the green flow for unsaturated case, each intersection makes its own plan with a difference of 60 seconds. So, the signal controller in intersection-one makes plan at time t=0, and 60 seconds later the signal controller in intersection-two makes its own plan. Since the simulator is also able to calculate measure-estimate incoming flows of the lanes, the signal controller in intersection-two makes a plan using the measurement of flow for next 60 seconds and the estimation for remaining 30 seconds for $f^2_{1,2}$. The latter flow is estimated by passing the signal times of the intersection-one to intersection-two.

Under the conditions described above, the solid lines in the Fig. 5 and Fig. 6 show the queue lengths versus time graph of intersection one and two respectively.

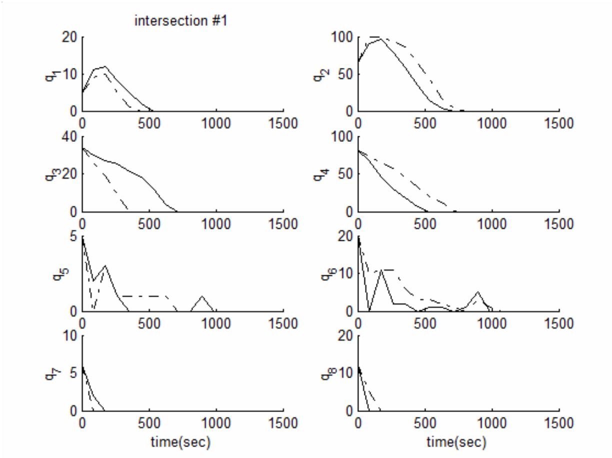

Fig. 5. Queue lengths versus time for intersection 1

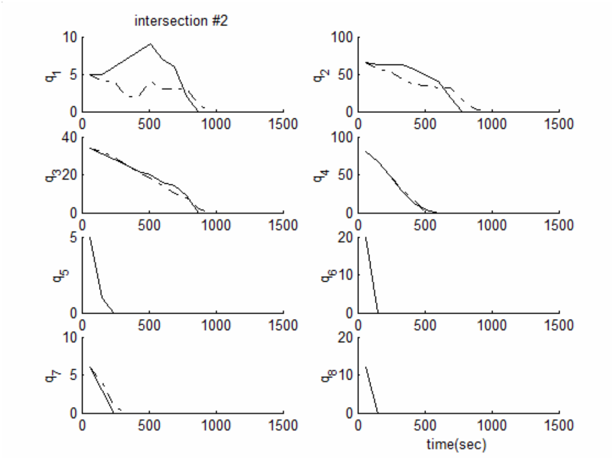

Fig. 6. Queue lengths versus time for intersection 2

Assume that the spillback threshold values of intersection-two are reached for the lane 1 and 2. Then set the weighting parameters of intersection-one as $w^1_j = 20$, $j = 1,3,5$. The dashed lines in Fig. 5 and Fig. 6 show the effect of these new weighting parameters. As seen from the figure (Fig. 6), the queue lengths for the lane 1 and lane 2 of intersection-two do not increase so much but gets zero later. But in this case the queue length of lane 2 of intersection-one reach up to its maximum value (Fig. 5). It is obvious that signalization and routing may be considered together (Khorrami and Ozguner, 1984) to handle downstream congestion problem in a better way.

4. CONCLUSIONS

A decentralized control is utilized for the traffic signal control of multi intersection case considering the local interactions of the intersections. The proposed signal controller considers not only the saturated traffic conditions but also the unsaturated traffic conditions by assuming non-fixed average incoming flow during a cycle. In the future work, the weighting parameters of the intersection signal controller may be selected more intelligently to handle the decentralized control in a better way.

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