Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Design and Real Time Implementation of a Fuzzy Tuned H_∞ Estimator in a Low Cost AHRS

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Abstract: In this paper a Fuzzy Tuned extended H_∞ (FTH_∞) estimator is designed and implemented in an Attitude Heading Reference System (AHRS), which is specialized to vehicular applications. In AHRS, 3-axis accelerometers are allotted to measure the earth's gravity field vector and then to update the roll and pitch angles obtained from gyros' dynamic. Therefore, the AHRS on an accelerated vehicle will be affected by large disturbances. Additionally, in ground vehicles, the measurements of 3-axis magnetometers are corrupted by both soft and hard iron time varying disturbances. The FTH_∞ estimator is an extended H_∞ filter in which only noise and attenuation bounds are tuned based on fuzzy linguistic if-then rules. The mentioned features make the estimator more reliable and suitable for hardware implementation. FTH_∞ estimator relies on a consistent fuzzy combination of two change detection tests; namely, likelihood ratio and averaged norm error. Real-time implementation of new attitude-heading estimator is performed on a TMS320VC5416 Digital Signal Processor (DSP). Incorporating this powerful and small size DSP with micro electro mechanical inertial and resistive magnetic sensors leads to a low cost, small size and low power consumption AHRS. Performance of the AHRS was evaluated in Monte Carlo simulations of vehicle's attitude-headings. The FTH_∞ estimator results in a superior performance compared to that of the extended H_∞ estimator in simulation and real tests.

1. INTRODUCTION

In this paper, the process of design and implementation of a Fuzzy Tuned H_∞ (FTH_∞) state estimator is considered for an Attitude Heading Reference System (AHRS) under both gradual and abrupt disturbance changes. H_∞ estimator is more robust than Kalman estimator in the presence of measurement noise uncertainties. This filter is optimal in terms of minimizing the $\infty-$norm of the gain between a set of disturbance inputs and the estimation error (Burl, 1999). The 2-norm bound of disturbance or covariance matrices (if exist) play an important rule in our state estimator designing. In other words, exactly tuning of these weight matrices helps to obtain better state estimation accuracy (Simon, 2006). This kind of flexibility in estimator design techniques is an open problem that is studied in this paper. Once an external disturbance affects the measurement sensors output, a change in innovation or residual signals may be detected. The changes of innovation signals before and after disturbance occurrence may be investigated using generalized likelihood ratio (Basseville *et al*, 1993). On the other hand, during vector transforming from an orthogonal coordinate system to another one, its norm will not be changed.

Attitude determination is often required in land vehicles, spacecrafts, aircrafts, marine vessels, and missiles. These wide applications derive from the important role of determining attitude angles, which are the main navigation parameters that define the system's orientation. The advent of low-cost inertial Micro Electro Mechanical Systems (MEMS) offers the opportunity for applying inertial navigation for a wide variety of new applications including ground vehicles. The MEMS accelerometers, gyros and magnetometers have large noise bounds in addition to bias and scale factor errors due to random walk drifts. An extended Kalman filter with adaptive gain was used to build a miniature attitude and heading reference system based on a stochastic model (Wang *et al*, 2004).

In this paper, to simplify hardware programming two independent FTH_∞ estimators are developed to obtain vehicle's orientation. At first estimator, 3-Axis Accelerometers (TAA) outputs are used to update roll and pitch angles that are obtained from Euler angles dynamic and rate gyros output. Once the attitude is determined, the measurements of a 3-Axis Magnetometer (TAM) and rate sensors are used to update heading angle. In non-accelerated maneuvers, the TAA measures components of the earth's gravity field vector along vehicle body coordinates. Therefore, once the system starts to accelerated movement, considerable biases will be observed in TAA outputs due to non-gravitational accelerations of the vehicle. On the other hand, unlike aerospace applications, in ground vehicles the TAM sensors are commonly affected by considerable soft- and hard-iron magnetic disturbances. These exogenous disturbances are generated due to steel made parts of the vehicle, its electrical devices, and other magnetic anomalies which may come from the environmental effects (Kao *et al*, 2006). However, these time-varying disturbances should be considered in estimation algorithm before correct vehicle heading can be determined to satisfy requirements of vehicle navigation (Keighobadi *et al*, 2007). The FTH$_\infty$ estimator is proposed to intelligently regarding the above mentioned disturbances in attitude-heading determination. This estimator is applied to construct a suitable AHRS for accelerated vehicles under magnetic disturbances. Two-norm bound of the measurement noise/disturbance matrix as a weighting factor and disturbance attenuation (performance) bound of the H$_\infty$ estimator is tuned based on the residual signals and averaged norm of the TAA/TAM output vector. A simple fuzzy fault/change detection system, which includes only four rules in the rule-base, is proposed for tuning the scale factor of noise bound matrix and the length of data history to update fuzzy system inputs. Two possible solutions for unknown changes are weighted cumulative sum (CUSUM) and the generalized likelihood ratio (GLR). The GLR test which is based on variations of the innovation may be used instead of maximum likelihood ratio (Basseville *et al*, 1993). Fuzzy combination of the GLR decision function, which is suitable to detect unknown changes, and a function of the measurement vector norm is considered to the attitude-heading determination system.

Nowadays, small-size Digital Signal Processors (DSPs) are able to execute more than 100 million instructions per second with a little power consumption. FTH$_\infty$ estimator is implemented using a TMS320VC5416 DSP on a MEMS AHRS. Performance of the AHRS is evaluated using simulations and wide-range tests of three kinds of vehicles. Monte Carlo simulations and real experiments revealed that the tracking capability of FTH$_\infty$ estimator on accurate estimation of vehicles' attitude and heading is significantly better than that of the standard H$_\infty$ estimator. Rest of the paper is organized follows: Section 2, presents the AHRS and disturbance modelling; Sections 3 describes the FTH$_\infty$ estimator and fault detection algorithm; In Section 4, the produced AHRS hardware details are given; Simulation and vehicular tests are provided in Section 5, and section 6 concludes the paper.

2. ATTITUDE HEADING REFERENCE SYSTEM

The low cost AHRS is a new solution to Schuler vertical reference tuning instead of high accuracy gyros and accelerometers. In a strap down AHRS, TAA is devoted to measure earth gravitational field components in vehicle body coordinates. Since any vehicle movement may be together with remarkable accelerations, the TAA outputs are affected by time varying disturbances. This results inaccurate attitude angles using vector matching between body and reference coordinate system. However, following equation describe the relation of earth gravity vector in body and North-East-Down (NED) coordinates. This relation shows the sources of error in attitude determination using the TAA measurements.

$$\dot{\mathrm{v}}^n = C_b^n(\varphi,\theta,\psi)(z^b + w_a) - (2\omega_{ie}^n + \omega_{en}^n)\times \mathrm{v}^n + g^n \tag{1}$$

where φ, θ and ψ are attitude-heading in the form of roll, pitch and yaw angles. C_b^n is Direction Cosine Matrix (DCM) from body to NED coordinates systems, z^b and w_a stand for measurements and noise vector of the TAA along vehicle's body coordinates axes, respectively. g^n is the earth's gravity vector in NED local level frame. Also, ω_{ie}^n and ω_{en}^n are rotation rates of the earth with respect to inertial frame and of the NED frame with respect to earth fixed frame, respectively. In addition, v^n stands for relative velocity of the vehicle. After accumulating all of non-gravitational accelerations of the vehicle including $2\omega_{ie}^n \times \mathrm{v}^n$, $\omega_{en}^n \times \mathrm{v}^n$ and $\dot{\mathrm{v}}^n$ -which are coriolis, centrifugal and kinematical accelerations respectively- in a so-called exogenous disturbance vector v_1, the following observation equation may be obtained through replacing the measurement vector y to z^b,

$$y(t) = C_n^b(t)(-g^n + v_1(t)) + w_a(t) \tag{2}$$

Once the roll and pitch angles are estimated, the heading angle may be estimated similarly using measurements coming from the TAM and rate gyros. In ground vehicles, it may not be possible to achieve a qualified heading angle without considering time varying disturbances from local magnetic field effects in estimation algorithm. Although these disturbances may be estimated in the form of time-varying parameters, attenuating their effects using an intelligently tuned robust H$_\infty$ filter is preferred due to probable singular modes of the parameter estimation algorithms. Therefore, the above mentioned disturbances are considered as exogenous inputs vectors in the H$_\infty$ filtering problem.

2.1 Attitude

Attitude-heading may be presented in the form of Euler angels; quaternion vector, direction cosine matrix, rotation vector and other recently developed models. By regarding to application issues, each of these representation methods may have some advantages with respect to others (Rogers, 2003). In ground vehicles, Euler angles may be selected because: (1) since the roll and pitch ranges are so much less than 90°, singular situations may not occur in our real system; (2) 3-dimensional Euler angles dynamic may be decoupled to a two-dimensional attitude and one-dimensional heading dynamics, which will result in less computational efforts and feasible hardware implementation; (3) duo to the physical insight of roll, pitch and heading angles, they have superiority of monitoring with respect to quaternion and other mathematical variables. The following attitude dynamic and corresponding observation equations, which are suitable for continuous H$_\infty$ estimator design, may be obtained through decoupling the Euler angles equations.

$$\dot{x} = f(x(t))\omega(t) + g(x(t))w_g(t) \tag{3}$$

$$\left\| w_g(t) \right\|_2 \le K_Q(t) \tag{4}$$

$$y(t) = h(x(t)) + w_a(t) + v_1(t) = h(x(t)) + v(t) \tag{5}$$

$$\left\| v \right\|_2 \le K_R \tag{6}$$

$$f(x) = g_w(x(t)) = \begin{bmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \end{bmatrix} \tag{7}$$

$$h(x) = \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\varphi \end{bmatrix} \tag{8}$$

where state vector, $x(t) = [\varphi(t)\ \theta(t)]^T$, includes roll and pitch angles. $f(x)$ and $g_w(x)$ (the time variable is suppressed) consist nonlinear smooth functions of state variables. $w_g = [w_x\ w_y\ w_z]^T$ and $\omega = [\omega_x\ \omega_y\ \omega_z]^T$ stand for the noise and measurement vectors coming from gyros of the Inertial Measurement Unit (IMU), respectively. $h(x)$ is the nonlinear observation vector and g is the acceleration due to gravity. In (5), the effects of the sensor noise and exogenous disturbances are considered in a new vector $v \in L^2(R_+, R^{n_v})$, whose energy bound will be determined using newly developed schemes in following sections.

Remark 1. Vector y consists measurements made by the IMU accelerometers only along axial and lateral axes such that the acceleration along normal axis of the vehicle is not considered due to affecting big dynamical loads in this direction.

2.2 Heading

Once the roll and pitch angles are estimated, vehicle heading angle will be determined using following dynamic system.

$$\dot{\psi} = \sin\varphi\sec\theta(\omega_y + w_y) + \cos\varphi\sec\theta(\omega_z + w_z) \tag{9}$$

$$y_m = \psi_m + w_m \tag{10}$$

$$\psi_m = \tan^{-1}\left(\frac{m_x\cos\varphi - m_z\sin\varphi}{m_x\cos\theta + m_y\sin\varphi\sin\theta + m_z\cos\varphi\sin\theta}\right) \tag{11}$$

where ψ_m and w_m shows the magnetic heading and its corresponding noise, respectively. m_x, m_y and m_z stand for measurements made by the TAM magnetic sensors that are inserted along IMU axes.

3. FUZZY TUNED H$_\infty$ ESTIMATOR

The H$_\infty$ worst case minimization problem for attitude system described by (3) and (5), results in the following state estimator considering the prescribed attenuation/performance bound, γ (Simon, 2006), (Burl, 1999).

$$\dot{\hat{x}} = f(\hat{x}(t)) + k(t)(y(t) - h(\hat{x}(t))) \tag{12}$$

$$k(t) = Q(t)H^T \tag{13}$$

$$\dot{Q}(t) = QF^T + FQ + GK_QK_Q^TG^T - Q(t)(H^TK_R^TK_RH - \gamma^{-2})Q(t) \tag{14}$$

$$Q(0) = 0 \tag{15}$$

where, F, H and G are linear matrices corresponding to $f(x)$, $h(x)$ and $g(x)$ of (3) and (5). The measurement equations of the H$_\infty$ estimator may be normalized with respect to 2-norm bounds of the exogenous disturbances. The weight matrices K_R and K_Q, are respectively 2-norm bounds of gyros and accelerometers/magnetometers of the AHRS. In this paper, by considering the fact that no stabilizing nonnegative definite solution exists for $K_R \ge \gamma$ (Green *et al*, 1995), FTH$_\infty$ estimator is designed for simultaneously tuning the noise matrix K_R and the attenuation bound γ. This new filtering method, which is not considered till now, declares faults or changes of disturbances in accelerometers and TAM sensors. In the FTH$_\infty$ estimator, adaptively tuning of K_R and γ, results in properly weighting data obtained from measurement equations against to those may come from the system dynamics. Pre-defined scale factor of the FTH$_\infty$ estimator bounds will be adaptively determined using the so called fuzzy decision making system. Therefore, the followings normalized FTH$_\infty$ estimator could be implemented in the attitude-heading determination system (the variable t is suppressed).

$$\dot{\hat{x}} = f(\hat{x}) + k_1(y_1 - h_1(\hat{x})) \tag{16}$$

$$\begin{aligned}\dot{Q}_1 = {} & Q_1F^T + FQ_1 + GK_QK_Q^TG^T - \\ & Q_1(H_1^TS_F^TK_R^TK_RS_FH_1 - \sup(S_F)^2\gamma^{-2})Q_1\end{aligned} \tag{17}$$

$$k_1 = Q_1H_1^T \tag{18}$$

$$y_1 = (S_F\ K_R)^{-1}y \tag{19}$$

$$h_1(x) = (S_F\ K_R)^{-1}h \tag{20}$$

$$H_1 = (S_F\ K_R)^{-1}H \tag{21}$$

where $\bullet_1$s stand for normalized versions of $\bullet$s .

3.1 Fuzzy Fault Detector

In this section, a fuzzy change/ fault detection mechanism is proposed to detect changes (due to disturbances) of estimated states which are obtained from the extended H$_\infty$ estimator. The basic idea behind this intelligent change detector, which is merely based on fuzzy *if-then* rules, is to cumulate advantages of both the innovation based and state estimate based methods conveniently. In addition, the fuzzy combination is more reliable because it turns away the probable drawbacks of each individual change detection schemes by extending them over.

Two input functions are considered in the fuzzy change detector based on these reasons: (1) any fault in sensors may lead to a non-white and biased innovation signal which in turn causes to change the GLR decision function; (2) norm of the TAA/TAM output vector may be deviated from norm of the earth's gravity and magnetic fields vector during affecting non-gravity accelerations and magnetic anomalies, respectively. Figure 1 shows more details of the fuzzy change detection system. Output vector of the fuzzy decision making system consists the weighting scale factor (S_F) of the H∞ estimator, and the length of recent data history for updating fuzzy system inputs. The first inputs of this fuzzy change detecting algorithm, which is the generalized likelihood ratio, will be employed as following (Basseville *et al*, 1993):

$$gl_k = \max_{l \le j \le k} \ln \frac{\sup_x \prod_{i=j}^{k} \mathrm{P}_{x_0+\Delta x}(y_i \mid x)}{\prod_{i=j}^{k} \mathrm{P}_{x_0}(y_i \mid x)} \tag{22}$$

where gl_k is the generalized likelihood ratio, $\mathrm{P}_{x_0+\Delta x}$ and P_{x_0} are the probability distribution functions of y_i after and before disturbance (change) occurrence, respectively. The following simple form of the gl_k may be obtained after some algebraic manipulations.

$$g_k = \max_{1 \le j \le k} \frac{k-j+1}{2}\left(z_j^k - h(\hat{x}_{k-1})\right)^T (K_R^T K_R)^{-1}\left(z_j^k - h(\hat{x}_{k-1})\right) \tag{23}$$

Second input of fuzzy decision making system is ANE of the TAA/TAM outputs in a variable length sliding window, which may be defined as follows:

$$\text{for attitude, } E\overline{n}_{lk} = \frac{1}{N}\sum_{i=0}^{N} n_{gl}\left(k-i\right) - g\,, \tag{24}$$

$$\text{and for heading, } E\overline{n}_{lk} = \frac{1}{N}\sum_{i=0}^{N} n_{ml}\left(k-i\right) - m \tag{25}$$

where n_{gl} and g are norm of the TAA outputs and of the earth's gravity vectors. Similarly, n_{ml} and m are norm of the TAM outputs and of the earth's magnetic vector. Fuzzy fault detection system uses a combination of two change detection indices to reveal change of states due to disturbance effects. This fuzzy system, because of its continuous outputs is significantly superior to the non-fuzzy change detection systems developed to specify start- and stop-points of a parameter change. In the meantime, the fuzzy approach is comprehensive and is designed in such a way to use both of the input functions in parallel. The fuzzy system has a clear and simple structure as follows:

$$O = \xi\left(gl_k, E\overline{n}_{lk}\right) \tag{26}$$

where components of defuzzified output vector O, which includes N and S_F, are the length of sliding window of states to update all of the fuzzy system inputs and the scale factor parameter for weighting FTH estimator. The *if-then* rules of fuzzy rule-base are in the following generic form:

If gl_k is A_1^l and $E\overline{n}_{lk}$ is A_2^l then S_F is B_1^l, N is B_2^l; where, A_i^l ($i = 1, 2$) and B_j^l ($j = 1, 2$) are fuzzy sets for linguistic input and output variables, respectively. Here we constructed 4 fuzzy rules, which may be simply implemented in manufactured hardware system. Therefore, by considering singleton fuzzifier, product implication engine and centre average defuzzifier, outputs of fuzzy system including N and S_F can be obtained as below (Wang, 1997):

$$o_j(x) = \frac{\sum_{l=1}^{4} \overline{y}_j^l \left(\prod_{i=1}^{2} \mu_{A_i^l}(x_i)\right)}{\sum_{l=1}^{4}\left(\prod_{i=1}^{2} \mu_{A_i^l}(x_i)\right)} \qquad j = 1, 2 \tag{27}$$

where $\overline{y}_j^l$ is centre of normal fuzzy set B_j^l, o_1 and o_2 stand for defuzzified N and S_F, respectively. The universe of discourse for fuzzy system inputs and outputs are determined as follows: $gl_k \in [0, 0.1]$, $E\overline{n}_{lk} \in [0, 0.3]$, $S_F \in [.8, 100]$ and $N \in [20, 120]$. The fuzzy rules are collected in the search Table 1 such that S, M and L stand for small, medium and large, respectively.

4. AHRS HARDWARE ARCHITECTURE

Real-time implementation of FTH∞ attitude-heading estimator is executed on a TMS320VC5416 fixed-point DSP as shown in Fig. 2. In addition to 128 KW (1 word = 16 bits) on-chip RAM, the DSP has several on-chip peripherals including software programmable wait state generator and programmable phase locked loop. The device also includes mechanisms to manage interrupts, repeated operations, and function calls (Texas Instruments, 2005). The AHRS block diagram in Fig. 3 shows the inserted sensors in the system. Magnetometers are Honeywell magneto-resistive sensors that are simple resistive bridge devices and only require a supply voltage to measure any ambient, or applied, magnetic field in the sensitive axis. The Motorola's MPX4115A temperature sensor is used to system thermal calibration. The IMU includes ADXL210E accelerometers and ADXRS150 gyros that are low-cost, low power, complete 2-axis with a digital output and each on a single monolithic IC. Noise floor of this complete rate gyrose is $0.05\,^\circ/\mathrm{s}\sqrt{\mathrm{Hz}}$ and is able to reject high vibrations (Analog Devices, 2004). The accelerometers measure a full-scale range of $\pm 10\mathrm{g}$ (Analog Devices, 2002). Six double-channel Analog to Digital Converters (ADC) are used to convert analog outputs of the AHRS sensors (Analog Devices, 2003). To control and to access the result, a serial peripheral interface was implemented for each ADC. To communicate with outside word and monitoring, a Universal Asynchronous Receiver Transmitter (UART) is implemented.

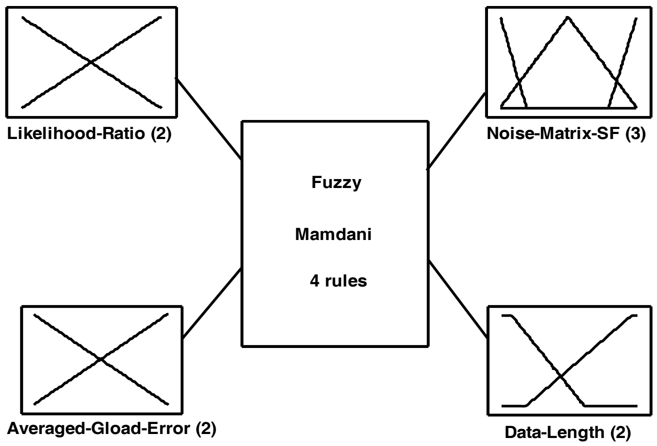

Fig. 1. Fuzzy fault detection and I/O membership functions

5. SIMULATION and VEHICULAR TESTING

In this section, the performance of the FTH$_\infty$ and extended H$_\infty$ estimators are evaluated using simulations and vehicle tests data. The considered vehicle for simulation may be accelerated up to $0.5\,g$ and affected by magnetic disturbances up to $1\,\mu\mathrm{T}$. Performance of the AHRS with FTH$_\infty$ estimator is evaluated in a Monte Carlo type simulation based on 1000 stochastic manoeuvres of the vehicle. The results are shown in Figs. 4 for roll, pitch and heading angles of vehicle in the sense of Root of Mean Square (RMS) error. As may be seen from this figure, the RMS of estimated attitude and heading errors using the FTH$_\infty$ estimator is superior to that of the H$_\infty$ estimator. AHRS performance strongly depends on calibration precision of both IMU and TAM to determine scale factors, biases and misalignment. The IMU is calibrated using a two DOF rotating table which could generate angular rates in the form of impulse and step signals. On the other hand, the TAM is calibrated using an attitude independent calibration algorithm after fixing the AHRS on the vehicle (Gebre *et al,* 2006).

Next, the proposed FTH$_\infty$ and standard H$_\infty$ estimators for AHRS are experimentally verified during tests with three kinds of vehicles including 'Peugeot 405', 'van MB140' and 'Paykan'. Fig. 5 shows the roll of Paykan vehicle in a mountain road obtained from: (1) an AHRS in which the FTH$_\infty$ estimator is loaded to its memory; (2) the AHRS using H$_\infty$ estimator; (3) stand alone mode of the IMU; and finally (4) an accurate INS/GPS as a reference system.As this figure shows, in most times, the estimated attitudes by both H$_\infty$ and FTH$_\infty$ estimators are approximately coincide to that obtained from reference INS/GPS. This should come from the fact that the TAA output vector is not affected by considerable non-gravitational accelerations due to slow dynamics of the vehicle under test. Unlike Paykan, the van MB140 is a high performance vehicle which may be accelerated up to .5g. Fig. 6 shows data of the AHRS sensors during accelerated movement on a sloped trajectory. Executing the FTH$_\infty$ estimator in AHRS, results in less attitude-heading errors than that of the H$_\infty$ as shown in Fig. 7.

Table 1. Search table of fuzzy rules

Fuzzy system inputs		Outputs	
gl_k	$E\overline{n}_{lk}$	S_F	N
S	S	S	L
L	S	M	M
S	L	M	L
L	L	L	M

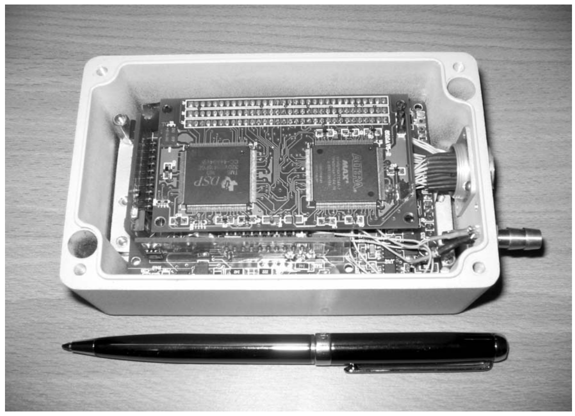

Fig. 2. Manufactured AHRS hardware

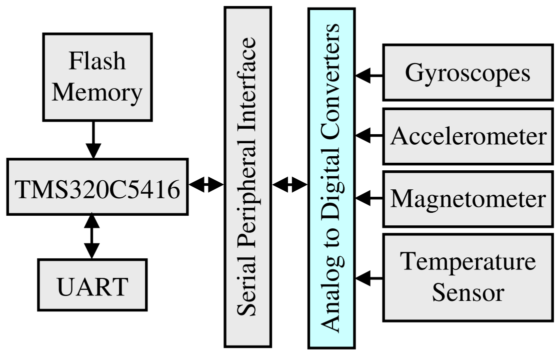

Fig. 3. Block diagram of AHRS hardware architecture

6. CONCLUSION

A low cost AHRS was configured on a high performance DSP. FTH$_\infty$ estimator was implemented via this processor to increase the system performance in accelerated vehicles, and in the presence of magnetic disturbances. Once the TAA/TAM sensors are affected by exogenous disturbances, fuzzy fault detector system determines their intensity using the GLR and ANEs simultaneously. Therefore, a considered scale factor for disturbance (noise) and attenuation bounds of the FTH estimator could be determined intelligently. The Monte Carlo simulations and several vehicles test using the produced AHRS revealed superior performance of the FTH$_\infty$ estimator with respect to that of the conventional H$_\infty$ estimator. By increasing the non-gravity accelerations and local magnetic fields affecting on the vehicles, more superiority of FTH$_\infty$ estimator with respect to H$_\infty$ estimator could be observed.

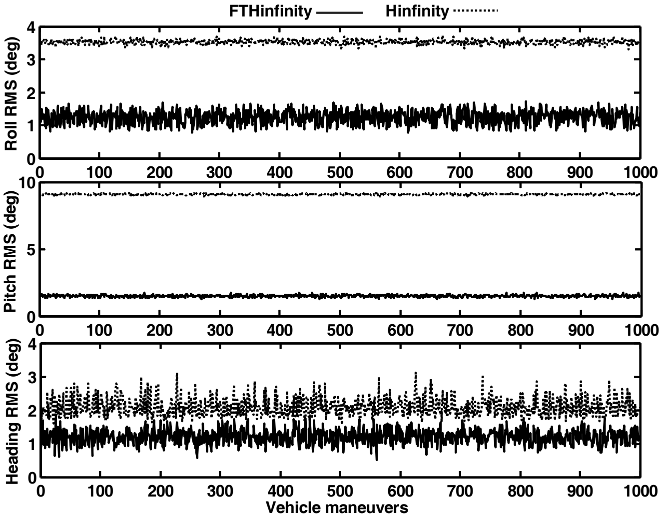

Fig. 4. RMS errors of attitude-headings in 1000 simulations

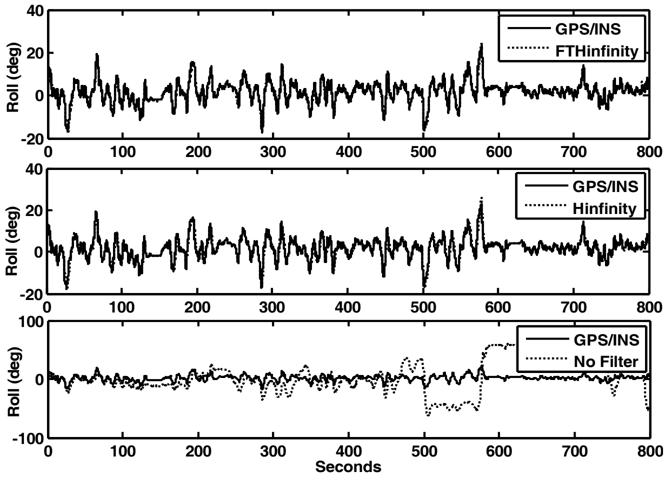

Fig. 5. Roll of AHRS in mountain road test using FTH$_\infty$, H$_\infty$ estimators, and IMU stand alone mode compared to INS/GPS

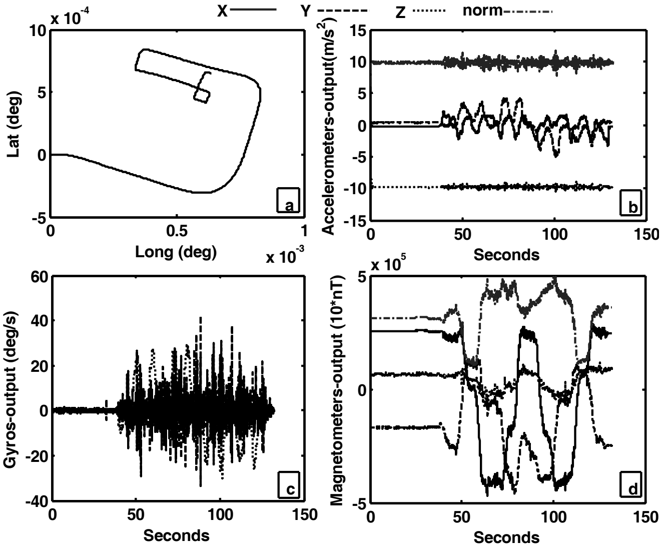

Fig. 6. Outputs of the accelerated AHRS by van MB140: (a) test path; (b) TAA data; (c) Gyros data; (d) TAM data

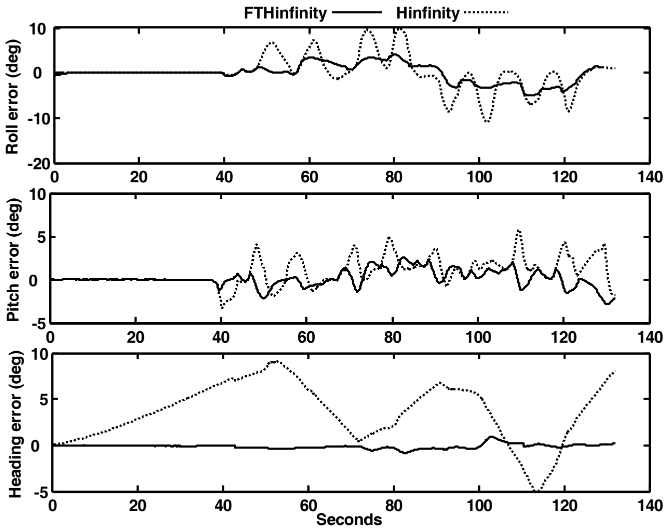

Fig. 7. Roll, pitch and heading errors of Toyota van using FTH and H$_\infty$ estimators from INS/GPS outputs

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