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Off-line robustification of explicit control laws

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Abstract: The paper deals with the predictive control for linear systems subject to constraints, leading to piecewise affine control laws. The main goal is to reduce the sensitivity of these schemes with respect to the model uncertainties. This objective can be attained by considering worst-case (min-max) formulations, but generally this is leading to fastidious on-line optimisation which may reduce the range of application. Here a two stage predictive strategy is proposed, which synthesize in a first instant an analytical (continuous and piecewise linear) control law based on the nominal model and secondly robustify the central controller (the controller obtained when no constraint is active). This robustification is then expanded to all the space of the piecewise structure by means of its corresponding disturbance model.

1. INTRODUCTION

The model predictive control (MPC) laws are optimization based techniques which allows constraints handling from the design stage. At each sampling time a finite horizon optimal control problem has to be solved. The analytical formulation of the optimum and its on-line evaluation avoids the important computational effort required for the real-time implementation. Solutions in this direction exist at least for two important classes of problems (linear and quadratic cost functions) subject to linear constraints due to the Abadie constraint qualification (Goodwin *et al.*, 2004). It must be said that these are in fact a part of a larger class of multiparametric convex programs (Bemporad *et al.*, 2002b) for which exact or approximate algorithms exist (Grancharova and Johansen, 2005; Bemporad and Filippi, 2006, Olaru and Dumur, 2004).

In the case of robust predictive control laws, the model uncertainties and the disturbances can be taken into account at the design stage. A popular methodology in this direction is the one based on a min-max criterium (in the case when the extreme combination of disturbances or uncertainties are known) (Kerrigan and Maciejowski, 2004; Bemporad *et al.*, 2002a; Olaru and Dumur, 2007) which comes finally to the resolution of a single multiparametric linear program. The structure of this ultimate optimization is however quite complex and large prediction horizons cannot be handled due to the exponential growth of disturbances realization to be taken into account. If the exact computation of explicit solutions is prohibitive, the construction of approximations can be an interesting alternative (Grancharova and Johansen, 2008).

In a slightly different manner by constructing an estimation mechanism (Goodwin *et al.*, 2004) for the constrained variables, one can obtain a robust control structure, but the multiparametric optimization remains intricate.

In (Olaru and Rodriguez-Ayerbe, 2006) it was presented a first study regarding the possible robustness improvement for the explicit affine feedback policy constructed upon predictive control strategy for linear systems. The simplest way to proceed is to consider an observer of the state variables (Goodwin *et al.*, 2004). With an observer, the dimension of the state space is kept, and so the piece-wise structure of controller does not change, and the same observer can be used in each region. The observer can also be viewed as a noise characterization for the prediction model. Nevertheless, the observer does not allow reaching all the space of stabilizing controllers.

The present paper presents an improved result based on the Youla-Kučera parametrization which span the space of stabilizing controllers. For a two-degree of freedom controller, one has access to all the stabilizing controllers that preserve the same input/output behavior, so the Youla-Kučera parameter offers more degrees of freedom than the use of an observer.

The robustification is made such that the state space dimension of the controller is augmented. The direct consequence is that the use of a same parameter in each region is not possible. The continuity between critical regions can be lost with severe degradation in stability and performances. The main contribution here is the reconstruction of the noise model induced by the Youla-Kučera parameter for the unconstrained case, and its use for the generation of the robust piece-wise controller corresponding to the constrained MPC case.

In the following, section 2 briefly recalls the constrained MPC control and section 3 the explicit solution to the associated mutiparametric optimisation problem. Section 4 the output control of the obtained piecewise controller and this robustification. Numerical examples are presented in section 5 and the final conclusions in section 6.

2. CONSTRAINED MPC FORMULATION

The design of a predictive control law is based on the existence of a numerical model of the system to be controlled. Within the general framework of linear systems theory, consider the following state space model:

$$x_{t+1} = Ax_t + Bu_t, \quad t \in Z^+ \tag{1}$$

with $x_t \in \Re^n$ the state vector at time *t*, $u_t \in \Re^m$ the control vector at time *t, A* and *B* matrices of adequate dimensions and the pair (A,B) assumed to be stabilisable.

At each sampling time, the current state vector (assumed to be measurable) $x_t = x_{t|t}$ is used to elaborate the open loop optimal control sequence $\mathbf{k}_u^*$:

$$\mathbf{k}_u^* = \begin{bmatrix} u_{t|t}^{\mathrm{T}} & \cdots & u_{t+N-1|t}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{2}$$

according to the following cost function:

$$\mathbf{k}_u^* = \arg\min_{\mathbf{k}_u} \left\| P\, x_{t+N|t} \right\|_p + \sum_{k=1}^{N-1} \left\{ \left\| Qx_{t+k|t} \right\|_p + \left\| R\, u_{t+k|t} \right\|_p \right\} \tag{3}$$

where $\| \; \|_p$ represents the norm $p = \{1, 2, \infty\}$ and the pair (Q, A) is assumed to be detectable. The prediction horizon N, the weighting terms $Q = Q^{\mathrm{T}} \geq 0$, $R = R^{\mathrm{T}} > 0$ and the final cost defined by $P = P^{\mathrm{T}} \geq 0$ are the tuning knobs of the control law.

The optimization of this cost function is performed subject to constraints imposed by the system dynamic, to functional constraints and to terminal or stability constraints:

$$\begin{cases} x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} \quad k \geq 0 \\ Cx_{t+k|t} + Du_{t+k|t} \leq \gamma, \\ 0 \leq k \leq N,\, x_{t+k|t} \in X,\, u_{t+k|t} \in U,\, x_{t+N|t} \in X_N \\ X_N = \left\{ x \in \Re^n \middle| L_{in} x \leq l_{in};\, L_{eq} x = l_{eq} \right\} \text{terminal domain} \end{cases} \tag{4}$$

All constraints previously mentioned are of polyhedral type, described by systems of linear equalities and inequalities. The finite set of constraints (4) can be restructured to obtain a formulation which will be directly usable by the optimization routine. Indeed, the predicted state vector can be written in a compact form:

$$\mathbf{x} = \begin{bmatrix} x_{t+1/t} \\ x_{t+2/t} \\ \vdots \\ x_{t+N/t} \end{bmatrix} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x + \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix} \mathbf{k}_u \tag{5}$$

with $x_{t|t} = x$, which enables to structure the optimization problem (3)-(4) by the relations:

- Case $p = 2$:

$$\mathbf{k}_u^* = \arg\min_{\mathbf{k}_u} 0{,}5\, \mathbf{k}_u^{\mathrm{T}} H \mathbf{k}_u + x^{\mathrm{T}} F\, \mathbf{k}_u$$
$$\text{subject to}: \begin{cases} A_{in}\mathbf{k}_u \leq B_{in}x + b_{in} \\ A_{eq}\mathbf{k}_u = B_{eq}x + b_{eq} \end{cases} \tag{6a}$$

- Case $p = 1, \infty$:

$$\mathbf{z}^* = \arg\min_{\mathbf{z}} c^{\mathrm{T}}\mathbf{z}$$
$$\text{subject to}: \begin{cases} A_{in}\mathbf{z} \leq B_{in}x + b_{in} \\ A_{eq}\mathbf{z} = B_{eq}x + b_{eq} \end{cases} \tag{6b}$$

with $\mathbf{z} = \{\mathbf{k}_u; \xi_1, \ldots, \xi_{N_\xi}\}$ and $\xi_1, \ldots, \xi_{N_\xi}$ auxiliary variables, the number N_ξ of these variables depending on the optimization horizon and the prediction model (Zadeh and Whalen 1962).

For both cases (6a)-(6b), the optimal argument includes the control sequence $\mathbf{k}_u^*$. Only the first part of this sequence is applied effectively to the system input, the complete procedure is iterated again at the next sampling time according to the receding horizon principle. Real time implementation is usually performed through on-line optimization procedures (linear or quadratic programming) in order to determine the optimum corresponding to a particular value of the state vector *x*.

3. EXPLICIT SOLUTION

Another approach considers (6) as a multiparametric optimization problem (see Pistikopoulos et al, 2007 for a review of the control problems under these framework) and describes the optimal solution as an explicit function of the state vector x :

$$f : \Re^n \to \Re^m \text{ so that } u_t^{MPC} = f(x) \tag{7}$$

Real time implementation is reduced in this case to the evaluation of this function.

Regarding the structure of the multiparametric problems it can be observed that the feasible domain is represented by a parameterized polyhedron. If bounded, then the optimum is given by a convex combination of parameterized vertices. If the optimal solution is not unique (usually the case of linear cost functions (6b)), the explicit solution is equiavalent to a point to set mapping (Olaru and Dumur, 2006b), and the continuity of the solution must be a crucial criterion when implementing the solution. Indeed, a continuous control law $u_t^{MPC} = f(x_t)$ avoids discontinuous variations on the control in case of disturbances appearing on the state vector.

The use of a dual representation of the feasible domain and projection mechanisms (see Olaru and Dumur, 2004 and 2005 for details) provides an insight on the topology of the optimisation problems and can be advantageous if there exist unbounded directions due to the fact that the generators representation offers the right tool for their description as well as for the control of the constraints redundancy.

Once the explicit solution of (6a or 6b) is obtained, we dispose of an analytic description of the control law (7). Several studies were dedicated to the piecewise affine characterisation (Bemporad *et al.*, 2002b; Seron *et al*, 2003; Olaru and Dumur, 2004; Mare and De Dona, 2005):

Indeed, the *explicit* predictive control law is described by a collection of piecewise affine function:

$$u_t^{MPC} = f(x_t) = \begin{cases} L_1 x_t + l_1 & if \quad x_t \in R_1 \\ \cdots & \cdots \\ L_k x + l_k & if \quad x_t \in R_k \\ \cdots & \cdots \end{cases} \tag{8}$$

with R_k polyhedral critical regions covering feasible states.

The structure of such a piecewise controller is shown in Figure 1. Once the look-up table of local laws is available, an efficient positioning mechanism (based on a search tree) can be constructed such that the on-line evaluation routine can find the optimal control action (TØndel *et al*., 2003). The effective implementation follows the scheme in Figure. 1.

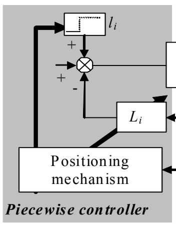

Fig. 1. Piecewise formulation for the MPC law under constraints

4. OUTPUT FEEDBACK - ROBUSTIFICATION

4.1 Observer based robustification

When the state x is not accessible, the simplest way is to obtain an estimate estate $\hat{x}$ with an observer (Goodwin *et al.* 2004). For a prediction observer we have:

$$\hat{x}_{t+1} = (A - KC)\hat{x}_t + Bu_t + Ky_t \tag{9}$$

while for an estimation observer:

$$\begin{aligned} \hat{x}_{t+1/t} &= (A - K_1 C)\hat{x}_{t/t-1} + Bu_t + K_1 y_t \\ \hat{x}_{t/t} &= (I - K_2 C)\hat{x}_{t/t-1} + K_2 y_t \end{aligned} \tag{10}$$

The gain K or (K_1, K_2) are allocating the observer poles. As is well known, the observer do not changes the tracking behaviour but modifies the disturbance rejection, so this gain can be used to robustify a given stabilizing controller. With the prediction observer (9), the obtained controller corresponds to:

$$\begin{aligned} \hat{x}_{t+1} &= (A - KC - BL)\hat{x}_t + Ky_t \\ u_t &= -L\hat{x}_t \end{aligned} \tag{11}$$

4.2 Youla-Kučera parameter based robustification

Youla-Kučera or Q parameter can be used to robustify the initial controller when no constraints are considered (Kouvaritakis *et al.* 1992). The advantage of this method is that we have access to the space of all stabilizing controllers. For a state space controller, the parametrization can be obtained by following the lines in (Boyd and Barratt 1991).

The estimation error $y'_t = y_t - C\hat{x}_t$ is used to obtain an additional signal u'_t, as shown in figure 2. The Q parameter corresponds to a stable system defined by A_Q, B_Q, C_Q, D_Q.

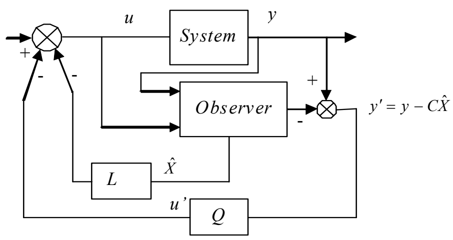

Fig. 2. State-space controller with the Q parameter

We have the following relations:

$$\begin{aligned} u_t &= -L\hat{x}_t - u'_t \\ u'_t &= C_Q x_{Q_t} + D_Q y'_t \\ X_{Q_{t+1}} &= A_Q x_{Q_t} + B_Q y'_t \\ y'_t &= y_t - C\hat{x}_t \end{aligned} \tag{12}$$

This corresponds to the following controller:

$$\begin{bmatrix} \hat{x}_{t+1} \\ X_{Q_{t+1}} \end{bmatrix} = \begin{bmatrix} A - KC - BL - BD_Q C & -BC_Q \\ -B_Q C & A_Q \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ X_{Q_t} \end{bmatrix} + \begin{bmatrix} K - BD_Q \\ B_Q \end{bmatrix} y_t \tag{13}$$

$$u_t = \begin{bmatrix} -L + D_Q C & -C_Q \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ X_{Q_t} \end{bmatrix} - D_Q y_t$$

As mentioned before, this kind of robustification disposes of more degrees of freedom than a robustification obtained through the use of an observer. It permits to access the entire space of stabilizing controllers and provides stability robustness while preserving nominal performance specifications. From the practical point of view, its construction can use classical optimization techniques, see (Rodriguez and Dumur, 2005; Rossiter 2003; Ansay *et al*., 1998; Yoon and Clarke, 1995; Kouvaritakis *et al*., 1992).

However, this method can not be applied to robustify the piecewise controller (8), as it is the case with observers presented in section 4.1. The raison is that the Q parameter increases the dimension of the controller and the continuity at the switching between the local affine laws of the piecewise controller is lost.

The idea developed in this paper is to obtain the disturbance model corresponding to the Q parameter and use this disturbance model to further regenerate the piecewise controller.

Note that the disturbance model allows us to reach the same unconstrained controller as the central controller in (13) obtained with the Q parameter. That is: the unconstrained controller for the initial model augmented with the disturbance model should be the same as the controller obtained with the initial model and the Q parameter.

The considered augmented model is:

$$\begin{cases} x_{t+1} = Ax_t + Bu_t + Ke_t \\ x_{v\,t+1} = A_v x_{v\,t} + B_v e'_t \end{cases} \text{and} \begin{cases} e_t = C_v X_{v\,t} + e'_t \\ y_t = Cx_t + e_t \end{cases} \tag{14}$$

With e'_t zero mean white noise. In this case the model becomes:

$$\begin{bmatrix} x_{t+1} \\ x_{v\,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & KC_v \\ 0 & A_v \end{bmatrix}}_{A_e} \begin{bmatrix} x_t \\ x_{v\,t+1} \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u_t + \underbrace{\begin{bmatrix} K \\ B_v \end{bmatrix}}_{K_e} e'_t$$
$$y_t = \underbrace{\begin{bmatrix} C & C_v \end{bmatrix}}_{C_e} \begin{bmatrix} x_t \\ x_{v\,t+1} \end{bmatrix} + e'_t \qquad (15)$$

This corresponds to an innovation representation, and K is the observer gain of the initial observer. The system is partially controllable, A_v being non controllable but observable. As the considered cost function (3) has a terminal constraint, the unconstrained controller will correspond to the infinity horizon optimal controller. This controller can be obtained solving the following Riccati equation:

$$P = A_e^T P A_e - (A_e^T P B_e)(B_e^T P B_e + R)^{-1} (A_e^T P B_e)^T + Q_r \qquad (16)$$

The controller gain is:

$$L_e = (B_e^T P B_e + R)^{-1} B_e^T P A_e \qquad (17)$$

And the controller considering a predictor observer is:

$$\hat{x}_{e\,t+1} = (A_e - K_e C_e - B_e L_e)\hat{x}_{e\,t} + K_e y_t$$
$$u_t = -L_e \hat{x}_{e\,t} \qquad (18)$$

The observer gain corresponds to K_e, as (15) is an innovation representation.

The problem is then to find A_v, B_v, C_v in order to have $(18) \equiv (13)$. This equivalence can be developed partitioning P, the solution of the Riccati equation (16), as the partition of A_e, $P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{pmatrix}$. Considering the weighting factor $Q_r = [C \;\; C_v]^T [C \;\; C_v]$, in order to ponder the output of the system, equation (16) becomes:

$$P_{11} = A^T P_{11} A - (A^T P_{11} B)(B^T P_{11} B + R)^{-1} (A^T P_{11} B)^T + C^T C$$
$$P_{12} = (A - BL)^T P_{12} A_v + (A - BL)^T P_{11} K C_v + C^T C_v \qquad (19)$$

And:

$$L_e = \begin{bmatrix} L & L_v \end{bmatrix} = (B^T P_{11} B + R)^{-1} B^T \begin{bmatrix} P_{11} A & P_{12} A_v + P_{11} K C_v \end{bmatrix} \qquad (20)$$

First equation of (19) corresponds to the Riccati equation of initial system, so L is the same as the one of the initial controller. Considering the equivalence $(18) \equiv (13)$ we can remark, that in the case when a prediction observer (9) is used, D_Q must be zero, because y_t is not used to estimate x_t. This imposes a structural constraint on the Q parameter. We can also remark, that with $D_Q = 0$ we can impose $Bv = B_Q$. After some developments, the following equations must be verified for the A_v, C_v, P_{12} matrices:

$$A_Q = A_v - B_v C_v$$
$$C_Q = (B^T P_{11} B + R)^{-1} B^T (P_{12} A_v + P_{11} K C_v) \qquad (21)$$
$$P_{12} = (A - BL)^T P_{12} A_v + (A - BL)^T P_{11} K C_v + C^T C_v$$

The equations (21) can be practically solved using optimization techniques in order to obtain the unknowns A_v, C_v, P_{12}. Interesting aspects related to the existence, the unicity and in general to the analytical solution of (21) are subject of ongoing research. Finally, the obtained extended model can be used to regenerate de piecewise controller obtained with the constrained MPC formulation.

6. EXAMPLE

Consider the position control of an induction motor. A simple model of an induction motor, between torque and position, for 1.0724 ms sampling period is:

$$H(q^{-1}) = \frac{y(t)}{u(t)} = \frac{\theta(t)}{\tau_{\text{ref}}(t)} = \frac{10^{-4}(0.821q^{-1} + 0.8206q^{-2})}{(1 - q^{-1})(1 - 0.998q^{-1})} \qquad (22)$$

Constraints in control amplitude are considered: $\tau_{\max} = 1.8$ and $\tau_{\text{ref}} \in [\tau_{\max}, -\tau_{\max}]$. To cancel steady-state errors towards load disturbances an integral action $u_t = u_{t-1} + \Delta u_t$ is added. The following estate space formulation is obtained:

$$x_{t+1} = A x_t + B \Delta u_t$$
$$y_t = C x_t$$

With:

$$A = \begin{bmatrix} 1.998 & -0.998 & 0.015625 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.015625 \\ 0 \\ 1 \end{bmatrix} \qquad (23)$$
$$C = \begin{bmatrix} 0.0052544 & 0.00525184 & 0 \end{bmatrix}$$

An initial MPC controller is designed with $Q = C^T C$ and $R = 0.001$. P matrix corresponds to the infinite horizon solution obtained with Q and R. The controller obtained with this infinity horizon without constraints is:

$$L = [6.807 \quad -5.992 \quad 0.397] \qquad (24)$$

The biggest positive invariant polyhedral set has been considered as terminal constraint. This can be obtained with (23) and the considered input constraints by using the maximal output admissible sets (Gilbert and Tan, 1991). This polyhedral set is shown in figure 3.

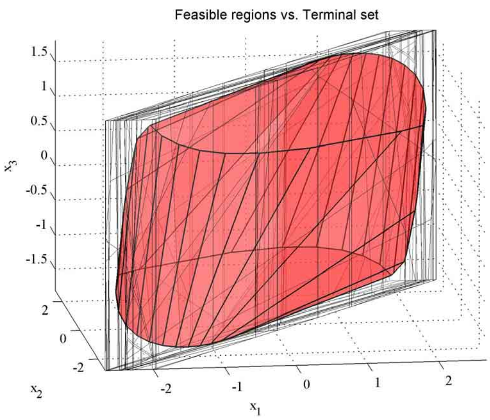

Fig. 3. Terminal set and the initial piecewise MPC controller regions.

Choosing N = 7 as prediction horizon in (3), with considered constrains, that is, constrains in the input $|u| < \tau_{\max}$ and in the terminal set shown in figure 3, a piecewise controller composed of 102 linear control laws is obtained solving the multiparametric optimization corresponding to the MPC problem (the central controller corresponding to (24)). The regions of this initial controller are shown in figure 3, in relation to the terminal set.

An observer as (9) has been calculated in order to place the poles of the observer at zero. As any disturbance is considered, this choice for the poles of the observer assures the fastest dynamic of the observer. The gain is:

$$K = [404.1 \quad 166.5 \quad 6091.6]^T \tag{25}$$

This initial central controller has been robustified by the means of the Q parameter. A parameter to improve the stability robustness towards additive uncertainty has been synthesised. We have found:

$$A_Q = \begin{bmatrix} 2.96 & -1.22 & 0.75 \\ 2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \quad B_Q = \begin{bmatrix} 128 \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

$$C_Q = \begin{bmatrix} -29.1 & 24.05 & -20.77 \end{bmatrix} \quad D_Q = 0$$

This parameter has been synthesised using techniques described in (Rodriguez and Dumur, 2005), and forcing $D_Q = 0$. As mentioned before, the synthesis of this robustification has been described by several authors, see (Kouvaritakis *et al.*, 1992; Yoon and Clarke, 1995; Ansay *et al.*, 1998; Rossiter 2003). Solving (21), with $Bv = B_Q$ we found:

$$A_v = \begin{bmatrix} 1.3 & -0.105 & -0.217 \\ 2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \quad C_v = \begin{bmatrix} -0.0109 \\ 0.00877 \\ -0.0075 \end{bmatrix}^T \tag{27}$$

This model is added to (23), and we obtain a representation as (15). With this new model the multiparametric optimization is solved, and a new piecewise controller composed of 1311 linear control laws is obtained. The central controller corresponds to: $Ln = [L \quad -29.105 \quad 24.05 \quad -20.77]$ and $Kn = [K \quad 128 \quad 0 \quad 0]^T$.

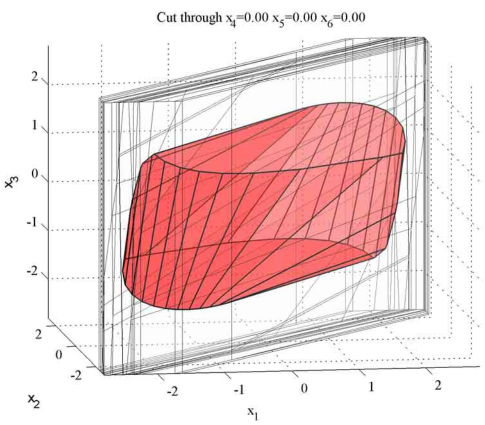

Fig. 4. Regions of piecewise robustified controller.

Figure 4 shows the obtained regions of the robustified controller in relation to the terminal set. The terminal set of this controller is of dimension 6, but as the first three variables are the same as in the initial controller, the terminal set with respect to these variables will correspond, as it can be see in figure 4. In this figure, we can also observe that the stabilize region with respect to these three variables has been augmented. This can be interpreted by means of a filtering of the control signal which avoids the constraints activation and ultimately, the infeasibility.

Figures 5 and 6 show the obtained simulations results considering a neglected dynamic in high frequency of the following characteristics $\omega_0 = 1000 rad/s \quad \xi = 0.3$ With this neglected dynamics, the plant representation is:

$$x_{t+1} = A_p x_t + B_p u_t$$
$$y_t = C_p x_t$$

With:

$$A_p = \begin{bmatrix} 2.775 & -0.7838 & 0.486 & -0.58 \\ 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} \quad B_p = \begin{bmatrix} 0.0078 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

$$C_p = \begin{bmatrix} 0.0008757 & 0.002044 & 0.00182 & 0.000632 \end{bmatrix}$$

The convergence for an initial state x_{ini} as (29) obtained with both controllers, are shown in figures 5 and 6:

$$x_{ini} = 0.15 \begin{bmatrix} 0.1 & -0.05 & 0 & 0.1 \end{bmatrix}^T \tag{29}$$

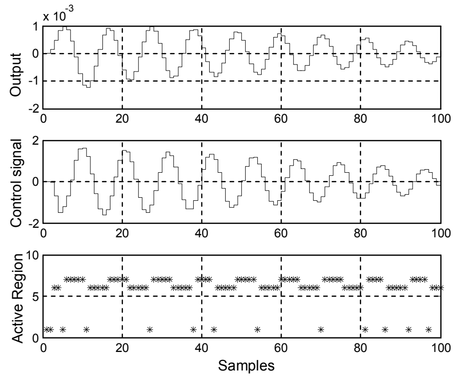

Fig. 5. Results for initial controller and uncertain model.

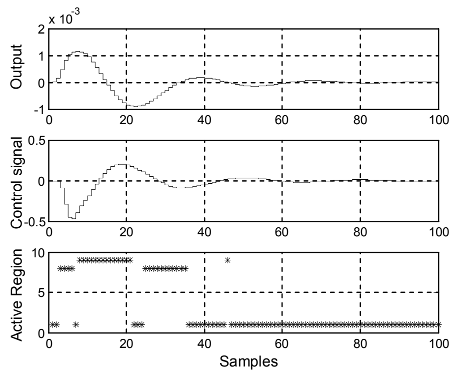

Fig. 6. Results for robustified controller and uncertain model.

In these figures we can observe the stable behaviour of the robustified controller and a dangerous sensitivity of the initial controller which may lead to infeasibility. The robustified controller has better behavior with respect to the uncertainties in high frequency. The continuity between the local linear controllers of the general piecewise affine control law is guaranteed.

6. CONCLUSIONS

The paper investigated the robustification methods for the control laws obtained in a constrained predictive control framework. The idea is to design in a first instance a piecewise controller which satisfies the basic demands in terms of tracking performances. In a second stage, the same predictive control structure (prediction horizon, weightings, etc.) is robustified using the model arguments accounting for the noise influence. The idea is similar to that of using a fixed observer, but exploring all the space of stabilizing controllers of the unconstrained system. This increases the number of degrees of freedom.

The robustification of initial unconstrained controller is made using the Youla-Kučera parametrization, and then this robustification is expanded to all the piecewise structure of the controller. For this, the disturbance model corresponding to the Youla-Kučera parameter is found, and use to regenerate the piecewise controller by preserving the same input/output behavior but with an increased robustness.

The limitations of the method are in the existence of the corresponding disturbance model of the Youla-Kučera parameter. This is transparent in the resolution of a non linear equation system. The robustification being done off-line, any infeasibility can be handled by retuning the MPC parameters.

From another point of view, the approach can be seen as an extension of the robustification methods for linear systems to the control laws under constraints.

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