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Port-based Simulation of Flexible Multi-body Systems

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Abstract: This paper is devoted to simulation aspects of complex multi-body systems resulting from the interconnection of rigid and flexible links. This work is the natural complement of Macchelli et al. [2006, 2007a], in which only the mathematical modeling aspects of such kind of devices have been discussed. This paper tries to show how the port Hamiltonian framework can be instrumental also for the easy implementation of efficient simulations if proper packages able to deal with the a-causality of port-based modeling techniques are used. In fact, once the main components (i.e. rigid and flexible links and kinematic pairs) have been created, the complete model just follows by port interconnection in a plug-and-play fashion. Then, it is the simulation engine that solves the causality of the overall scheme and generate the simulation code. The main steps are illustrated in detail with an example.

Keywords: modeling, simulation, robot dynamics, flexible robots, port Hamiltonian systems

1. INTRODUCTION

In recent years, the port Hamiltonian approach has shown its potentialities for modeling and control purposes of complex dynamical systems. Here, the word “complex” means “made of interconnected subsystems with different physical properties”. The resulting “network” is mathematically described by a Dirac structure (Dalsmo and van der Schaft [1999]), a generalization of the Kirchhoff laws of circuit theory. According to the port Hamiltonian formalism, finite (Maschke and van der Schaft [1992]) and infinite dimensional systems (van der Schaft and Maschke [2002]) are characterized by a common *interface*, the power port. This makes the interconnection easier and allows the creation of dynamical models described by ODEs, PDEs and algebraic constraints. It is clear that a manipulator with flexible links belongs to this class.

Following Maschke [1996], Stramigioli [2001], Macchelli et al. [2006], the mechanism is decomposed in its fundamental parts, i.e. rigid and flexible links and kinematic pairs, and its structure described by means of a set of oriented graphs. Based only on the analysis of these graphs, the interconnection equations involving the port variables of the links can be written. Then, the dynamics of the mechanism follows automatically from the interconnection equations and the model of each component. The Hamiltonian description of the rigid link presented in Stramigioli [2001] is adopted, while the flexible link representation is based on the 3D nonlinear flexible beam in port Hamiltonian form discussed in Macchelli et al. [2006, 2007a].

The modularity of the approach can be useful also for simulation purposes. If a modular, object-oriented physical system modeling software package is adopted, beside the mathematical derivation of the model, also the numerical simulation of complex mechanisms can be carried out simply by port interconnection, thus freeing the user from the solution of the causality of each sub-system. In this paper, the simulation package 20-Sim© is used since it is able to deal with bond-graph models, but also other software or modeling languages can be adopted. For example, in Ferretti et al. [2005], the multi-body system dynamics is implemented in the Modelica language.

2. SHORT BACKGROUND ON LIE GROUPS

In order to present the notation adopted in this paper, some basic concepts on Lie groups and Lie algebras are briefly discussed. More details in Stramigioli [2001], Selig [2005]. Denote by $\mathbb{E}_i$ and $\mathbb{E}_j$ a couple of 3-dimensional Euclidean spaces and define a pair of rigid bodies B_i and B_j as subsets of $\mathbb{E}_i$ and $\mathbb{E}_j$ respectively. The relative position of $\mathbb{E}_i$ with respect to $\mathbb{E}_j$ is $h_i^j \in SE_i^j(3)$, with $SE_i^j(3)$ the set of positive isometries from $\mathbb{E}_i$ to $\mathbb{E}_j$.

If $\mathcal{I}$ is an open interval of $\mathbb{R}$, it is possible to consider curves in $SE_i^j(3)$ parametrized by $\tau \in \mathcal{I}$. The differentiable function $h_i^j : \mathcal{I} \to SE_i^j(3)$ is a relative motion and its derivative with respect to τ is $\dot{h}_i^j \in T_{h_i^j} SE_i^j(3)$. It is convenient to transport this vector to the tangent space at the identity of the group, i.e. to a Lie algebra. It is possible to map $\dot{h}_i^j$ either to an element $t_i^{i,j} \in se_i(3)$ or to an element $t_i^{j,j} := t_i^j \in se_j(3)$ by means of the invertible maps $\pi^i_{h_i^j}$ and $\pi^j_{h_i^j}$. More precisely, we have that

$$t_i^{i,j} = h_j^i \circ \dot{h}_i^j =: \pi^i_{h_i^j}\left(\dot{h}_i^j\right) \tag{1a}$$

and

$$t_i^j = \dot{h}_i^j \circ h_j^i =: \pi^j_{h_i^j}\left(\dot{h}_i^j\right) \tag{1b}$$

with $t_i^{i,j}$ and $t_i^j = \mathrm{Ad}_{h_i^j} t_i^{i,j}$ the twist of body B_i with respect to B_j, respectively expressed in $\mathbb{E}_i$ and $\mathbb{E}_j$.

Once $t_i^{i,j} \in se_i(3)$ is given, it is possible to compute the relative motion $h_i^j(\tau)$ of space $\mathbb{E}_i$ with respect to $\mathbb{E}_j$ that passes through $h_i^j(0)$ for $\tau = 0$ with a *velocity* equal to $t_i^{i,j}$. The motion is given by

$$h_i^j(\tau) = h_i^j(0) \circ e^{t_i^{i,j}\tau} \tag{2}$$

where e denotes the group exponential. The group exponential maps an element of the algebra to an element of the group. Consequently, the map $\mathrm{Ad}_{e^{t_i^{i,j}\tau}}$ is well defined as its differential in $\tau = 0$. More precisely, the following linear map within the algebra is well defined:

$$\mathrm{ad}_{t_i^{i,j}} = \frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{Ad}_{e^{t_i^{i,j}\tau}} \Big|_{\tau=0} \tag{3}$$

The same considerations hold for $t_i^j \in se_j(3)$.

Given $h_i^j \in SE_i^j(3)$, it is also possible to consider the co-vectors belonging to $T^*_{h_i^j} SE_i^j(3)$ which, applied to elements of $T_{h_i^j} SE_i^j(3)$, result in a scalar. Based on this duality property, it is possible to define the generalized force between two Euclidean spaces $\mathbb{E}_i$ and $\mathbb{E}_j$ in h_i^j as an element of $T^*_{h_i^j} SE_i^j(3)$. As in (1) for $\dot{h}_i^j$, this element can be intrinsically mapped to $w_i^{i,j} \in se_i^*(3)$ or to $w_i^j \in se_j^*(3)$ by means of the adjoint maps associated to

$$\chi^i_{h_i^j} = \left(\pi^i_{h_i^j}\right)^{-1} \quad \text{and} \quad \chi^j_{h_i^j} = \left(\pi^j_{h_i^j}\right)^{-1} \tag{4}$$

The resulting quantities represent the wrench between B_i and B_j expressed in $\mathbb{E}_i$ and $\mathbb{E}_j$ respectively. Clearly,

$$w_i^{i,j} = \mathrm{Ad}^*_{h_i^j} w_i^j \tag{5}$$

3. MAIN COMPONENTS

3.1 Rigid body

Consider a rigid body in the 3D space and denote by $\mathbb{E}_i$ a reference system connected with it and by $\mathbb{E}_0$ an inertial reference frame. Position and orientation of the rigid body with respect to $\mathbb{E}_0$ are mathematically described by the canonical transformation $h_i^0 \in SE_i^0(3)$. As reported in Maschke [1996], Stramigioli [2001], the port Hamiltonian description of the rigid body motion is

$$\begin{aligned}
\begin{pmatrix} \dot{h}_i^0 \\ \dot{m}^i \end{pmatrix} &= \begin{pmatrix} 0 & \chi_{h_i^0} \circ \mathrm{Ad}_{h_i^0} \\ -\mathrm{Ad}^*_{h_i^0} \circ \chi^*_{h_i^0} & m^i \wedge \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial h_i^0} \\ \frac{\partial H}{\partial m^i} \end{pmatrix} \\
&\quad + \begin{pmatrix} 0 \\ \mathrm{Ad}^*_{h_i^0} \end{pmatrix} w_i^0 \\
t_i^0 &= \begin{pmatrix} 0 & \mathrm{Ad}_{h_i^0} \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial h_i^0} \\ \frac{\partial H}{\partial m^i} \end{pmatrix}
\end{aligned} \tag{6}$$

where $m^i \in se_i^*(3)$ is the momentum corresponding through the inertia tensor I to $t_i^0 \in se_i(3)$. The inertia tensor I defines a quadratic form $\langle \ | \ \rangle_I$ on $se_i(3) \times se_i(3)$, while $Y = I^{-1}$ defines a quadratic form $\langle \ | \ \rangle_Y$ on $se_i^*(3) \times se_i^*(3)$. In (6), the function H represents the energy (Hamiltonian) of the rigid body and it is given by

$$H(h_i^0, m^i) = \frac{1}{2} \left\langle m^i \mid m^i \right\rangle_Y + V(h_i^0)$$

where V is the potential. Moreover, $m^i \wedge$ is a mapping from $se_i(3)$ to $se_i^*(3)$ defined as $(m^i \wedge) t_i^{i,0} = \mathrm{ad}^*_{t_i^{i,0}} m^i$, with $\mathrm{ad}_{t_i^{i,0}}$ introduced in (3). The couple of signals $w_i^0 \in se_i^*(3)$ and $t_i^0 \in se_i(3)$ defines the power port of the rigid body. Here, w_i^0 is the wrench acting on the body and expressed in $\mathbb{E}_0$, while t_i^0 is the twist of $\mathbb{E}_i$ with respect $\mathbb{E}_0$ and expressed in $\mathbb{E}_0$. The dual product of these quantities provides the power flow through the port.

3.2 Flexible link

Consider a slender flexible beam of length L and with an unstressed configuration which is *not* required to be a straight line. Following Golo et al. [2003], Simo [1985], Macchelli et al. [2007a], if $s \in [0, L]$ denotes the position along the link in the unstressed configuration, assume that the configuration in the space of the cross section with respect to an inertial reference $\mathbb{E}_0$ is given by $h_b^0(s) \in SE_b^0(3)$, where the subscript b denotes the *body reference* $\mathbb{E}_b(s)$ attached to the cross section. The unstressed configuration is described by $\hat{h}_b^0(s) \in SE_b^0(3)$.

The distributed port Hamiltonian formulation of the flexible link dynamics is (Macchelli et al. [2007a]):

$$\begin{cases} \partial_t q = \mathrm{d}\delta_p \mathcal{H} + \mathrm{ad}_{(q+\hat{n})} \delta_p \mathcal{H} \\ \partial_t p = \mathrm{d}\delta_q \mathcal{H} - \mathrm{ad}^*_{(q+\hat{n})} \delta_q \mathcal{H} + p \wedge \delta_p \mathcal{H} \end{cases} \tag{7a}$$

with boundary terms given by

$$\begin{aligned} f^B(0) &= \delta_p \mathcal{H} \mid_{s=0} & e^B(0) &= \delta_q \mathcal{H} \mid_{s=0} \\ f^B(L) &= \delta_p \mathcal{H} \mid_{s=L} & e^B(L) &= \delta_q \mathcal{H} \mid_{s=L} \end{aligned} \tag{7b}$$

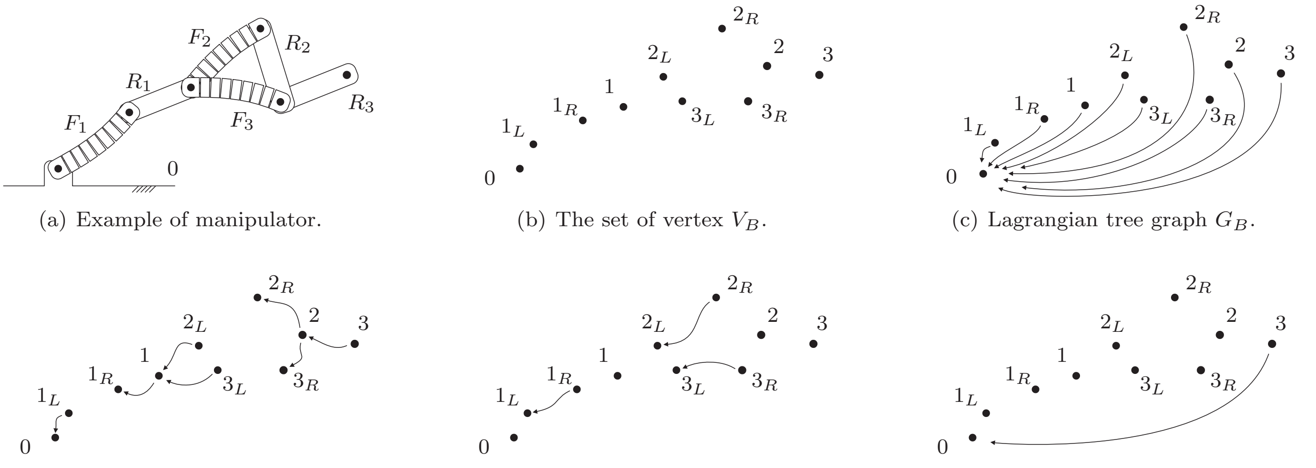
In (7), $\mathcal{H}$ is the total Hamiltonian, given by the sum of two contributions: the kinetic energy and the potential elastic one due to deformation, i.e.

$$\mathcal{H}(p, q) = \frac{1}{2} \int_{\mathcal{Z}} * \Big(\langle *p \mid *p \rangle_Y + \langle *q \mid *q \rangle_{C^{-1}} \Big) \tag{8}$$

Note that the state (energy) variables associated with the flexible link are the infinitesimal deformation q and momentum p, expressed in body reference. More in details, the state space is defined as $\mathcal{X} = \Omega^1_{se(3)}(\mathcal{Z}) \times \Omega^1_{se^*(3)}(\mathcal{Z})$, with q and p $se(3)$-valued and $se^*(3)$-valued one-forms respectively. From a *physical* point of view, it is necessary that these quantities are 1-forms because they are *densities*. If I denotes inertia tensor of the cross section and $p \in se^*(3)$ the momentum of the cross section which corresponds to $t_b^{0,b} \in se_b(3)$ via the tensor I, the first contribution in (8) denotes the kinetic energy density, where $Y = I^{-1}$. Note that $*p$ is a function of $s \in \mathcal{Z}$ with values in $se_b^*(3)$. In the same way, C is the compliance tensor, with inverse C^{-1}, which defines a quadratic form on $se(3)$ taking into account the potential elastic energy due to deformation (Selig and Ding [2001]). Finally, in (7), $\hat{n} = \left(\hat{h}_b^0\right)^{-1} \mathrm{d}\hat{h}_b^0$ is the "twist" in body reference that provides the direction of the unstressed configuration while δ and d denote the variational and exterior derivative (van der Schaft and Maschke [2002]) respectively. Moreover, the following energy balance relation is satisfied:

$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = \left\langle e^B(L), f^B(L) \right\rangle - \left\langle e^B(0), f^B(0) \right\rangle \tag{9}$$

This relation states an obvious property of this physical system, i.e. the fact that the variation of internal energy equals the total power flow at its boundary. Since no



(a) Example of manipulator. (b) The set of vertex V_B. (c) Lagrangian tree graph G_B.

(d) Kinematic pairs graph G_K. (e) Flexible links graph G_F. (f) Interaction graph G_I.

Fig. 1. Mechanism topology, Macchelli et al. [2006]. A manipulator with flexible links and the corresponding Lagrangian tree, kinematic pairs, flexible links and interaction graphs.

dissipative effect is considered, if the boundary energy flow is set to zero (i.e. in the case of a flexible beam clamped at both its extremities) energy is conserved.

3.3 Kinematic pair

Consider two rigid bodies $\mathbb{E}_i$ and $\mathbb{E}_j$. A kinematic pair between $\mathbb{E}_i$ and $\mathbb{E}_j$ is a set of vector fields $k_i^j = \{v_1, \dots, v_p\}$ on the manifold $SE_i^j(3)$. Given a relative configuration $h_i^j \in SE_i^j(3)$, the number of linear independent vectors $v_1(h_i^j), \dots, v_p(h_i^j)$ in $T_{h_i^j} SE_i^j(3)$ is called degree of freedom of the pair in h_i^j. The kinematic pair k_i^j allows a relative motion in the direction $v \in T_{h_i^j} SE_i^j(3)$ if there exist real numbers α_k such that $v = \sum_k \alpha_k v_k$, with $v_k \in k_i^j$.

A kinematic pair is called bilateral in the configuration h_i^j if allowing a motion in the v direction implies allowing in the $-v$ one. In the remaining part of the paper, only holonomic kinematic pairs that do not depend on h_i^j are considered. These kinematic pairs are called *lower pairs*. For any lower pair, it is possible to define a configuration sub-manifold $\mathcal{Q}_i^j$ which describes the allowed configurations of $\mathbb{E}_i$ with respect to $\mathbb{E}_j$. Moreover, given a lower pair and a direction $v \in T_{h_i^j} SE(3)$, it is possible to associate to v two elements t_j^i and t_i^j belonging respectively to $se_i(3)$ and $se_j(3)$. More details in Stramigioli [2001].

The Lie algebras $se(3)$ and $se^*(3)$ are vector spaces that do not depend on the relative configuration h_i^j. So, it makes sense to define the lower pair as a constant subspace of $se_i(3)$ or of $se_j(3)$. These subspaces describe the *allowed twists* for the pair in space $\mathbb{E}_i$ and $\mathbb{E}_j$ respectively and characterize the relative motion. They are indicated with $\mathcal{T}_{i,j}^A$ and $\mathcal{T}_{j,i}^A$. Their complements, which are not unique due to the fact that there is no intrinsic metric in $se(3)$, are called *forbidden twists* subspaces and denoted by $\mathcal{T}_{i,j}^F$ and $\mathcal{T}_{j,i}^F$ respectively. Consequently, we have that

$$se_i(3) = \mathcal{T}_{i,j}^A \oplus \mathcal{T}_{i,j}^F \qquad se_j(3) = \mathcal{T}_{j,i}^A \oplus \mathcal{T}_{j,i}^F \tag{10a}$$

If the dual algebras $se_i^*(3)$ and $se_j^*(3)$ are considered, it is possible to intrinsically define the subspace of the constraint wrenches $\mathcal{W}_{i,j}^C \subset se_i^*(3)$ dual to $\mathcal{T}_{i,j}^A$:

$$\mathcal{W}_{i,j}^C = \left\{ w_j^i \in se_i^*(3) \;\middle|\; \left\langle w_j^i, t_j^i \right\rangle = 0 \;\forall t_j^i \in \mathcal{T}_{i,j}^A \right\}$$

where $\langle\ ,\ \rangle$ denotes the duality product. In the same way, it is possible to define $\mathcal{W}_{j,i}^C$. The *actuation subspaces* $\mathcal{W}_{i,j}^A$ and $\mathcal{W}_{j,i}^A$ are some complement of $\mathcal{W}_{i,j}^C$ and $\mathcal{W}_{j,i}^C$ such that

$$se_i^*(3) = \mathcal{W}_{i,j}^A \oplus \mathcal{W}_{i,j}^C \qquad se_j^*(3) = \mathcal{W}_{j,i}^A \oplus \mathcal{W}_{j,i}^C \tag{10b}$$

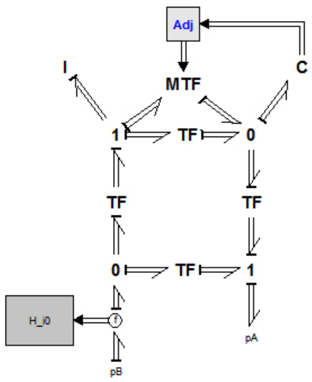
The input control field is a base of the actuation wrenches.

4. MECHANISM TOPOLOGY AND INTERCONNECTION EQUATIONS

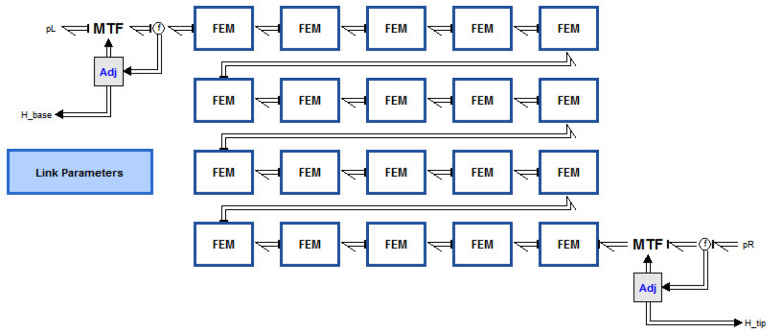
An oriented graph G is a pair (V, E), with V the set of vertexes and $E \subset V \times V$ the set of edges. Given $e = (v_l, v_r) \in E$, the edge e interconnects v_l and v_r, with orientation going from v_l to v_r. Moreover, it is useful to introduce the functions $l, r : E \to V$ defined as $l(e) = v_l$ and $r(e) = v_r$.

The dynamical equations of a generic manipulator made of n_r rigid and n_f flexible links interconnected by n_k kinematic (i.e. lower) pairs can be systematically written once the mechanism is described by a set of oriented graphs. An example of mechanism is given in Fig. 1(a). Each graph takes into account a particular aspect of the structure, e.g. the relative position of the links or the coupling introduced by a kinematic pair, Stramigioli [2001]. The vertexes represent reference frames associated with each rigid link and with the extremities of each flexible link. Since the rigid body constraint is not valid for flexible links, a pair of reference frames is required for describing its configuration in space. The neutral (or inertial) reference $\mathbb{E}_0$ is denoted by 0.

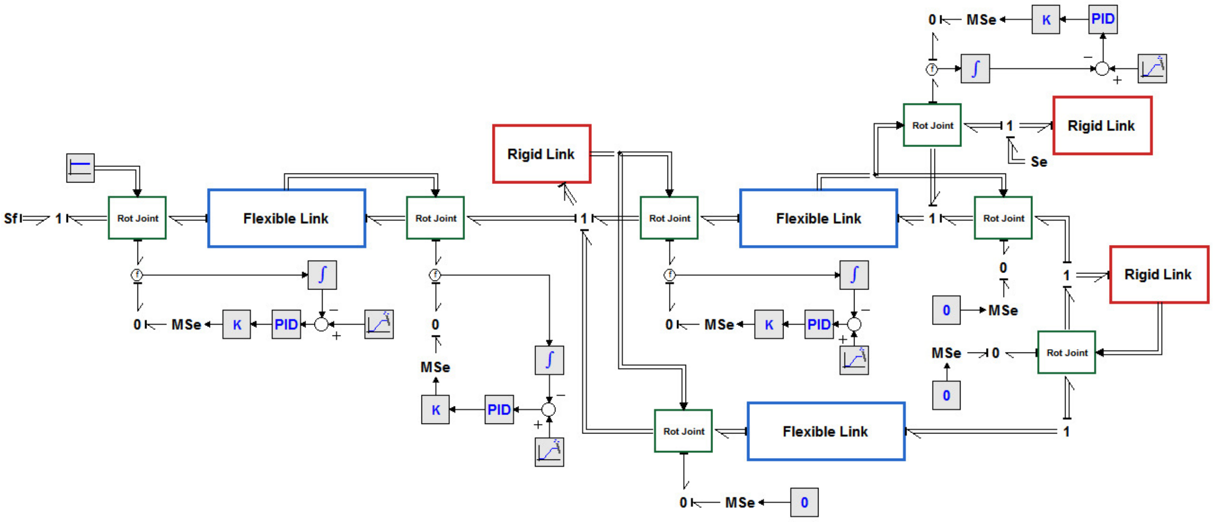
Denote with $V_{B,r}$ and $V_{B,f}$ the set of vertexes associated to rigid and flexible links. More precisely, if the robot is made of n_r rigid links and n_f flexible links, we assume that $V_{B,r} = \{1, \dots, n_r\}$ and that $V_{B,f} = \{1_L, 1_R, \dots, n_{fL}, n_{fR}\}$, where i_L and i_R denote the extremities of the i-th flexible link corresponding to $s = 0$



(a) Finite element dynamics.



(b) Implementation of the finite element approximation of the link ($N = 20$ elements).



(c) Complete 20-Sim© scheme for the mechanism of Fig.1.

Fig. 2. Modeling the flexible link in 20-Sim©.

and $s = L_i$ respectively. Finally, define the set of all the vertexes as $V_B = V_{B,r} \cup V_{B,f}$, as reported in Fig. 1(b). Then, the following oriented graphs can be defined:

- **Lagrangian tree graph.** $G_B = (V_B \cup \{0\}, E_B)$ Each edge in E_B is oriented from $v \in V_B$ to 0 and represents the configuration of the reference system associated to v with respect to the inertial frame (see Fig. 1(c)).
- **Kinematic pairs graph.** $G_K = (V_B \cup \{0\}, E_K)$ Each edge $e = (i, j) \in E_K$ represents the kinematic pair k_i^j which allows a relative motion between the references $\mathbb{E}_i$ and $\mathbb{E}_j$.
- **Flexible links graph.** $G_F = (V_{B,f}, E_F)$ Each edge $e = (n_L, n_R) \in E_F$ interconnects a pair of vertexes representing the extremities of the n-th flexible link (see Fig. 1(e)).
- **Interaction graph.** $G_I = (V_B \cup \{0\}, E_I)$ Each of the n_i edges $e = (i, j) \in E_I$ indicates the place where an external system is able to interact with the mechanism. In particular, it denotes the fact the environment applies a wrench between $\mathbb{E}_i$ and $\mathbb{E}_j$.

The *kinematic graph* $G_{KIN} = (V_B \cup \{0\}, E_F \cup E_K)$ is necessary to compute the direct kinematics of the mechanism, while interconnection equations follow from the *total graph* $G_T = (V_B \cup \{0\}, E_K \cup E_F \cup E_I)$. Note that G_B is a spanning tree of G_T.

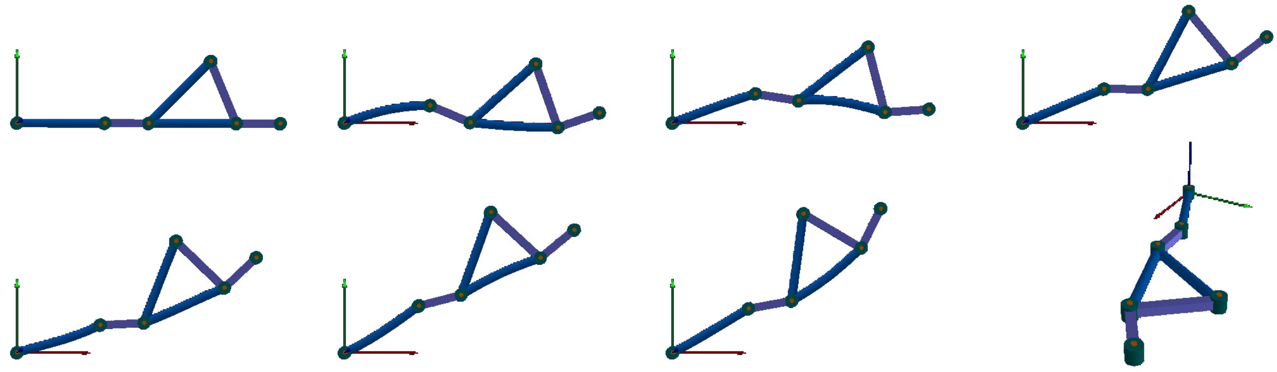
Once the main components have been presented in Sect. 3, the next step is to describe the mechanical structure. Given the total graph G_T, for any cycle C we have (Stramigioli [2001])

$$\sum_{i \in C} \mathrm{Ad}_{h^k_{r(i)}} t^{r(i)}_{l(i)} = 0 \tag{11}$$

where k is a fixed vertex belonging to the cycle. Dually, the next relation relates wrenches and it is simply the action and reaction principle. For every vertex k, if $\bar{C}_k$ denotes the set of edges adjacent to k, i.e. the co-cycle, then

$$\sum_{i \in \bar{C}_k} \mathrm{Ad}^*_{h^{r(i)}_k} w^{r(i)}_{l(i)} = 0 \tag{12}$$

where either $r(i) = k$ or $l(i) = k$. Relations (11) and (12) have to be written for every cycle and co-cycle in G_T. In Macchelli et al. [2006], it is how the complete dynamical model follows from these interconnection equations. For simulation purposes, if a graphical editor is available, these relations are automatically computed by the numerical solver. The next section is devoted to illustrate the procedure on the example of Fig. 1(a).



(a) First part of the transitory. (b) Final configuration.

Fig. 3. Illustrative example.

5. SIMULATION OF COMPLEX MULTI-BODY SYSTEMS

5.1 Port-based simulation

Following the same rationale behind the derivation of the mathematical model of a multi-body system, the simulation of whatever complex mechanism can be carried out simply by port interconnection of its main components, i.e. rigid and flexible links with kinematic pairs. Differently form signal-based approaches, which require to explicitly solve the constraints and the causality of each sub-system, the port-based approach leaves these steps to the simulation package that elaborates the topology of the system. These concepts have been implemented, for example, in the package 20-Sim©, which has been adopted in this paper in order to validate the proposed modeling and simulation methodology.

5.2 Main components

Rigid link The simulation of the rigid link dynamics basically requires the implementation of (6) via a bond graph scheme. Within this formalism, the rigid link can be described by an inertia element $\mathbb{I}$ characterized by 6 dof. Main difficulty is that the dynamics is written in terms of quantities expressed in a moving reference frame (the body frame) in order to keep the inertia tensor constant. Consequently, the port variables, i.e. the twist of the rigid body and the applied wrench, which are expressed with respect to a fixed reference frame (usually the base frame), have to be *transformed* via the group adjoint map.

Flexible link The flexible link is modeled as an infinite dimensional system in the form (7a) where the boundary conditions are given by (7b), thus defining a pair of power ports at the extremities of the link. Consequently, a finite dimensional approximation is necessary for carrying out simulations with, possibly, varying boundary conditions originated, for example, by state-feedback controllers. Classical discretization procedures generally assume that boundary conditions are given but, in this case, a boundary interaction (e.g. the torques/forces applied at the extremities by actuators) has to be taken into account. In Golo et al. [2004], a novel spatial discretization procedure based on a particular type of mixed finite element and able to provide a finite dimensional input/output approximated system has been introduced. From the analysis of the geometric structure of the distributed port Hamiltonian system, a finite dimensional *a-causal* port Hamiltonian approximation that satisfies the same energy balance relation of its infinite dimensional counterpart can be obtained without any particular hypotheses on the boundary conditions. This technique has been generalized to a wider range of distributed systems with 1D spatial domain in Bassi et al. [2006] and it has been applied to the flexible link model (7) in Macchelli et al. [2007b]. Once the spatial domain of the flexible link has been divided into N parts, the infinite dimensional dynamics is approximated on each portion of the domain by means of a finite dimensional system whose bond graph and 20-Sim© implementation is reported in Fig. 2(a). This nonlinear system can be written in port Hamiltonian form and it is characterized by a couple of power ports, i.e. a pair twist/wrench, which represent finite dimensional counterpart of the boundary conditions (7b). These power ports are used to interconnect all of these *components* so that the whole finite dimensional approximation of the link dynamics can be obtained. The result is the bond graph reported in Fig. 2(b).

Kinematic pair The kinematic pair can be simulated once the decomposition (10) is implemented. Following Karnopp et al. [2006], this can be practically realized by using a rigid spring and a damper on the "components" of the relative velocity which are constrained by the kinematic pair. On the other hand, 20-Sim© offers the possibility of explicitly introduce flow (i.e. velocity) constraints, thus freeing the user from the particular internal implementation. Consider, for example, a rotational joint in a planar motion and denote by (v_x, v_y, ω) the relative twist and by (f_x, f_y, τ) the wrench. Then, the kinematic pair can implemented with the following code:

```
fcon=[vx,vy];
econ=constraint(fcon);
[fx,fy]=econ;
```

The instruction `econ=constraint(fcon)` computes the effort `econ` such that the dual flow `fcon` is set equal to 0. This command directly interacts with the numerical integration method, thus providing, in principle, better performances, flexibility and robustness. Clearly, the port behavior at (ω, τ) is specified, for example, by the electric motor interconnected to the joint.

5.3 An illustrative example

The proposed modeling and simulation methodology for complex multi-body systems is illustrated through an example in which a serial mechanical structure and a closed kinematic chain made of rigid and flexible links interconnected by means of rotational joints coexist. More precisely, the simulation of the mechanism of Fig. 1(a) is illustrated. Once the mathematical model of rigid and flexible links, together with the kinematic pairs, have been implemented within the simulation software, the complete description of the mechanism can be obtained simply by port interconnection and the resulting scheme is reported in Fig. 2(c).

In Fig. 3(b), the 3D description of the mechanism is reported, while a simple animation is presented in Fig. 3(a). The actuators have been interconnected to the joints denoted by q_1, q_2, q_3 and q_4, which correspond to the kinematic pairs $(1_L, 0)$, $(1, 1_R)$, $(3_L, 1)$ and $(3, 2)$ (see Fig. 1(d)). Under the effect of a decentralized PD controller, the mechanism is stabilized around a desired configuration. The effect of such a controller, in fact, can be interpreted as the result of a spring and a damper and can asymptotically stabilize the mechanical system, De Luca and Siciliano [1996]. Vibration have been induced by the flexible links and transient can be made faster if the dissipative effect of the damper is increased. The parameters of the elastic links have been chosen in such a way that *large* deformation appear in order to show the performances of the flexible link model. Once the vibrations have been fully damped, the mechanism reaches the steady state configuration of Fig. 3(b).

6. CONCLUSIONS AND FINAL REMARKS

In this paper, a systematic procedure for the simulation of manipulators with flexible links has been presented. Starting from a limited set of components, i.e. rigid and flexible links and kinematic pairs, and based on a finite elements and port based approximation of the flexible link, it has been shown how it is possible to construct simulations of complex mechanism simply by port interconnection using appropriate software packages, such as 20-Sim© . In this paper, an illustrative example has been provided, together with useful indications and suggestion on how all the main components (rigid bodies, flexible links and kinematic pairs) can be implemented. Future work will deal with the implementation of the proposed methodology for example within the package 20-Sim©, which is is now able to handle models of rigid mechanisms by using a systematic procedure similar to what has been discussed in the present paper. An open issue is the development of control schemes that take advantage of the infinite dimensional and nonlinear description of the link dynamics.

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