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Bias-compensated Least Squares Method in Closed Loop Environment

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Abstract: In this paper, a bias-compensated least squares (BCLS) method in the closed loop environment is proposed. It is assumed that the observation noise is a white gaussinan signal while there are no process noises. It is also assumed that the plant is controlled by a linear time invariant controller and that the closed loop system is asymptotically stable. The proposed estimator is unbiased and it does not require the reference input be informative. An iterative redesign of the prefilters is also considered in order to achieve a minimum variance estimator. The proposed BCLS method is applied for the iterative redesign of the prefilters in order to reduce the computational cost.

1. INTRODUCTION

There are increasing demands for the identification under the feedback control(Ljung et al. [1974], Gustavsson et al. [1977], Ljung [1999]) due to the safty or economic reasons(Forssell and Ljung [1999]). The difficulty of the closed loop identification arises from the fact that the input and the output of the plant correlates with the noise because of the feedback loop. In order to remove the asymptotic bias caused by the correlations, a special treatment will be required for the closed loop identification. Another problem is the computational cost because unbiased estimation of the plant parameters becomes a nonlinear optimization problem and requires an iterative algorithm. Therefore, reduction of the iteration or reduction of the computational cost will be required.

In order to obtain an unbiased estimate in the closed loop environment, the instrumental variable (IV) methods (Gilson and Van den Hof [2001, 2005], Gilson et al. [2006]) are proposed, which require a few iterations to obtain unbiased estimate. These methods are based on the indirect approach of the closed loop identification or at least the instrumental variables are produced based on the reference input signal. Thus, the estimation error depends on the informativeness of the reference input. For less informative reference input, direct approach of the closed loop identification will be required.

Unbiased estimate is also obtained by using the prediction error method with an appropriate choise of the prefilters. Because such suitable prefilters are based on the noise model, the iterative redesign of the prefilters will be required as in the IV methods mentioned above. Unlike the IV methods, this approach requires many iterations because of the asymptotic bias of the estimate at each iteration. On the other hand, bias compensated least squares (BCLS) method(Sagara and Wada [1977]) is a method based on the analysis of the noise effect on the estimated parameters and on the estimation of the noise variance. Therefore, it is expected to reduce the number of iteration by applying the bias compensation to the estimate at each iteration.

The main purpose of the closed loop identification is to improve the control performance by obtaining a more accurate plant model. Thus, the concern is laid on the estimation of the plant dynamics rather than on estimation of the noise dynamics. In such circumstances, it will be reasonable to estimate the plant model based on the output error (OE) model. In this paper, OE model is considered.

In section 2, the problem is formulated and the least squares method with prefilters is briefly summarized in section 3. The asymptotic bias of the least squares estimate in the closed loop environment and the noise variance are analysed in sections 4 and 5, respectively. Section 6 proposes an iterative algorithm with bias compensation in the closed loop environment. Section 7 shows numerical examples in order to illustrate the bias and the variance of the proposed method. Section 8 concludes the paper.

Notation:

Let $E\{x\}$ denote an expectation of random variable x.

Let q denote a shift operator *i.e.* $qx[k] = x[k+1]$.

2. PROBLEM FORMULATION

Consider a single-input single-output (SISO) n-th order discrete time plant:

$$y[k] = \frac{b_p(q)}{a_p(q)} u[k] + \nu[k], \tag{1}$$

$$a_p(q) = q^n + a_1 q^{n-1} + \cdots + a_n, \tag{2}$$

$$b_p(q) = \qquad b_1 q^{n-1} + \cdots + b_n, \tag{3}$$

where $u[k] \in \mathrm{R}$, $y[k] \in \mathrm{R}$, and $\nu[k] \in \mathrm{R}$ are the input, the output, and the observation noise, respectively. The polynomial $a_p(q)$ is monic and of the n-th order, while $b_p(q)$ is polynomial whose degree is less than n.

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The plant to be estimated is assumed to be controlled by the following feedback compensator:

$$u[k] = \frac{b_c(q)}{a_c(q)}(y[k] - r[k]), \tag{4}$$

$$a_c(q) = q^m + a_{c1}q^{m-1} + \cdots + a_{cm}, \tag{5}$$

$$b_c(q) = b_{c0}q^m + b_{c1}q^{m-1} + \cdots + b_{cm}, \tag{6}$$

where $r[k] \in \mathrm{R}$ is a reference input.

The following assumptions are made for the plant, the noise, and the I/O data.

(A1) $a_p(q)$, $b_p(q)$ does not have a common zero outside of the open unit disc.
(A2) an upper bound of the plant degree is known to be n.
(A3) the observation noise $\nu[k]$ is a zero mean white gaussian signal with variance

$$E\{\nu[k]\nu[l]\} = \sigma_\nu^2 \delta_{kl}, \tag{7}$$

where δ_{kl} denotes a Kronecker delta.
(A4) the I/O data is collected in the closed loop environment and the feedback loop is asymptotically stable.
(A5) the reference input $\{r[k]\}$ is independent of the observation noise $\{\nu[k]\}$.

From the assumption (A3), the persistently excitation(PE) condition will be satisfied even if $r[k] = 0$ because the closed loop is driven by a white gaussian signal.

Let the characteristic polynomial of the closed loop be denoted by

$$d_{cl}(q) = a_p(q)a_c(q) + b_p(q)b_c(q), \tag{8}$$

all the zeros of $d_{cl}(q)$ lie on the open unit disc from the assumption (A4).

Problem: Estimate the unknown coefficient of $a_p(q)$ and $b_p(q)$ from the I/O data $\{u[k], y[k]\}$ $(k = 1, \ldots, N)$.

3. LEAST SQUARES ESTIMATE

In this section, least squares estimate together with prefileters are briefly summarized.

Define the characteristic polynomial of the prefilter as

$$f(q) = q^n + f_1 q^{n-1} + \cdots + f_n, \tag{9}$$

where all the zeros of $f(q)$ are selected to lie on the unit disc. Define the filtered output $y_f[k]$ and the filtered input $u_f[k]$ as follows:

$$y_f[k] = \frac{q^n}{f(q)}y[k] \quad \text{and} \quad u_f[k] = \frac{q^n}{f(q)}u[k]. \tag{10}$$

Multiplying the both side of eq. (1) by $a_p(q)/f(q)$, we obtain

$$y[k] = \frac{f(q) - a_p(q)}{f(q)}y[k] + \frac{b_p(q)}{f(q)}u[k] + \frac{a_p(q)}{f(q)}\nu[k]. \tag{11}$$

From the equation above, the following linear regression formula is obtained:

$$y[k] = \varphi[k]^\top \theta + \varepsilon[k], \tag{12}$$

where

$$\begin{aligned} \theta &= [\theta_1^\top, \theta_2^\top]^\top \\ &= [f_1 - a_1, \ldots, f_n - a_n | b_1, \ldots, b_n]^\top, \end{aligned} \tag{13}$$

$$\begin{aligned} \varphi[k] = [&y_f[k-1], \ldots, y_f[k-n], \\ &u_f[k-1], \ldots, u_f[k-n]]^\top, \end{aligned} \tag{14}$$

$$\varepsilon[k] = \frac{a_p(q)}{f(q)}\nu[k]. \tag{15}$$

Adopting the least squares criterion:

$$J(\hat{\theta}) = \sum_{k=1}^{N}\left[y[k] - \hat{\theta}^\top \varphi[k]\right]^2, \tag{16}$$

the parameter minimizing $J(\hat{\theta})$ is given by

$$\hat{\theta}_{LS,N} = (\Phi_N^\top \Phi_N)^{-1}\Phi_N^\top y_N, \tag{17}$$

where

$$\Phi_N = [\varphi[1], \ldots, \varphi[N]]^\top, \tag{18}$$

$$y_N = [y[1], \ldots, y[N]]^\top. \tag{19}$$

Substituting eq. (12) for eq. (17), another representation of the least squares estimator is given by

$$\hat{\theta}_{LS,N} = \theta + (\Phi_N \Phi_N)^{-1}\Phi_N^\top \varepsilon_N. \tag{20}$$

where

$$\varepsilon_N = [\varepsilon[1], \ldots, \varepsilon[N]]^\top \tag{21}$$

Becase the equation error $\varepsilon[k]$ is not white and has correlation with the regression vector $\varphi[k]$ in general, the least squares estimator $\hat{\theta}_{LS,N}$ has an asymptotic bias.

4. ASYMPTOTIC BIAS IN CLOSED LOOP ENVIRONMENT

In this section, the asymptotic bias of the least squares estimator (20) in the closed loop environment is investigated.

The output and the input of the plant are given by

$$y[k] = \frac{b_p(q)b_c(q)}{d_{cl}(q)}r[k] + \frac{a_p(q)a_c(q)}{d_{cl}(q)}\nu[k], \tag{22}$$

$$u[k] = \frac{a_p(q)b_c(q)}{d_{cl}(q)}r[k] - \frac{a_p(q)b_c(q)}{d_{cl}(q)}\nu[k]. \tag{23}$$

In order to calculate the expectation of the asymptotic bias, the expectations $E\{y_f[k-i]\varepsilon[k]\}$ and $E\{u_f[k-i]\varepsilon[k]\}$, $i = 1, \ldots, n$ are to be calculated. Because $\{r[k]\}$ and $\{\nu[k]\}$ are independent from the assumption (A5), the expectation of the correlation between the equation error and the r-dependent part of each regressor becomes zero. Thus, we assume $r[k] = 0$ without loss of generarity in this section. The noise dependent parts of the filtered output $y_f[k-i]$ and the filtered input $u_f[k-i]$ are given by

$$y_f[k-i] = \frac{q^{n-i}a_c(q)}{d_{cl}(q)}\varepsilon[k], \tag{24}$$

$$u_f[k-i] = -\frac{q^{n-i}b_c(q)}{d_{cl}(q)}\varepsilon[k]. \tag{25}$$

Eqs. (24), (25) together with eq. (15) have a state space representation as follows:

$$\bar{X}[k+1] = \bar{A}\bar{X}[k] + \bar{b}\nu[k], \tag{26}$$

$$\varphi[k] = [\,C_{cl} \quad 0\,]\bar{X}[k], \tag{27}$$

$$\varepsilon[k] = [\,0 \quad h\,]\bar{X}[k] + \nu[k], \tag{28}$$

where $\bar{A}$ and $\bar{b}$ are defined by

$$\bar{A} = \begin{bmatrix} A_{cl} & b_{cl}h \\ 0 & F \end{bmatrix}, \tag{29}$$

$$\bar{b} = \begin{bmatrix} b_{cl} \\ g \end{bmatrix}, \tag{30}$$

and (A_{cl}, b_{cl}, C_{cl}) and $(F, g, h, 1)$ are the system matrices of the state space representations of the transfer functions as follows:

$$\left[\begin{array}{c|c} A_{cl} & b_{cl} \\ \hline C_{cl} & 0 \end{array}\right] = \frac{\begin{bmatrix} s(q)a_c(q) \\ -s(q)b_c(q) \end{bmatrix}}{d_{cl}(q)}, \tag{31}$$

$$s(q) = [q^{n-1}, \ldots, q, 1]^\top, \tag{32}$$

$$\left[\begin{array}{c|c} F & g \\ \hline h & 1 \end{array}\right] = \frac{a_p(q)}{f(q)}. \tag{33}$$

From eqs. (26), (27) and (28), and taking into account that $E\{\bar{X}[k]\nu[k]\} = 0$, $E\{\varphi[k]\varepsilon[k]\}$ can be calculated as

$$E\{\varphi[k]\varepsilon[k]\} = [\,C_{cl} \quad 0\,]E\{\bar{X}[k]\bar{X}^\top[k]\} \begin{bmatrix} 0 \\ h^\top \end{bmatrix}. \tag{34}$$

Covariance matrix of $\bar{X}[k]$ is given by

$$E\{\bar{X}[k]\bar{X}^\top[k]\} = P\sigma_\nu^2, \tag{35}$$

where $P = P^\top > 0$ is a solution of the Lyapunov equation

$$P = \bar{A}P\bar{A}^\top + \bar{b}\bar{b}^\top. \tag{36}$$

Finally, we obtain

$$E\{\varphi[k]\varepsilon[k]\} = C_{cl}P_{12}h\sigma_\nu^2, \tag{37}$$

where $P_{12} \in R^{(n+m)\times n}$ is a 1-2 block of P.

The asymptotic bias of the least squares estimator in the closed loop environment is given by the following theorem.

Theorem 1. Consider the closed loop defined by eqs. (1) and (4) together with the assumptions (A1) to (A5). The expectation of the least squares estimate defined by eq. (17) is given by

$$E\left\{\lim_{N\to\infty} \hat{\theta}_{LS,N}\right\}$$
$$= \theta + \lim_{N\to\infty}\left(\frac{1}{N}\Phi_N^\top\Phi_N\right)^{-1} C_{cl}P_{12}h^\top\sigma_\nu^2, \tag{38}$$

where P_{12} is a 1-2 block of P defined by eqs. (29) to (33) and (36).

Proof: It is obvious from the discussions above and the fact that the correlation between $(\Phi_N^\top\Phi_N)/N$ and $(\Phi_N^\top\varepsilon_N)/N$ goes to zero as $N \to \infty$.

Remark 2. When F and g are realized as a controller canonical form, h becomes a coefficient vector of $a_p(q) - f(p)$. This implies that there are no asymptotic bias when the prefilters are designed to satisfy $f(q) = a_p(q)$ if the plant is stable.

Remark 3. When the plant has an unstable pole, define $\bar{a}_p(q)$ such that $a_p(q)/\bar{a}_p(q)$ becomes an inner function. Then the equation error can be represented as

$$\varepsilon[k] = \frac{\bar{a}_p(q)}{f(q)}\nu^\dagger[k], \quad \nu^\dagger[k] = \frac{a_p(q)}{\bar{a}_p(q)}\nu[k]. \tag{39}$$

Because $|a_p(e^{j\omega})/\bar{a}_p(e^{j\omega})| = \text{const.}$, the power spectrum of $\nu^\dagger[\cdot]$ is proportional to that of $\nu[\cdot]$, which concludes $\nu^\dagger[\cdot]$ is a white noise. Therefore, the asymptotic bias becomes zeros if the design parameters of the prefilters are chosen as $f(q) = \bar{a}_p(q)$ when the plant is unstable.

Consider the case when the plant is unstable. Let (F, g) in eq. (33) be a controller canonical form and define $h^\dagger$ be a coefficient vector of $\bar{a}_p(q) - f(q)$ where $\bar{a}_p(q)$ is defined in remark 3. Then, $(F, g, h^\dagger, 1)$ is a realization of $\bar{a}_p(q)/f(q)$. And define $P^\dagger$ be a positive definite solution of the Lyapunov equation

$$P^\dagger = \bar{A}^\dagger P^\dagger(\bar{A}^\dagger)^\top + \bar{b}\bar{b}^\top \tag{40}$$

where

$$\bar{A}^\dagger = \begin{bmatrix} A_{cl} & b_{cl}h^\dagger \\ 0 & F \end{bmatrix}. \tag{41}$$

Then, the expectation $E\{\varphi[k]\varepsilon[k]\} = C_{cl}P_{12}h^\top\sigma_\nu^2$ has another representation

$$E\{\varphi[k]\varepsilon[k]\} = C_{cl}P_{12}^\dagger(h^\dagger)^\top\sigma_{\nu^\dagger}^2 \tag{42}$$

where $P_{12}^\dagger$ is a 1-2 block of $P^\dagger$ and $\sigma_{\nu^\dagger}^2$ is a variance of $\nu^\dagger$.

From the discussions above, an optimal prefilter which achieves unbiased estimate will be defined such that the characteristic polynomial of the prefilter becomes $f(q) = a_p(q)$ when the plant is stable and $f(q) = \bar{a}_p(q)$ when the plant is unstable. However, in order to obtain an optimal prefilter, an iterative redesign of the prefilter and the re-calculation of the regression vector will be required.

Remark 4. The matrix P_{12} can be obtained by solving the following Sylvester equation instead of solving the Lyapunov equation (36):

$$P_{12} = A_{cl}P_{12}F^\top + b_{cl}h^\top P_{22}F^\top + b_{cl}g^\top \tag{43}$$

where P_{22} is a solution of the Lyapunov equation

$$P_{22} = FP_{22}F^\top + gg^\top, \tag{44}$$

which is independent of the plant parameters when (F, g) is realized as a controller canonical form.

5. ESTIMATION OF THE NOISE VARIANCE

Based on the similar idea of Sagara and Wada [1977], the noise variance σ_ν^2 is to be estimated from the residuals.

Define the residuals as

$$e[k] = y[k] - \hat{\theta}_{LS,N}^\top\varphi[k], \tag{45}$$

$$e_N = [e[1], \ldots, e[N]]^\top. \tag{46}$$

Then, from eq. (20), the residuals becomes

$$e_N = \varepsilon_N - \Phi_N\left(\frac{1}{N}\Phi_N^\top\Phi_N\right)^{-1}\left(\frac{1}{N}\Phi_N\varepsilon_N\right)$$

Its mean squared error becomes

$$\frac{1}{N}e_N^\top e_N = \frac{1}{N}\varepsilon_N^\top \varepsilon_N - \left(\frac{1}{N}\Phi_N^\top \varepsilon_N\right)^\top \left(\frac{1}{N}\Phi_N^\top \Phi_N\right)^{-1} \left(\frac{1}{N}\Phi_N^\top \varepsilon_N\right). \quad (47)$$

The expectation of the first term of the r.h.s can be calculated as

$$E\left\{\frac{1}{N}\varepsilon_N^\top \varepsilon_N\right\} = (1 + hP_{22}h^\top)\sigma_\nu^2 \quad (48)$$

where $P_{22} \in R^{n\times n}$ is a the positive definite solution of the Lyapunov equation (44). The expectation of the second term of the r.h.s. can be calculated by using eq. (37) when N goes to infinity. Thus, the following theorem is obtained.

Theorem 5. The expectation of the squared error of the residuals is given by

$$E\left\{\lim_{N\to\infty}\frac{1}{N}e_N^\top e_N\right\} = (1 + hP_{22}h^\top)\sigma_\nu^2 - hP_{12}^\top C_{cl}^\top \lim_{N\to\infty}\left(\frac{1}{N}\Phi_N^\top \Phi_N\right)^{-1} C_{cl}P_{12}h^\top \sigma_\nu^4. \quad (49)$$

Proof: It is obvious from the discussions above.

From the theorem above, the expectation of the noise variance is given by

$$E\{\lim_{N\to\infty}\sigma_\nu^2\} = \lim_{N\to\infty}\frac{2\gamma_N}{\beta + \sqrt{\beta^2 - 4\alpha_N\gamma_N}} \quad (50)$$

where

$$\alpha_N = NhP_{12}^\top C_{cl}^\top (\Phi_N^\top \Phi_N)^{-1} C_{cl}P_{12}h^\top, \quad (51)$$

$$\beta = 1 + hP_{22}h^\top, \quad (52)$$

$$\gamma_N = \frac{1}{N}e_N^\top e_N. \quad (53)$$

6. BCLS IN CLOSED LOOP ENVIRONMENT

Based on the analysis in the previous two sections, a bias compensated least squares method in closed loop environment is to be proposed.

Let A_{cl} and b_{cl} be realized as a controller canonical form. Then $C_{cl} \in R^{2n\times(n+m)}$ becomes as follows:

$$C_{cl} = \begin{pmatrix} 1 & a_{c1} & \cdots & a_{cm} & & 0 \\ & \ddots & \ddots & & \ddots & \\ 0 & & 1 & a_{c1} & \cdots & a_{cm} \\ -b_{c0} & -b_{c1} & \cdots & -b_{cm} & & 0 \\ & \ddots & \ddots & & \ddots & \\ 0 & & -b_{c0} & -b_{c1} & \cdots & -b_{cm} \end{pmatrix}. \quad (54)$$

Also, let F and g be realized as a controller canonical form.

The proposing estimate is give by the following algorithm.

Algorithm 1. Iterative redesign of the prefilters with BCLS in Closed Loop Environment

(1) Initialize the prefileter parameter $f^{(0)}(q)$ for example $f^{(0)}(q) = q^n$. Let $i = 0$.

(2) Define $y_f^{(i)}$ and $u_f^{(i)}$ by using $f^{(i)}(q)$ as in eq. (10) and define $\varphi^{(i)}[k]$ and $\Phi_N^{(i)}$ as in eqs. (14) and (18). Calculate $\hat{\theta}_{LS,N}^{(i)}$ and $\gamma_N^{(i)}$ as in eqs. (17) and (53).

(3) Define $\hat{A}_{cl}^{(i)}$, $\hat{h}^{(i)}$, $\bar{A}^{(i)}$ as the estimates of $\bar{A}$, A_{cl}, h in eqs. (29), (31), and (33) by using $\hat{\theta}_{LS,N}^{(i)}$.

(4) Calculate $\hat{P}^{(i)}$ as a positive definite solution of the Lyapunov equation

$$\hat{P}^{(i)} = \bar{A}^{(i)}\hat{P}^{(i)}(\bar{A}^{(i)})^\top + \bar{b}\bar{b}^\top \quad (55)$$

and define

$$\hat{\alpha}_N^{(i)} = \hat{h}^{(i)}(\hat{P}_{12}^{(i)})^\top C_{cl}^\top((\Phi_N^{(i)})^\top \Phi_N^{(i)})^{-1} C_{cl}\hat{P}_{12}^{(i)}(\hat{h}^{(i)})^\top, \quad (56)$$

$$\hat{\beta}^{(i)} = 1 + \hat{h}^{(i)}P_{22}(\hat{h}^{(i)})^\top \quad (57)$$

where $\hat{P}_{12}^{(i)}$ is a 1-2 block of $\hat{P}^{(i)}$.

(5) Estimate the variance of the noise as

$$\hat{\lambda}_\nu^{(i)} = \frac{2\gamma_N^{(i)}}{\hat{\beta}^{(i)} + \sqrt{(\hat{\beta}^{(i)})^2 - 4\hat{\alpha}_N^{(i)}\gamma_N^{(i)}}}, \quad (58)$$

and update the parameter estimate as

$$\hat{\theta}_{BC,N}^{(i)} = \hat{\theta}_{LS,N}^{(i)} - N((\Phi_N^{(i)})^\top \Phi_N^{(i)})^{-1} C_{cl}\hat{P}_{12}^{(i)}(\hat{h}^{(i)})^\top \hat{\lambda}_\nu^{(i)} \quad (59)$$

(6) Define $\hat{a}_p^{(i)}(q)$ and $\hat{b}_p^{(i)}(q)$ be the estimates of the denominator and the numerator of the plant by using the estimated parameter $\hat{\theta}_{BC,N}^{(i)}$.

(7) Let $f^{(i+1)} = \hat{a}_p^{(i)}(q)$ for a stable plant, or let $f^{(i+1)} = \hat{\bar{a}}_p^{(i)}(q)$ for an unstable plant. Increase i by 1 and go to step (2).

Remark 6. The least squares estimate $\hat{\theta}_{LS,N}$ is obtained in the framework of the direct approach in closed loop identification and does not require the information on the compensator. However, information on the compensator is required for the compensation of the bias. This means the proposed method is no more a direct approach in closed loop identification.

7. NUMERICAL EXAMPLE

7.1 Illustration of the theorems

In order to illustrate theorems 1 and 5, the least squares estimate (17) is compensated by using eq. (38) and σ_ν^2 is estimated by using eq. (50) where the true values of P_{12} and h are used in this subsection.

Consider the following first order plant:

$$y[k] = \frac{b}{q-a}u[k] + \nu[k] = \frac{0.5}{q-0.8}u[k] + \nu[k],$$

where the variance of the noise $\nu[k]$ is $\sigma_\nu^2 = 1$. The control input $u[k]$ is defined by the following feedback compensator:

$$u[k] = \frac{0.2289}{q-0.09004}(r[k] - y[k]). \quad (60)$$

The reference input $r[k]$ is set to be 0. The number of the samples is $N = 4096$.

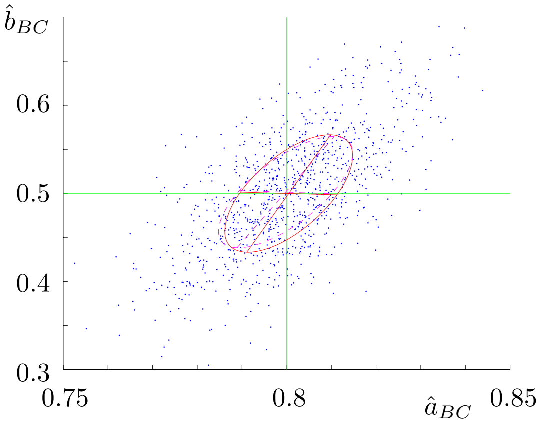

Fig. 1. BCLS estimate by using true P_{12}, P_{22}, and h

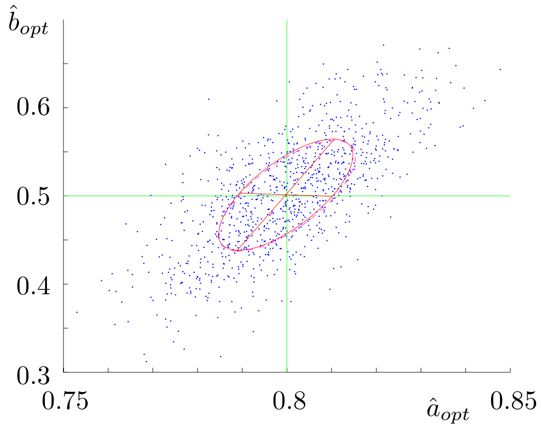

Fig. 2. Parameter estimate by using the optimal prefilter

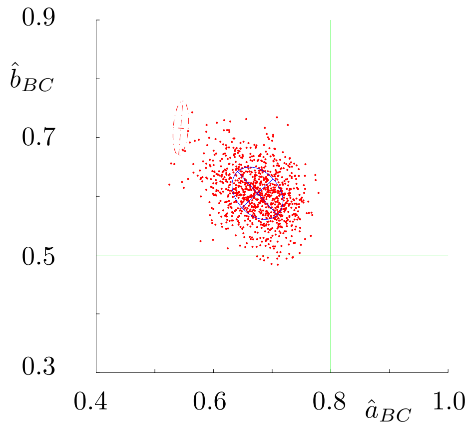

Fig. 3. BCLS estimate

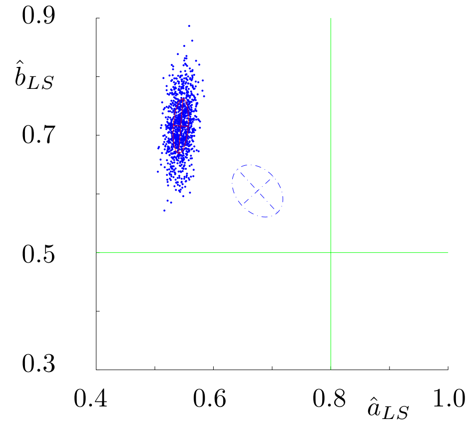

Fig. 4. LS estimate

The estimate $\hat{\theta}_{BC,N}$, in which the design parameter of the prefilter is set $f = 0.5$, is compared with the optimal estimate $\hat{\theta}_{opt,N}$, in which the design parameter of the prefilter is set $f = 0.8$.

One thousand pairs of I/O data are prepared for the estimation of the plant parameters. The proposed estimate and the optimal estimate are plotted in figs. 1 and 2, respectively. The estimated parameter for each pair of I/O data is plotted by dot and the ellipses calculated by the estimated covariance matrices are drawn by solid lines. In fig. 1, the ellipse defined by the covariance of the optimal estimate is plotted by a dashed line, while in fig. 2, the ellipse defined by Cramér-Rao bound is plotted by a dashed line.

From figs. 1 and 2, each of the proposed estimate and the optimal estimate has almost zero bias. Fig. 1 shows that the covariance of the proposed estimate is slightly larger than that of the optimal estimate, while fig. 2 shows that the covariance of the optimal estimate is almost the same as the Cramér-Rao bound. As a result, the covariance of the proposed estimate achieves almost the minimum value.

7.2 Estimation of the bias

In order to estimate the noise variance and the bias of the least squares estimate, P_{12} and h have to be estimated. As a result, the bias compensated least squares estimate may be biased especially when the reference input is less informative. However, the bias of the BCLS estimate is expected to be smaller than that of the LS estimate. In this subsection, the bias of the LS estimate is compared with the bias of the BCLS estimate in which P_{12} and h in eqs. (38) and (50) are estimated based on the LS estimate $\hat{\theta}_{LS,N}$.

The plant to be estimated, the feedback controller, the number of I/O data, the noise variance, and the design parameter of the prefilter f are the same as in the previous subsection. In this subsection, the reference input $r[k]$ is produced by a random binary sequence with the magnitude 0.5 and the bandwidth 1/16. The estimated results of the BCLS and of the LS methods are shown in fig.3 and in fig.4, respectively.

The average of $\hat{a}_{BC}$ is 0.6751 and is still biased. However, the bias is improved compared to the average of $\hat{a}_{LS}$ 0.5442. Thus, if $\hat{a}_{BC}$ is used as the next design parameter of the prefilter f, the convergence of the iterated algorithm will be accelerated.

7.3 Iterative redesign of the prefilters

In this section, the proposed iterative algorithm is compared with the iterative algorithm without bias compensation and the IV method(Gilson et al. [2006]).

The plant to be estimated, the feedback controller, the number of I/O data, the noise variance, and the reference

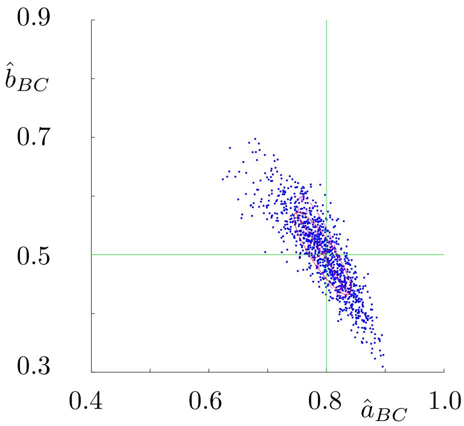

Fig. 5. Parameter estimate of the proposed method

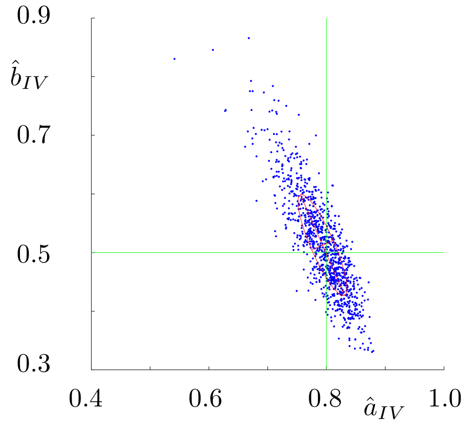

Fig. 6. Parameter estimate by using the IV method

input $r[k]$ are the same as in the subsection 7.2. The initial design parameter of the prefilter is $f = 0.7$ in each method. The iteration stops when the norm of the parameter improvement becomes less than 0.001. In the IV method, a noise model is not estimated because the OE model is used in this simulation.

The estimated results of the proposed method and the IV method are shown in figs.5 and 6, respectively. The estimated result of the iterative algorithm without bias compensation is almost the same as the proposed method. So, the graph of the result is omitted here. In fig.5, the ellipse defined by the covariance matrix calculated by the proposed estimates is plotted in a solid line, while the ellipse calculated by the IV estimates is plotted in a dotted line. The covariance matrix of the proposed method is slightly improved compared to that of the IV method.

The computational costs of the three methods are compared in table 1. The averaged number of iterations in the prosed method is improved compared to that of the iterative algorithm without bias compesnsation. The computational cost consuming part of the proposed method is the calculation of the filtered I/O in step (2), which is also required in the method without bias compensation. Therefore, the reduction of the number of iterations results in the reduction of the CPU time as in table 1. The number of iterations required for the IV method is very small. However, the IV method requires the calculation of the instrumental variables and their filtered values as well as

Table 1. Number of iteration and CPU time

method	averaged number of iterations	CPU time [s] for 1000 estimates
Proposed method	10.15	32.5828
iterative redesign of prefilters without BC	15.88	42.7188
IV method	3.355	68.2813

the filtered I/O for each iteration. This results in a large CPU time in the IV method.

8. CONCLUSION

The asymptotic bias of the least squares estimate in the closed loop environment and the variance of the residuals are analysed. An estimation method with iterative redesign of the prefilters in which the BCLS method is applied for each iteration is proposed. Numerical example shows that the computational cost is reduced by applying the bias compensation at each iteration. It is also shown in the numerical simulation that the variance of the proposed method is smaller than that of the IV method. Variance analysis of the proposed estimate is a future work.

REFERENCES

U. Forssell and L. Ljung. Closed-loop identification rivisited. *Automatica*, 35:1215–1241, 1999.

M. Gilson and P. Van den Hof. On the relation between a bias-eliminated least-squares(BELS) and an IV estimator in closed-loop identification. In *Automatica*, volume 37, pages 1593–1600, 2001.

M. Gilson and P. Van den Hof. Instrumental variable methods for closed-loop system identification. In *Automatica*, volume 41, pages 241–249, 2005.

M. Gilson, H. Garnier, P. Young, and P. Van den Hof. A refined IV method for closed-loop system identification. In *Proc. of 14th IFAC SYSID*, pages 903–908, 2006.

I. Gustavsson, L. Ljung, and T. Söderström. Identification of Processed in Closed Loop — Identifiability and Accuracy Aspects—. *Automatica*, 13:59–75, 1977.

L. Ljung. *System Identification 2nd ed. —Theory for the User—*. Prentice Hall, 1999.

L. Ljung, I. Gustavsson, and T. Söderström. Identification of Linear Multivariable Systems Operating Under Linear Feedback Control. *IEEE Tr. on Automat. Contr.*, AC-19(6):836–840, 1974.

S. Sagara and K. Wada. On-line modified least-squares parameter estimation of linear discrete dynamic systems. *Int. J. Control*, 25:329–343, 1977.