

## Localization based on observations linear in log range<sup>\*</sup>

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Abstract Received signal strength (RSS) is used in wireless networks as a ranging measurement for positioning and localization services. This contribution studies conceptually different networks, where neither transmitted power or the path decay constant can be assumed to be known. The application in mind is a rapidly deployed network consisting of a number of sensor nodes with low-bandwidth communication, each node consisting of a number of sensor types measuring RSS. Typical sensors measure acoustic, seismic, magnetic and IR power emitted from a target. First, a model linear in the unknown nuisance parameters (transmitted power and path loss constant) is presented and validated from real data. Then, the separable least squares principle is applied to the non-linear least squares (NLS) cost function, after which a cost function of only the unknown position is obtained. Results from field trials are presented to validate the method.

Keywords: sensor networks, estimation, Kalman filtering

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### 1. INTRODUCTION

This contribution considers a sensor network scenario, where each sensor unit has a multitude of sensors measuring received signal strength (RSS) from one target. The considered problem focuses on target localization, but the reverse problem of navigation of one sensor from several beacons, or targets, with known position is also covered by reversing the role of transmitters and receivers. An underlying assumption is that communication constraints between the sensor units make any algorithm based on the signal waveform (like coherent detection) infeasible. Communication only allows for sending RSS measurements to other sensor units.

Explicit algorithms and performance bounds are derived for energy-based measurements, including sensors of radio, acoustic, seismic, infra-red (IR) or microwave energy. Localization from RSS is of course a fairly well studied problem, see the surveys Patwari et al. [2003], Gezici et al. [2005], Gustafsson and Gunnarsson [2005] and the papers Li and Hu [2003], Huang et al. [2000], though the major part of literature addresses the related problem of localization from time of arrival (TOA) and time-difference of arrival (TDOA) measurements. While TOA measures range and TDOA range differences computed from propagation time, energy based localization utilizes the exponential power decay of the involved signals. Measuring the received power in decibels, the measurements are proportional to the logarithm of distance, and this is the main difference to time-based localization approaches. Dedicated approaches to this problem assume that the path loss exponent is known Li and Hu [2003], Huang et al. [2000], or include the RSS measurements as a general non-linear relation Gustafsson and Gunnarsson [2007]. Several *ad-hoc* methods to eliminate nuisance parameters have

been proposed in this context, including taking pairwise differences or ratios of observations.

The theory of RSS based localization is here extended to the case where both the path loss exponent and transmitted power are unknown nuisance parameters. These nuisance parameters are removed from a set of RSS measurements using the separable least squares principle, after which the resulting problem is non-linear in target state parameters only, and a standard low-dimension non-linear least squares (NLS) problem, where efficient numerical algorithms exist. Algorithms of different complexity and performance are outlined for this framework. Tracking algorithms are also described, which are based on stating the localization NLS problem formulation as the measurement relation in an extended Kalman filter.

The fundamental performance bound implied by the Cramér-Rao lower bound enables efficient analysis of sensor network architecture, management and resource allocation. This bound has been analyzed thoroughly in the sensor network literature, primarily for TOA, TDOA and angle-of-arrival (AOA), Patwari et al. [2003], Gezici et al. [2005], Gustafsson and Gunnarsson [2005], but also for RSS Koorapaty [2004], Qi and Kobayashi [2003] and with specific attention to the impact from non-line-of-sight Qi and Kobayashi [2002b,a]. Numerical explicit algorithms and Cramér-Rao lower bounds (CRLB) for both stationary and moving target are derived for the NLS problem formulation.

The proposed approach is applied to measurements from field tests, and preliminary results of this is reported in the last section.

### 2. RECEIVED SIGNAL STRENGTH MEASUREMENTS

This section formulates the *received signal strength (RSS)* from each sensor type as an exponential decay as a model, and validates the assumptions on data from field trials.

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### 2.1 Model

Consider a sensor network, where each sensor unit is located at position  $p_k$  measuring a variety of RSS using different sensors. The received power from each sensor type  $i$  at unit  $k$  is assumed to *in average* follow an exponential decay rate

$$\bar{P}_{k,i} = \bar{P}_{0,i} \|x - p_k\|^{n_{p,i}}. \quad (1)$$

The bar on  $\bar{P}$  indicates power in linear scale. In the sequel, power will be given in logarithmic scale and the bar will be dropped.

Both the transmitted energy  $P_{0,i}$  and path loss constant  $n_{p,i}$  are assumed unknown. Further, these are different for each sensor type, but spatially constant in the local environment where the sensor network operates.

$$P_{k,i} = P_{0,i} + n_{p,i} \underbrace{\log(\|x - p_k\|)}_{\triangleq c_k(x)}. \quad (2)$$

The fundamental log range (LR) term  $c_k(x)$  is here introduced.

Non-line of sight (NLOS) is a major issue in radio based localization. Basically, NLOS invalidates the exponential model (1). NLOS is less of an issue for seismic, magnetic and acoustic waves, partly because of their different nature and partly because of the rather limited range of operation.

### 2.2 Validation from Field Trials

Sensor measurements from field trials are used to illustrate the validity of the log range linear model. The sensor unit, equipped with an acoustic and a seismic sensor, is located a few meters from a road, see Figure 1.

The positions of the vehicle and sensor are known in this model validation case, that is,  $c_k(x)$  is known, and (2) becomes a standard least squares problem in  $P_{0,i}$  and  $n_{p,i}$ .

Figure 1 visualizes the received signal energy as a function of the vehicle position  $x$  along the road, where the origin is defined as the closest point to the sensor.

Figure 2 illustrates that the log range linear model is very reasonable, and that the data fits the model in (3). For example, the exponent estimates at the specific field trial environment are  $n_{1,1} = -2.3$  for the acoustic sensor, and  $n_{1,2} = -2.6$  for the seismic sensor. Note also from Figure 2 that the noise level is fairly independent of range in logarithmic scale, which confirms the assumption that noise is additive to the logarithmic RSS measurements.

## 3. SENSOR FUSION MODEL

The previous section shows that the received signal strength can be seen as a function of log range which is linear in the environmental nuisance parameters. This model will be referred to as the *log range linear model (LRLM)*.

The sensor error has zero mean (otherwise, the mean can be incorporated in the nuisance parameter  $P_{0,i}$ ) and variance  $\text{Var}(e_{k,i}) = \sigma_{P,i}^2$  independent of range. This model implies that the energy-based measurements get worse quality in a linear scale with increasing distance Patwari et al. [2003]. Both mean and variance are assumed constant for each sensor unit, but different for different sensor types. In this section,  $\sigma_{P,i}^2$  is assumed known or accurately

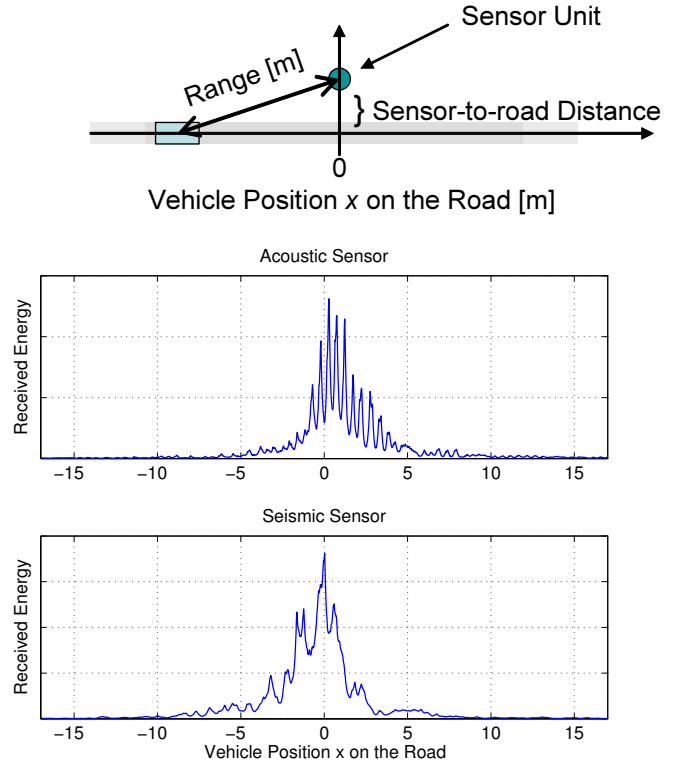


Figure 1. Sensor layout, and received sensor energy in linear scale at the acoustic and seismic sensor, respectively.

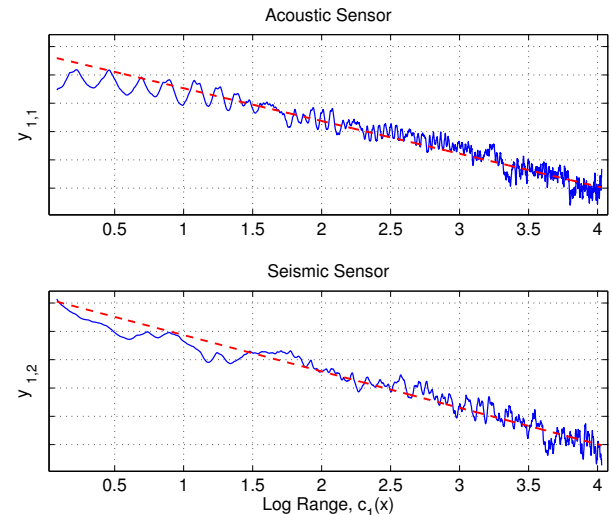


Figure 2. Received sensor energy in log scale, together with a fitted linear relation as modeled in (3).

estimated from measurements when no target is present. Later on, more details on how this can be done is given.

That is, the model (2) is linear in the nuisance parameters  $P_{0,i} = \log \bar{P}_{0,i}$  and  $n_{p,i}$ ,

$$y_{k,i} = P_{0,i} + n_{p,i} c_k(x) + e_{k,i}. \quad (3)$$

Collecting all relations of the kind (3) for different sensor types  $i = 1, 2, \dots, M$  and sensor units  $k = 1, 2, \dots, N$  yields a non-linear equation system

$$\mathbf{y} = \mathbf{h}(x) + \mathbf{e}, \quad (4a)$$

$$\mathbf{y} = \begin{pmatrix} y_{1,1} \\ y_{2,1} \\ y_{3,1} \\ \vdots \\ y_{N,M} \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} e_{1,1} \\ e_{2,1} \\ e_{3,1} \\ \vdots \\ e_{N,M} \end{pmatrix}, \quad (4b)$$

$$\mathbf{h} = \begin{pmatrix} P_{0,1} + n_{p,1}c_1(x) \\ P_{0,2} + n_{p,1}c_2(x) \\ P_{0,2} + n_{p,1}c_3(x) \\ \vdots \\ P_{0,M} + n_{p,M}c_N(x) \end{pmatrix}, \quad (4c)$$

$$\text{Cov}(\mathbf{e}) = \mathbf{R} = \text{diag}(\sigma_{P,1}^2(x)I_N, \dots, \sigma_{P,M}^2(x)I_N). \quad (4d)$$

Solving this in the least squares sense is the subject of the next section.

#### 4. NLS ESTIMATION USING THE LRLM

Let  $\theta_i = (n_{p,i}, P_{0,i})^T$  denote the unknown parameters in the LRLM for sensor type  $i$ , and  $\theta = (\theta_1, \theta_2, \dots, \theta_M)^T$  the vector of unknowns for all sensor types. The non-linear least squares approach aims at minimizing the sum of squared errors between observations and the model with respect to target location  $x$  and the nuisance parameters in  $\theta$ . This can be stated as

$$(\hat{x}, \hat{\theta}) = \arg \min_{x, \theta} V(x, \theta), \quad (5a)$$

$$V(x, \theta) = \sum_{i=1}^M \sum_{k=1}^N \frac{(y_{k,i} - h(c_k(x), \theta_i))^2}{\sigma_{P,i}^2}, \quad (5b)$$

$$h(c_k(x), \theta_i) = \theta_{i,1} + \theta_{i,2}c_k(x), \quad (5c)$$

$$c_k(x) = \log(\|x - p_k\|). \quad (5d)$$

The goal in this section is to eliminate the nuisance parameters  $\theta_i$ , including the path loss constant  $n_{p,i}$  and transmission power  $P_{0,i}$ , and the unknown noise variances  $\sigma_{P,i}$  for  $i = 1, 2, \dots, M$ .

##### 4.1 Elimination of Nuisance Parameters by Separable Least Squares

Using the separable least squares (LS) principle, the environmental parameter,  $n_{p,i}$ , and transmission power,  $P_{0,i}$ , can be eliminated explicitly from (5) for each sensor type  $i$ . The algebraic minimizing argument of (5b) is given by

$$\hat{\theta}_i(x) = \left[ \underbrace{\begin{pmatrix} N & \sum_{k=1}^N c_k(x) \\ \sum_{k=1}^N c_k(x) & \left(\sum_{k=1}^N c_k(x)\right)^2 \end{pmatrix}}_{R(x)} \right]^{-1} \times \underbrace{\begin{pmatrix} \sum_{k=1}^N y_{k,i} \\ \sum_{k=1}^N c_k(x)y_{k,i} \end{pmatrix}}_{f_i(x)} \quad (6)$$

Note that the parameter estimate depends on the target location  $x$ . The matrix  $R(x)$  and vector  $f_i(x)$  are introduced in (6) to get more compact notation in the following.

Note also that the matrix  $R(x)$  is just a function of sensor geometry and target position. The matrix inversion can be eliminated to get

$$R(x) = \frac{1}{N \sum_{k=1}^N c_k^2(x) - \left(\sum_{k=1}^N c_k(x)\right)^2} \times \begin{pmatrix} \sum_{k=1}^N c_k^2(x) & -\sum_{k=1}^N c_k(x) \\ -\sum_{k=1}^N c_k(x) & N \end{pmatrix} \quad (7)$$

With this matrix defined, the covariance matrix is given by

$$\text{Cov}(\hat{\theta}_i(x)) = \sigma_{P,i}^2 R(x). \quad (8)$$

The variance of the LRLM after plugging in the parameter estimate is thus given by

$$\text{Var}(h(c_k(x), \hat{\theta}(x))) = \sigma_{P,i}^2 (1, c_k(x)) R(x) (1, c_k(x))^T. \quad (9)$$

##### 4.2 Sensor Noise Variance Estimation

Further, the minimum of the sum of least squares for sensor type  $i$  can be taken as an estimate of the measurement variance as

$$\hat{\sigma}_{P,i}^2(x) = \frac{1}{N-2} \sum_{k=1}^N (y_{k,i} - h(c_k(x), \hat{\theta}_i(x)))^2 \quad (10)$$

$$= \frac{1}{N-2} \left( \sum_{k=1}^N y_{k,i}^2 - f_i^T(x) \hat{\theta}_i(x) \right), \quad (11)$$

where the normalization with  $N-2$  accounts for the number of freedom lost by the minimization, and is needed to get an unbiased variance estimate. The last equality is a consequence of the LS theory, and will be used in the NLS formulation below.

##### 4.3 LRLM NLS Formulation

The NLS formulation in (5) is now algebraically equivalent to the following reduced NLS problem in target location  $x$

$$\hat{x} = \arg \min_{x, \theta} V(x, \theta, \sigma_P) = \arg \min_x V(x, \hat{\theta}(x), \sigma_P), \quad (12a)$$

$$V(x, \hat{\theta}(x), \sigma_P) = \sum_{i=1}^M \sum_{k=1}^N \frac{(y_{k,i} - h(c_k(x), \hat{\theta}_i(x)))^2}{\sigma_{P,i}^2} = \sum_{i=1}^M \frac{\sum_{k=1}^N y_{k,i}^2 - f_i^T(x) \hat{\theta}_i(x)}{\sigma_{P,i}^2}, \quad (12b)$$

The new weighting in the sum of least squares accounts for both measurement noise and the estimation uncertainty in the nuisance parameters. Typically, far away sensor nodes  $k$  or uncertain sensor types  $i$  get larger uncertainty in the parameters and thus automatically a smaller weight in the criterion.

Note that the noise variance has to be known in the NLS approach above. The simple idea of plugging in the estimate does not work, since

$$V(x, \hat{\theta}(x), \hat{\sigma}_{P,i}^2) = M(N-2). \quad (13)$$

The maximum likelihood (ML) approach can be used to circumvent this problem, as described in Section 5.2.

## 5. LOCALIZATION ALGORITHMS

In summary, in the previous section we have derived the LR model

$$\mathbf{y} = \mathbf{h}(x, \hat{\theta}(x)) + \mathbf{e}, \quad (14a)$$

$$\mathbf{h}_{k,i} = h(c_k(x), \hat{\theta}_i(x)), \quad (14b)$$

$$\text{Cov}(\mathbf{e}) = \mathbf{R} = \text{diag}(\sigma_{P,1}^2(x)I_N, \dots, \sigma_{P,M}^2(x)I_N). \quad (14c)$$

Here,  $\hat{\theta}_i(x)$  is given in (6),  $c_k(x)$  in (5d),  $h(c_k(x), \hat{\theta}_i)$  in (5c), and  $R(x)$  in (7). The purpose here is to outline possible implementation strategies.

### 5.1 Estimation Criteria

The derivation in Section 4 was motivated by NLS. However, the same elimination of nuisance parameters can be applied to more general maximum likelihood (ML) approaches, with a Gaussian assumption or with other assumptions on sensor error distributions, as summarized in Table 1. The Gaussian ML (GML) approach is useful when the variances of the individual measurements include important localization information in themselves.

Table 1. Optimization criteria  $V(x)$

<b>NLS</b>	$V^{NLS}(x) = (\mathbf{y} - \mathbf{h}(x))^T \mathbf{R}^{-1}(x) (\mathbf{y} - \mathbf{h}(x))$
<b>GML</b>	$V^{GML}(x) = (\mathbf{y} - \mathbf{h}(x))^T \mathbf{R}^{-1}(x) (\mathbf{y} - \mathbf{h}(x)) + \log \det \mathbf{R}(x)$
<b>ML</b>	$V^{ML}(x) = \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))$

### 5.2 Eliminating the noise variance

The GML approach is also needed in the case where the noise variance is unknown. Minimizing the GML cost with respect to  $\sigma_{P,i}$  gives a result similar to (12b),

$$\min_{\sigma_P} V^{GML}(x, \sigma_P) = \sum_{i=1}^M N \log \left( \sum_{k=1}^N y_{k,i}^2 - f_i^T(x) \hat{\theta}_i(x) \right). \quad (15)$$

The logarithm intuitively decreases the difference in weighting between the different sensor types compared to the case of known noise variances in (12b).

### 5.3 Estimation Algorithms

As in any estimation algorithm, the classical choice is between a gradient and Gauss-Newton algorithm, see Dennis Jr. and Schnabel [1983]. The basic forms are given in Table 2. Here,  $\mathbf{H}(x) = \nabla_x \mathbf{h}(x)$  for NLS and GML and  $\mathbf{H}(x) = \nabla_x \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))$  for ML. These local search algorithms generally require good initialization, otherwise the risk is to reach a local minimum in the loss function  $V(x)$ . Today, simulation based optimization techniques may provide an alternative.

Table 2. Estimation algorithms for Table 1.

<b>Grid-based</b>	$\hat{x} = \arg \min_{x=x^{(k)}} \sum_{i=1}^M \hat{\sigma}_{P,i}^2(x)$
<b>Steepest descent</b>	$\hat{x}_k = \hat{x}_{k-1} + \mu_k \cdot \mathbf{H}^T(\hat{x}_{k-1}) \mathbf{R}^{-1}(y - \mathbf{H}(\hat{x}_{k-1}) \hat{x}_{k-1})$
<b>Newton-Raphson</b>	$\hat{x}_k = \hat{x}_{k-1} + \mu_k \left( \mathbf{H}^T(\hat{x}_{k-1}) \mathbf{R}^{-1} \mathbf{H}(\hat{x}_{k-1}) \right)^{-1} \cdot \mathbf{H}^T(\hat{x}_{k-1}) \mathbf{R}^{-1}(y - \mathbf{H}(\hat{x}_{k-1}) \hat{x}_{k-1})$

### 5.4 Gradient Derivation

In these numerical algorithms, the gradient  $\mathbf{H}(x) = \nabla_x \mathbf{h}(x)$  of the model with respect to the position is instrumental, and it is the purpose here to derive the necessary equations.

First, it is easier to apply the chain rule to the expression

$$c_k(x) = \log(\|x - p_k\|) = \frac{1}{2} \log(\|x - p_k\|^2), \quad (16)$$

though the result is the same in the end. The gradient is then immediate as

$$\frac{dc_k(x)}{dx} = \frac{x - p_k}{\|x - p_k\|^2}. \quad (17)$$

The gradient of the NLS loss function  $\bar{V}(x, \hat{\theta}(x))$  becomes a function of the gradients of  $\hat{\theta}_i(x)$  and  $R(x)$ . These are all tedious but straightforward applications of the chain rule, not reproduced here. However, the point is that everything that is needed in the optimization algorithms surveyed in the next section are symbolic functions in target location  $x$  and sensor locations  $p_k$  only.

### 5.5 Model Validation and Target Detection

Assume that the noise in the LRLM is Gaussian distributed. The NLS loss function at the true target location  $x^o$  is then  $\chi^2(M(N-2))$  distributed. This can be used for model validation, and also for testing the hypotheses that there is a target.

### 5.6 Fundamental Performance Bounds

The *Fisher Information Matrix (FIM)* provides a fundamental estimation limit for unbiased estimators referred to as the *Cramér-Rao Lower Bound (CRLB)* Kay [1993]. This bound has been analyzed thoroughly in the literature, primarily for AOA, TOA and TDOA, Patwari et al. [2003], Gezici et al. [2005], Gustafsson and Gunnarsson [2005], but also for RSS Koorapaty [2004], Qi and Kobayashi [2003] and with specific attention to the impact from non-line-of-sight Qi and Kobayashi [2002b,a].

For the log range model 3, the  $2 \times 2$  Fisher Information Matrix  $J(x)$  is defined as

$$J(x) = \mathbb{E} \left( \nabla_x^T \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x)) \nabla_x \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x)) \right) \quad (18a)$$

$$\nabla_x \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x)) = \left( \frac{\partial \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))}{\partial x_1} \quad \frac{\partial \log p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))}{\partial x_2} \right) \quad (18b)$$

where  $p$  is the two-dimensional position vector and  $p_{\mathbf{e}}(\mathbf{y} - \mathbf{h}(x))$  the likelihood given the error distribution.

In case of Gaussian measurement errors  $p_{\mathbf{e}}(e) = N(0, \mathbf{R}(x))$ , the FIM equals

$$J(x) = \mathbf{H}^T(x) \mathbf{R}(x)^{-1} \mathbf{H}(x), \quad (19a)$$

$$\mathbf{H}(x) = \nabla_x \mathbf{h}(x). \quad (19b)$$

This form is directly applicable to the log range linear model (3).

In comparison, if the nuisance parameters were known, the FIM can be obtained in the Gaussian measurement error case as the following sum over sensors

$$\begin{aligned}
 J(x) &= \sum_{i=1}^M \sum_{k=1}^N \nabla_x h(c_k(x), \theta_i) \\
 &= \sum_{i=1}^M \sum_{k=1}^N \frac{\theta_{i,1}^2}{\sigma_{P,i}^2 \|x - p_k\|^2} (x - p_k)(x - p_k)^T \quad (20)
 \end{aligned}$$

Plausible approximative scalar information measures are the trace of the FIM and the smallest eigenvalue of FIM

$$J_{\text{tr}}(x) \triangleq \text{tr } J(x), \quad J_{\text{min}}(x) \triangleq \min \text{ eig } J(x). \quad (21)$$

The former information measure is additive as FIM itself, while the latter is an under-estimation of the information useful when reasoning about whether the available information is sufficient or not. Note that in the Gaussian case with a diagonal measurement error covariance matrix, the trace of FIM is the squared gradient magnitude.

The Cramér-Rao Lower Bound is given by

$$\text{Cov}(\hat{x}) = \text{E}(x^o - \hat{x})(x^o - \hat{x})^T \geq J^{-1}(x^o), \quad (22)$$

where  $x^o$  denotes the true position. The CRLB holds for any unbiased estimate of  $\hat{x}$ . It is in general hard to tell if an estimate is unbiased or not, and the unbiasedness of the estimation problem in this paper is still an open question. A bias mostly increase the obtained mean square error, but not always. For instance, the data independent estimate  $\hat{x} = 0$  is biased with zero variance and a small MSE when the true position is close to the origin. Nevertheless, we will use the CRLB as an indicator of how an attainable MSE varies over space for a given network deployment. Further, even if the estimate is unbiased, the lower bound may not be an attainable bound. It is known that asymptotically in the number of sensor nodes, the ML estimate is  $\hat{x} \sim \text{N}(x^o, J^{-1}(x^o))$  Lehmann [1991] and thus reaches this bound, but this may not hold for finite amount of data.

The right hand side of (22) gives an idea of how suitable a given sensor configuration is for positioning. It can also be used for *sensor network design*. However, it should always be kept in mind though that this lower bound is quite conservative and relies on many assumptions.

In practice, the root mean square error (RMSE) is perhaps of more importance. This can be interpreted as the achieved position error in meters. The CRLB implies the following bound:

$$\begin{aligned}
 \text{RMSE} &= \sqrt{\text{E}((x_1^o - \hat{x}_1)^2 + (x_2^o - \hat{x}_2)^2)} \\
 &= \sqrt{\text{tr Cov}(\hat{x})} \geq \sqrt{\text{tr } J^{-1}(x^o)} \quad (23)
 \end{aligned}$$

If RMSE requirements are specified, it is possible to include more and more measurements in the design until (23) indicates that the amount of information is enough.

An example of CRLB is illustrated in Figure 3. Such plots can be used as guidelines for the sensor node layout, before the network is deployed.

## 6. RESULTS FROM FIELD TRIALS

Extensive experiments have been performed to evaluate the potential of the proposed algorithm. Different targets (military and civilian vehicles, pedestrians *etc*), trajectories and sensor types and node configurations have been tested. We here in detail present the result for a motorcycle passing a network cluster. The trajectory and sensor node layout are illustrated in Figure 4, where these are overlaid a satellite image. The sensor observations are downsampled to 2 Hz before estimation, all sensor types and nodes are

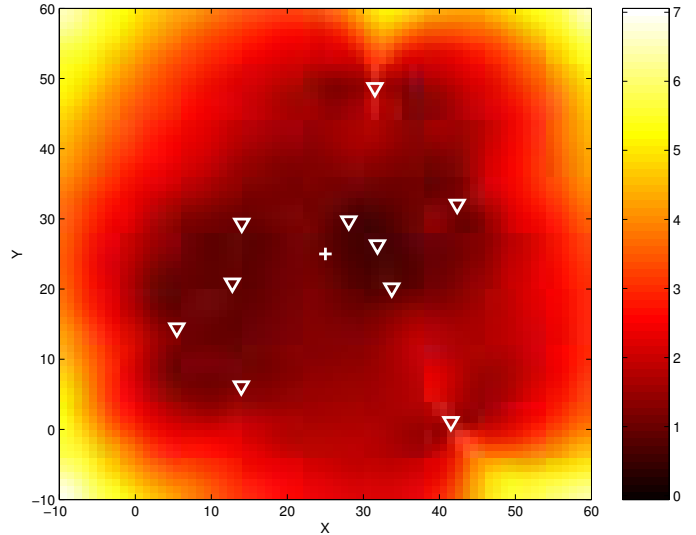


Figure 3. Spatial variations of CRLB indicating how positioning accuracy will vary with the position.

carefully calibrated, and the vehicle is equipped with GPS satellite navigation for validation of the performance.



Figure 4. Sensor node locations and sample trajectory for motor cycle.

The thick cross indicates a point of particular interest. The NLS cost function in Figure (12) as a function of horizontal position is shown in Figure 5 for this particular target location. Though the target is in the center of the network, there are still many minima. This is not a problem for a grid based global minimization algorithm, but might imply problems for local gradient based methods. In such case, a filtering approach should be taken, where the prediction from previous time is used as a starting point. We will not discuss the natural extension to filtering in this contribution, see Gustafsson and Gunnarsson [2007] for details on this.

Figure 6 shows the estimation performance of minimizing (12) as illustrated in 5 at each time instant. The absolute error appears to increase linearly in distance. Though the result is consistent with theory, the estimation accuracy is not acceptable for tracking. The obvious next step is to fuse measurements from more sensor types, and apply a Kalman filter based on a motion model.

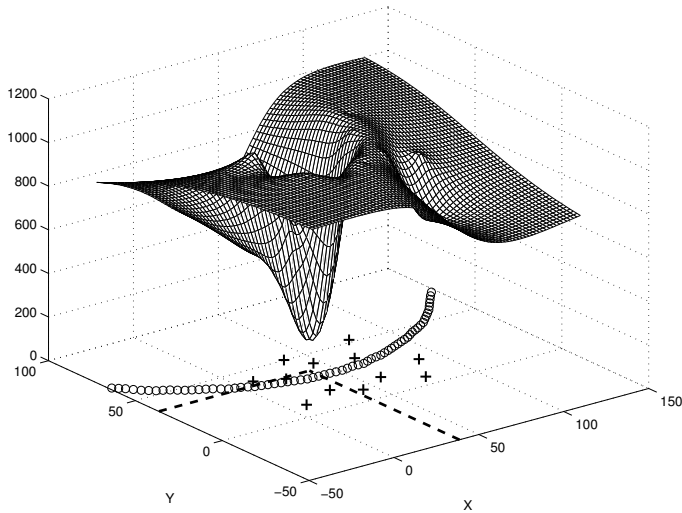


Figure 5. NLS cost function for microphone sensors as a function of position for a certain target location indicated with dashed lines in the contour plot and the thick cross in Figure 4.

### 7. CONCLUSIONS

Conventional received signal strength (RSS) based algorithms as found in the literature of wireless or acoustic networks assume either that the transmitted power is known, or that the path loss constant is known from calibration. We have considered a network that is rapidly deployed in an unknown environment where the path loss constant is unknown, or may vary with time. Also, the transmitted power is inherently unknown is the localization and tracking applications under consideration. For localization, both transmission power and path loss constant are nuisance parameters unique for each target and sensor type, but constant over the sensor nodes.

The nonlinear least squares (NLS) algorithm offers a suitable framework for positioning in this kind of sensor networks, where the RSS measurements suffer from unknown transmitted power and where also the environmental path loss constant is unknown. Marginalization of the nuisance parameters using the separable least squares principle leads to a NLS cost function of only two unknowns (horizontal position), where global grid based methods can be used for minimization. Results from field trials confirm the usability of the proposed method.

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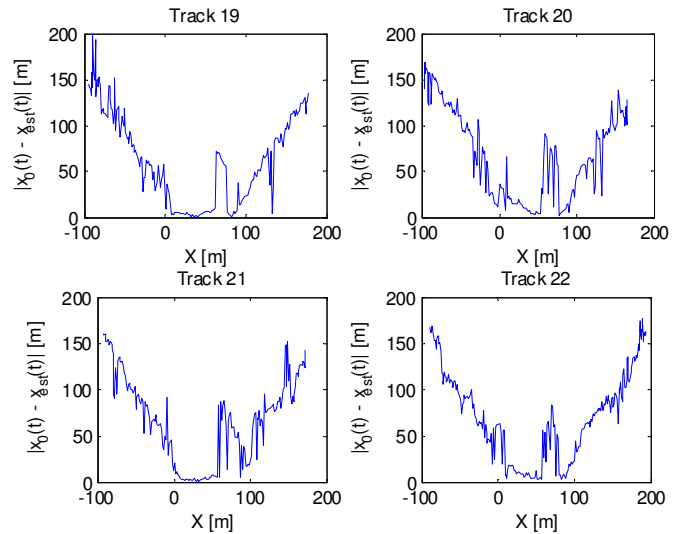


Figure 6. Absolute error (solid) in the grid-based localization algorithm.

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