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The control and identification algorithm for devices with differential inductive sensors

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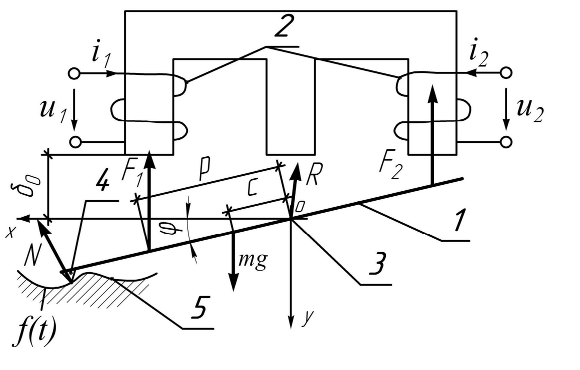
Abstract: The design algorithms of closed loop control system for surface profile measurement device are proposed. Control inputs are used to construct closed loop system, which oriented on identification of desired parameter that is estimated by high gain observer then. As a result, the designed nonlinear system has high selective properties with respect to registered value. The dynamic compensator approach is used to reduce the estimation error.

1. INTRODUCTION

With evolution of scientific and technological progress and aggravation of competitive activity an increase of the requirements with respect to the quality of completed production takes place. Some of the main parameters, which characterize quality of completed production, are roughness, waviness. It is well known that modern contact devices for measuring roughness are based on open loop system. The traditional approach is based on tracing surface by the diamond needle of the sensor (Biryukov G.S. *et al*., 1987). The needle oscillation is transformed then with the help of sensitive transducer into variation of coil inductances. These inductances are connected into bridge scheme, one of the diagonal of which is supplied by the high frequency AC voltage. The primitive construction of the sensor is presented on the fig. 1. In the next step the signal from the sensor is amplified, detected, and filtered and so on similar to signal processing in the highway of the radio transmission. It is obviously that this approach has significant disadvantage. Parametric and additive disturbances can intervene to the output owing to linear character and the open loop of the processing. Moreover, each element of open loop system adds its own dynamics to the process of signal transmission. Other words, this approach is assigned for measuring values, which are varied slowly. Meanwhile, there is a need to design systems, which allow measuring surface profile with fast dynamic (Morrison E., 1996). Algorithms for design of close loop control system, which have no shortcomings listed up, are proposed in this paper. Basic algorithm consists in using control inputs to form close loop system, which is oriented on necessary parameter, which characterizes the surface profile. Then this parameter is identified by high-gain observer. The synthesized nonlinear system has high selective properties with respect to registered value. Simulation results represent the efficiency of proposed control and identification algorithm. The rate of necessary parameter identification is of order of transient time in the control object.

2. PROBLEM STATEMENT

Let us consider the construction of the primary converter (fig. 1).



1 – rotor, 2 – coils, 3 – mount point, 4 – diamond needle, 5 – controlled profile

Fig. 1. The construction of the primary converter

The differential equations of the sensor can be expressed in the following form (Zhuravlev Yu.N, 2003)

$$\begin{cases} \dot{\varphi} = \omega\,; \\ \dot{\omega} = \dfrac{mgc}{J_0} + \dfrac{pk}{2J_0}\left(\dfrac{i_2^2}{(\delta_0 - p\varphi)^2} - \dfrac{i_1^2}{(\delta_0 + p\varphi)^2}\right) + \dfrac{M(t)}{J_0}\,; \\ \dfrac{di_1}{dt} = \dfrac{pi_1\omega}{\delta_0 + p\varphi} - \dfrac{i_1R_1(\delta_0 + p\varphi)}{k} + \dfrac{u_1(\delta_0 + p\varphi)}{k}\,; \\ \dfrac{di_2}{dt} = -\dfrac{pi_2\omega}{\delta_0 - p\varphi} - \dfrac{i_2R_2(\delta_0 - p\varphi)}{k} + \dfrac{u_2(\delta_0 - p\varphi)}{k}\,, \end{cases} \quad (1)$$

where m is mass of rotor, k is constant design coefficient, J_0 is inertia moment, δ_0 is nominal air gap, $M(t)$ is the reaction moment of N.

It is necessary to note that for such sensor the next inequality is fulfilled

$$\delta_0 >> p\varphi . \tag{2}$$

One can exclude from system (1) the mechanical equations and get

$$\begin{cases} \dot{\varphi} = \omega ; \\ \dfrac{di_1}{dt} = -i_1\alpha_1(\varphi,\omega) + b_1(\varphi)u_1 ; \\ \dfrac{di_2}{dt} = -i_2\alpha_2(\varphi,\omega) + b_2(\varphi)u_2 , \end{cases} \tag{3}$$

where $\alpha_1(\varphi,\omega) = -pi_1\omega/(\delta_0 + p\varphi) + R_1(\delta_0 + p\varphi)/k$; $\alpha_2(\varphi,\omega) = pi_2\omega/(\delta_0 - p\varphi) + R_2(\delta_0 - p\varphi)/k$; $b_1(\varphi) = (\delta_0 + p\varphi)/k$; $b_2(\varphi) = (\delta_0 - p\varphi)/k$.

The sensor is moved across controlled surface with constant velocity during measurement cycle. The diamond needle 4 touches with roughness and the position of rotor, determined by angle φ, is changed. Therefore the parameter φ is proportional to the height of controlled profile. Thus, the problem of identification for the value φ can be established in view of the system (3) according to measurements of coil currents i_1, i_2 .

3. SYNTHESIS OF THE CONTROL STRUCTURE

It is well known that modern devices for measurement roughness are based on the principle of amplitude modulation of high frequency alternative voltage, which supplies one of diagonal of differential inductive sensor. This approach is used during many years and has good reputation. Therefore let us use oscillations as the basic mode for operation. So one can consider the system (3) and choose the control inputs u_1, u_2 in such a way, that limit cycle are excited in the following form

$$i_1^2 + i_2^2 = C , \tag{4}$$

where C is any constant.

Besides, if the equation (4) is fulfilled, we get that variables i_1, i_2 are changed according to *sine* and *cosine* laws with amplitude determined by constant C. And in this case i_1 and i_2 are orthogonal functions and can be used as reference signals for high gain identifier.

It is necessary to note that the non-stationary system

$$di_1/dt = \beta(t)i_2 ; di_2/dt = -\beta(t)i_1 , \tag{5}$$

where $\beta(t)$ is continuously differentiable, fulfils the equation (4).

Let us choose the control inputs in combined form

$$\begin{cases} u_1 = \mu b_2 i_2 + \gamma_1 ; \\ u_2 = -\mu b_1 i_1 + \gamma_2 , \end{cases} \tag{6}$$

where $\mu - const$, γ_1, γ_2 are new control inputs.

The substitution (6) to (3) leads to

$$\begin{cases} \dfrac{di_1}{dt} = -\alpha_1 i_1 + \mu\, b_1 b_2 i_2 + b_1\gamma_1 ; \\ \dfrac{di_2}{dt} = -\alpha_2 i_2 - \mu\, b_1 b_2 i_1 + b_2\gamma_2 . \end{cases} \tag{7}$$

Assumption 1. The variables φ and ω are assumed equal to sufficiently small values. Therefore the parameters α_i, b_i $(i = 1,2)$ are differed slightly from their nominal quantities

$$\alpha_i^* = R_i\delta_0/k > 0,\ b_i^* = b^* = \delta_0/k > 0\ (i = 1,2) . \tag{8}$$

One can neglect φ и ω and get linearized system in the form

$$\begin{cases} \dfrac{di_1}{dt} = -i_1\alpha_1^* + (b^*)^2\mu i_2 + b^*\gamma_1 ; \\ \dfrac{di_2}{dt} = -i_2\alpha_2^* - (b^*)^2\mu i_1 + b^*\gamma_2 . \end{cases} \tag{9}$$

The characteristic equation of (9)

$$p^2 + (\alpha_1^* + \alpha_2^*)p + (b^*)^4\mu^2 = 0 . \tag{10}$$

Let us choose the parameter μ so that roots of (10) are complex conjugate quantities, i.e.

$$D = (\alpha_1^* + \alpha_2^*)^2 - 4(b^*)^4\mu^2 << 0 .$$

Thus we get the criterion of choice of the parameter μ

$$\mu >> \sqrt{(\alpha_1^* + \alpha_2^*)/2}\,/(b^*)^2 = \sqrt{k^3(R_1 + R_2)/2\delta_0^3} . \tag{11}$$

It is necessary to note that differential inductive transformer is designed so that it's inductive reactance greater than resistance for some frequency. Therefore we can choose the parameter μ to guaranty (11) always. For example, one can accept μ as

$$\mu = 2\pi f_{nom}/(b^*)^2 = 2\pi f_{nom}k^2/\delta_0^2 . \tag{12}$$

With the help of first term in (6) designed system obtain the oscillator property. Now we must choose second control components γ_1, γ_2 in such a way to guaranty that steady limit cycle are excited in (7).

Let us consider function $V(i)$

$$V(i) = (i_1^2 + i_2^2 - C)/2 , \tag{13}$$

where $i = (i_1 \quad i_2)^T$.

It's derivative according to (7)

$$\dot{V}(i) = -\alpha_1 i_1^2 - \alpha_2 i_2^2 + b_1 i_1\gamma_1 + b_2 i_2\gamma_2 .$$

Let us choose γ_1, γ_2 so that next condition is fulfilled

$$\dot{V} = -V(i)s(i) , \tag{14}$$

where $s(i)$ is any positive definite function, which is equal to zero in the origin of coordinates.

The class of control inputs is determined by (14). We can choose γ_1, γ_2 in the following continuously differentiable form

$$\begin{cases} \gamma_1 = \alpha_1 C i_1 / \left(b_1\left(i_1^2 + i_2^2\right)\right); \\ \gamma_2 = \alpha_2 C i_2 / \left(b_2\left(i_1^2 + i_2^2\right)\right), \end{cases} \tag{15}$$

where C is constant which determine the amplitude of steady oscillations.

One can see that $\dot{V}$ according to (15) is equal to

$$\dot{V} = -2\left(\alpha_1 i_1^2 + \alpha_2 i_2^2\right) V(i) / \left(i_1^2 + i_2^2\right).$$

So we get that $\dot{V}$ is negative and equal to zero in the origin of coordinates. In this case (4) is invariant set of the system (3) with control inputs (6), (15).

4. CONTROL ALGORITHM

We get valuable control structure in the previous chapter. However it's hard to realize (6) and (15) accurately in real devices because of unknown parameters $\alpha_1, \alpha_2, b_1, b_2$. Let us show that any limit cycle is excited in the system (3) with the help of control inputs in the form (6), (15) with some positive parameters $\hat{\alpha}_1, \hat{\alpha}_2, \hat{b}_1, \hat{b}_2$ instead of $\alpha_1, \alpha_2, b_1, b_2$. So according to reasoning we choose u_1, u_2 as

$$\begin{cases} u_1 = \mu \hat{b}_2 i_2 + \hat{\alpha}_1 C i_1 / \left(\hat{b}_1\left(i_1^2 + i_2^2\right)\right); \\ u_2 = -\mu \hat{b}_1 i_1 + \hat{\alpha}_2 C i_2 / \left(\hat{b}_2\left(i_1^2 + i_2^2\right)\right). \end{cases} \tag{16}$$

Assumption 2. The variables $\alpha_1, \alpha_2, b_1, b_2, \hat{\alpha}_1, \hat{\alpha}_2, \hat{b}_1, \hat{b}_2$ are changed slowly in comparison with currents i_1, i_2. In other words we can say that $\alpha_1, \alpha_2, b_1, b_2, \hat{\alpha}_1, \hat{\alpha}_2, \hat{b}_1, \hat{b}_2$ are quasi-stationary parameters.

One can make coordinates transformation in the form

$$i_1 = \sqrt{\mu\, b_1 \hat{b}_2}\, i_1^*, \; i_2 = \sqrt{\mu\, b_2 \hat{b}_1}\, i_2^*. \tag{17}$$

and get the differential equation of the transformed system

$$\begin{cases} \dfrac{di_1^*}{dt} = -\left(\alpha_1 + \dfrac{p\omega}{2k\mu b_1 \hat{b}_2}\right) i_1^* + \mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}\, i_2^* + \dfrac{b_1 \hat{\alpha}_1 C i_1^*}{\hat{b}_1 \mu\left(b_1 \hat{b}_2 i_1^{*2} + b_2 \hat{b}_1 i_2^{*2}\right)}; \\ \dfrac{di_2^*}{dt} = -\left(\alpha_2 - \dfrac{p\omega}{2k\mu b_2 \hat{b}_1}\right) i_2^* - \mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}\, i_1^* + \dfrac{b_2 \hat{\alpha}_2 C i_2^*}{\hat{b}_2 \mu\left(b_1 \hat{b}_2 i_1^{*2} + b_2 \hat{b}_1 i_2^{*2}\right)}. \end{cases} \tag{18}$$

According to assumption 1 the next inequality is right

$$\left| - p\omega / \left(2k\mu b_1 \hat{b}_2\right) + p\omega / \left(2k\mu b_2 \hat{b}_1\right) \right| << \alpha_1 + \alpha_2 . \tag{19}$$

Let us consider the reference system in the form (5)

$$\begin{cases} \dfrac{di_1^*}{dt} = \mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}\; i_2^*; \\ \dfrac{di_2^*}{dt} = -\mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}\; i_1^*. \end{cases}$$

So the general solution of this system is

$$\begin{cases} i_1^* = I \sin \theta \;; \\ i_2^* = I \cos \theta \;, \end{cases} \tag{20}$$

where I is constant determined by initial condition.

The averaging procedure (Grebennikov E.A., 1986) is used for investigation (18). According to this method, expressions (20) are used as formulas for the new variables A, θ. The substitution (20) into (18) leads to

$$\begin{cases} \dot{I} = -\left(\alpha_1 + \dfrac{p\dot{\varphi}}{2k\mu b_1 \hat{b}_2}\right) I \sin^2\theta - \left(\alpha_2 - \dfrac{p\dot{\varphi}}{2k\mu b_2 \hat{b}_1}\right) I \cos^2\theta + \\ + \dfrac{b_1 \hat{\alpha}_1 C \sin^2\theta / \hat{b}_1 + b_2 \hat{\alpha}_2 C \cos^2\theta / \hat{b}_2}{\mu I\left(b_1 \hat{b}_2 \sin^2\theta + b_2 \hat{b}_1 \cos^2\theta\right)}; \\ I\dot{\theta} = -\left(\left(\alpha_1 + \dfrac{p\dot{\varphi}}{2k\mu b_1 \hat{b}_2}\right) + \left(\alpha_2 - \dfrac{p\dot{\varphi}}{2k\mu b_2 \hat{b}_1}\right)\right) I \sin\theta \cos\theta + \\ + \mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}\, I + \dfrac{C \sin\theta \cos\theta}{\mu I\left(b_1 \hat{b}_2 \sin^2\theta + b_2 \hat{b}_1 \cos^2\theta\right)}\left(\dfrac{b_1 \hat{\alpha}_1}{\hat{b}_1} + \dfrac{b_2 \hat{\alpha}_2}{\hat{b}_2}\right). \end{cases}$$

We can use averaging procedure and get

$$\begin{cases} \dot{I} = -\alpha \dfrac{I}{2} + \dfrac{\beta}{I}; \\ \dot{\theta} = \mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}. \end{cases}$$

where $\beta = \dfrac{C\left(\sqrt{b_1 \hat{b}_2}\hat{\alpha}_1 + \sqrt{b_2 \hat{b}_1}\hat{\alpha}_2\right)}{\mu \hat{b}_1 \hat{b}_2\left(\sqrt{b_1 \hat{b}_2} + \sqrt{b_2 \hat{b}_1}\right)}, \alpha = \dfrac{1}{2}\left(\alpha_1 + \alpha_2 + \dfrac{p\dot{\varphi}}{2k\mu}\left(\dfrac{1}{b_1 \hat{b}_2} - \dfrac{1}{b_2 \hat{b}_1}\right)\right).$

Thus the solution of this equation takes form

$$I(t) = \sqrt{\beta / \alpha + C_1 e^{-2\alpha t}}$$

where C_1 is determined by initial conditions.

It is obviously that $I(t)$ is stable on the assumption of (19). In the steady state we get

$$\begin{cases} I(t) \to \sqrt{\beta / \alpha}\;; \\ \theta = \mu\sqrt{b_1 b_2 \hat{b}_1 \hat{b}_2}\; t\,. \end{cases} \tag{21}$$

It means that limit cycle arises in the state space of (18). Other word (18) converges to reference system in first approximation.

For simplicity let us choose $\hat{b}_1 = 1, \hat{b}_2 = 1$ and select control inputs in the form

$$u_1 = \mu i_2 + \hat{\alpha}_1 C i_1 / \left(i_1^2 + i_2^2\right); u_2 = -\mu i_1 + \hat{\alpha}_2 C i_2 / \left(i_1^2 + i_2^2\right), \quad (22)$$

where $\mu = 2\pi f_{nom} / b^* = 2\pi f_{nom} k / \delta_0$, denote parameters $\hat{\alpha}_1, \hat{\alpha}_2$ as estimations of α_1, α_2 obtained with the help of observer.

Taking into consideration assumption 1 and (22) we get in the steady state that

$$I \to \sqrt{\beta / \alpha} \approx \sqrt{C / \mu} \Rightarrow \begin{cases} i_1 \approx \sqrt{b^* C} \sin\theta \, ; \\ i_2 \approx \sqrt{b^* C} \cos\theta \, . \end{cases} \quad (23)$$

accurate within small terms φ and ω.

5 IDENTIFICATION ALGORITHM SYNTHESIS

We make control design in the previous chapter and get orthogonal function i_1, i_2 with the help of control inputs u_1, u_2. Thus linear independence of i_1, i_2 guaranties that closed loop system (3), (22) is identifiable. Now we are ready to identification algorithm synthesis. Firstly, the design problem of basic observer is solved. Then dynamic compensator is used to reduce the estimation error of profile height.

5.1 Basic observer design

One can rewrite closed loop system (3), (22) as

$$\begin{cases} \dot{i}_1 = \theta_1 x_1 \, ; \\ \dot{i}_2 = \theta_2 x_2 \, , \end{cases} \quad (24)$$

where $x_1 = (i_1 \;\; u_1)^T, x_2 = (i_2 \;\; u_2)^T, \theta_1 = (-\alpha_1 \;\; b_1)^T$, $\theta_2 = (-\alpha_2 \;\; b_2)^T$.

The difference between coefficients b_1 and b_2 is

$$b_1 - b_2 = 2p\varphi / k \Rightarrow \quad \varphi = (b_1 - b_2) k / (2p).$$

It is obviously that identification of parameter $b_1 - b_2$ covers the problem of surface profile estimation. So $b_1 - b_2$ is proportional to φ and consequently to the height of roughness.

Let us use the structure of this system as basis for observer, which is built in the form proposed in paper (Utkin V.I., 1981)

$$\begin{cases} \dot{\hat{i}}_1 = \hat{\theta}_1 x_1 + M_1 \bar{i}_1 \, ; \\ \dot{\hat{\theta}}_1 = l_1 M_1 \bar{i}_1 x_1^T \, . \end{cases} \qquad \begin{cases} \dot{\hat{i}}_2 = \hat{\theta}_2 x_2 + M_2 \bar{i}_2 \, ; \\ \dot{\hat{\theta}}_2 = l_2 M_2 \bar{i}_2 x_2^T \, . \end{cases} \quad (25)$$

where M_1, M_2 are big coefficients, $\hat{i}_1, \hat{\theta}_1, \hat{i}_2, \hat{\theta}_2$ are estimations of appropriate variables, l_1, l_2 are positive coefficients. We can suppose that $\dot{\theta}_1 \approx 0, \dot{\theta}_2 \approx 0$ according to assumption 2 and write the equations with respect to the errors $\bar{i} = \left(i_1 - \hat{i}_1 \;\; i_2 - \hat{i}_2\right)^T = \left(\bar{i}_1 \;\; \bar{i}_2\right)^T$, $\bar{\theta}_1 = \theta_1 - \hat{\theta}_1$, $\bar{\theta}_2 = \theta_2 - \hat{\theta}_2$

$$\begin{cases} \dot{\bar{i}}_1 = \bar{\theta}_1 x_1 - M_1 \bar{i}_1 \, ; \\ \dot{\bar{\theta}}_1 = -l_1 M_1 \bar{i}_1 x_1^T \, . \end{cases} \qquad \begin{cases} \dot{\bar{i}}_2 = \bar{\theta}_2 x_2 - M_2 \bar{i}_2 \, ; \\ \dot{\bar{\theta}}_2 = -l_2 M_2 \bar{i}_2 x_2^T \, . \end{cases}$$

It is obviously that if $M_1, M_2 \to \infty$ one get

$$\begin{cases} M_1 \bar{i}_1 \to \bar{\theta}_1 x_1 ; \\ \dot{\bar{\theta}}_1 = -l_1 \bar{\theta}_1 x_1 x_1^T \, . \end{cases} \qquad \begin{cases} M_2 \bar{i}_2 \to \bar{\theta}_2 x_2 ; \\ \dot{\bar{\theta}}_2 = -l_2 \bar{\theta}_2 x_2 x_2^T \, . \end{cases} \quad (26)$$

If we select Lyapunov function in the form

$V = \bar{\theta}_1 \bar{\theta}_1^T / 2.$

Then its derivative along the trajectories of the system (26) is

$\dot{V} = \dot{\bar{\theta}}_1 \bar{\theta}_1^T = -l_1 \left(\bar{\theta}_{11} i_1 + \bar{\theta}_{12} u_1\right)^2$

where $\bar{\theta}_{1j}$ $(j = 1,2)$ are corresponding elements of $\bar{\theta}_1$.

It is well known [Utkin] that if the functions i_1, u_1 are linearly independent, then derivative of Lyapunov function as negative everywhere except the origin of coordinates.

One can consider coordinates transformation

$$\begin{cases} i_1 = i_1 & ; \\ u_1 = \mu \, i_2 + \hat{\alpha}_1 C i_1 / \left(i_1^2 + i_2^2\right), \end{cases}$$

and rewrite this equation taking into account linear independence of i_1, i_2 and (23)

$$\begin{pmatrix} i_1 \\ u_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \hat{\alpha}_1 C / \left(i_1^2 + i_2^2\right) & \mu \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \hat{\alpha}_1 C / \left(I b^*\right) & \mu \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}.$$

As it is seen the transformation matrix is nonsingular. Therefore variables i_1, u_1 are linearly independent.

Appropriate selecting the coefficients M_i, l_i $(i = 1, 2)$ provides required time rate of getting to the trivial solution. The estimation of profile height is

$\hat{h}(t) = p\hat{\varphi} = \left(\hat{\theta}_{12} - \hat{\theta}_{22}\right) k / 2 = \left(\hat{b}_1 - \hat{b}_2\right) k / 2 \, .$

5.2 Dynamic compensator design for identification problem

According to assumption 2, $\dot{\theta}_1, \dot{\theta}_2$ is bounded function. Let us consider case when $\dot{\theta}_1 \neq 0, \dot{\theta}_2 \neq 0$. Then we can write (26) under this condition

$$\begin{cases} M_1 \bar{i}_1 \to \bar{\theta}_1 x_1 ; \\ \dot{\bar{\theta}}_1 = \dot{\theta}_1 - l_1 \bar{\theta}_1 x_1 x_1^T \, . \end{cases} \qquad \begin{cases} M_2 \bar{i}_2 \to \bar{\theta}_2 x_2 ; \\ \dot{\bar{\theta}}_2 = \dot{\theta}_2 - l_2 \bar{\theta}_2 x_2 x_2^T \, . \end{cases} \quad (27)$$

The derivative of Lyapunov function in this case is

$\dot{V} = \dot{\bar{\theta}}_1 \bar{\theta}_1^T = -l_1 \left(\bar{\theta}_{11} i_1 + \bar{\theta}_{12} u_1\right)^2 + \bar{\theta}_{11} \dot{\theta}_{11} + \bar{\theta}_{12} \dot{\theta}_{12} \, .$

Since $\dot{V}$ is negative defined with $\dot{\theta}_1 = 0$, we obtain due to the boundedness of $\dot{\theta}_1$ that estimations errors $\bar{\theta}_1$ converge to a bounded vicinity of zero determined by value of $\dot{\theta}_1, \dot{\theta}_2$.

The terms $\dot{\theta}_1, \dot{\theta}_2$ in (27) can be treated as unknown disturbances. Let us use dynamic compensator approach (Utkin V.A., 2006) to reduce estimation errors.

We can consider unknown profile function $f(x)$ as continuously differentiable function of time and present it in the form of some series. So according to reasoning let us approximate parameters $\alpha_1, b_1, \alpha_2, b_2$ as Tailor series

$$\alpha_1(t) = \sum_{i=1}^{\infty} \alpha_{1i} t^{i-1},\ \alpha_2(t) = \sum_{i=1}^{\infty} \alpha_{2i} t^{i-1},\ b_1(t) = \sum_{i=1}^{\infty} \beta_{1i} t^{i-1},\ b_2(t) = \sum_{i=1}^{\infty} \beta_{2i} t^{i-1}.$$

One can leave only first two terms in these equations and establish the models of disturbances according to dynamic compensation method as

$$\begin{cases} \dot{\theta}_1 = W\theta_1\ ; \\ A_1 = C\theta_1\ , \end{cases} \qquad \begin{cases} \dot{\theta}_2 = W\theta_2\ ; \\ A_2 = C\theta_2\ , \end{cases} \tag{28}$$

where $\theta_1 = (\alpha_1\ \alpha_{11}\ b_1\ b_{11})^T, \theta_2 = (\alpha_2\ \alpha_{21}\ b_2\ b_{21})^T$,

$$W = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Let us extend system (24) under (27)

$$\begin{cases} \dot{i}_1 = A_1^T x_1\ ; \\ \dot{\theta}_1 = W\theta_1\ ; \\ A_1 = C\theta_1\ , \end{cases} \qquad \begin{cases} \dot{i}_2 = A_2^T x_2\ ; \\ \dot{\theta}_2 = W\theta_2\ ; \\ A_2 = C\theta_2\ , \end{cases}$$

and modify the observer equation (25)

$$\begin{cases} \dot{\hat{i}}_1 = \hat{A}_1^T x_1 + M_1 \bar{i}_1 & ; \\ \dot{\hat{\theta}}_1 = W\hat{\theta}_1 + LNx_1 M_1 \bar{i}_1\ ; \\ \hat{A}_1 = C\hat{\theta}_1 & , \end{cases} \qquad \begin{cases} \dot{\hat{i}}_2 = \hat{A}_2^T x_2 + M_2 \bar{i}_2 & ; \\ \dot{\hat{\theta}}_2 = W\hat{\theta}_2 + LNx_2 M_2 \bar{i}_2\ ; \\ \hat{A}_2 = C\hat{\theta}_2 & , \end{cases} \tag{29}$$

where $N = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}^T, L = \begin{pmatrix} l_1 & 0 & 0 & 0 \\ 0 & l_1 l_2 & 0 & 0 \\ 0 & 0 & l_1 & 0 \\ 0 & 0 & 0 & l_1 l_2 \end{pmatrix}$ is positive definite matrix.

The equations with respect to the errors is

$$\begin{cases} \dot{\bar{i}}_1 = \bar{A}_1^T x_1 - M_1 \bar{i}_1 & ; \\ \dot{\bar{\theta}}_1 = W\bar{\theta}_1 - LNx_1 M_1 \bar{i}_1\ ; \\ \bar{A}_1 = C\bar{\theta}_1 & . \end{cases} \qquad \begin{cases} \dot{\bar{i}}_2 = \bar{A}_2^T x_2 - M_2 \bar{i}_2 & ; \\ \dot{\bar{\theta}}_2 = W\bar{\theta}_2 - LNx_2 M_2 \bar{i}_2\ ; \\ \bar{A}_2 = C\bar{\theta}_2 & . \end{cases}$$

If $M_1, M_2 \to \infty$ we get

$$\begin{cases} M_1 \bar{i}_1 \to \bar{A}_1^T x_1 & ; \\ \dot{\bar{\theta}}_1 = W\bar{\theta}_1 - LNx_1 \bar{A}_1^T x_1\ ; \\ \bar{A}_1 = C\bar{\theta}_1 & . \end{cases} \qquad \begin{cases} M_1 \bar{i}_1 \to \bar{A}_1^T x_1 & ; \\ \dot{\bar{\theta}}_1 = W\bar{\theta}_1 - LNx_1 \bar{A}_1^T x_1\ ; \\ \bar{A}_1 = C\bar{\theta}_1 & . \end{cases}$$

Let us consider one of these systems

$$\begin{cases} \dot{\bar{\theta}}_{11} = \bar{\theta}_{12} - l_1 i_1^2 \bar{\theta}_{11} - l_1 i_1 u_1 \bar{\theta}_{13}\ ; \\ \dot{\bar{\theta}}_{12} = -l_2 l_1 i_1^2 \bar{\theta}_{11} - l_2 l_1 i_1 u_1 \bar{\theta}_{13}\ ; \\ \dot{\bar{\theta}}_{13} = \bar{\theta}_{14} - l_1 \bar{\theta}_{13} u_1^2 \bar{\theta}_{13} - l_1 i_1 u_1 \bar{\theta}_{11}\ ; \\ \dot{\bar{\theta}}_{14} = -l_2 l_1 u_1^2 \bar{\theta}_{13} - l_2 l_1 i_1 u_1 \bar{\theta}_{11}\ . \end{cases} \tag{30}$$

If $l_1 \to \infty$ we get

$$\begin{cases} l_1 i_1^2 \bar{\theta}_{11} \to \bar{\theta}_{12} - l_1 i_1 u_1 \bar{\theta}_{12}; \\ l_1 \bar{\theta}_{13} u_1^2 \bar{\theta}_{13} \to \bar{\theta}_{14} - l_1 i_1 u_1 \bar{\theta}_{11}; \end{cases}$$

The substitution of this system to (30) leads to

$$\begin{cases} \dot{\bar{\theta}}_{12} = -l_2 \bar{\theta}_{12}; \\ \dot{\bar{\theta}}_{14} = -l_2 \bar{\theta}_{14}. \end{cases}$$

Thus from (30) we get following hierarchy

$$\begin{cases} \bar{\theta}_{12} \to 0; \\ \bar{\theta}_{14} \to 0. \end{cases} \Rightarrow \begin{cases} \dot{\bar{\theta}}_{11} \to -l_1 i_1^2 \bar{\theta}_{11} - l_1 i_1 u_1 \bar{\theta}_{13}; \\ \dot{\bar{\theta}}_{13} \to -l_1 \bar{\theta}_{13} u_1^2 \bar{\theta}_{13} - l_1 i_1 u_1 \bar{\theta}_{11}. \end{cases} \Rightarrow \begin{cases} \bar{\theta}_1 \to 0. \\ \bar{i}_1 \to 0. \end{cases}$$

Appropriate selecting the coefficients $M_i, l_i\ (i = 1, 2)$ provides required time rate of getting to the trivial solution.

Thus we show that if $\alpha_1, b_1, \alpha_2, b_2$ are changed according to (28) then the errors of identification of these parameters asymptotically converge to zero. Actually (28) means that unknown profile function $f(t)$ is approximated by linear function in each moment of time as it is shown on figure 2.

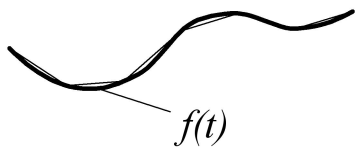


Fig. 2. Approximation of surface profile

We proved that for some positive $\hat{\alpha}_1, \hat{\alpha}_2$ trajectories of system (3), (22) converge to limit cycle. The estimations $\hat{\alpha}_1, \hat{\alpha}_2$ obtained with the help of observer (25), (29) can change sign during transient time. So according to reasoning, let us modify (22) in the following way

$$u_1 = \mu i_2 + |\hat{\alpha}_1| C i_1 / (i_1^2 + i_2^2); u_2 = -\mu i_1 + |\hat{\alpha}_2| C i_2 / (i_1^2 + i_2^2). \tag{31}$$

6. SIMULATION

In this chapter we present simulation results which show efficiency of proposed control and identification algorithm. The conditions of experiment are next

Resistances of coils, ohm $R_1 = 90$, $R_2 = 80$.

Nominal air gap, m $\delta_0 = 0{,}003$.

Constructive parameter, m/H $k = 0{,}24 \cdot 10^{-3}$.

Constructive parameter, m $p = 0{,}02$.

Other parameters $\mu = 60000 / 12{,}5$, $C = 10^{-4}$.

The initial conditions, A $i_1(0) = 0{,}001$, $i_2(0) = -0{,}007$.

1). The height of profile is changed according to the law

$$h(t) = 10^{-6}\, sin(1000t) + 10^{-6}\, sin(550t),$$

where t is time.

The simulation results of system (3) with control inputs (31) and observer (25) are represented on the figure 3.

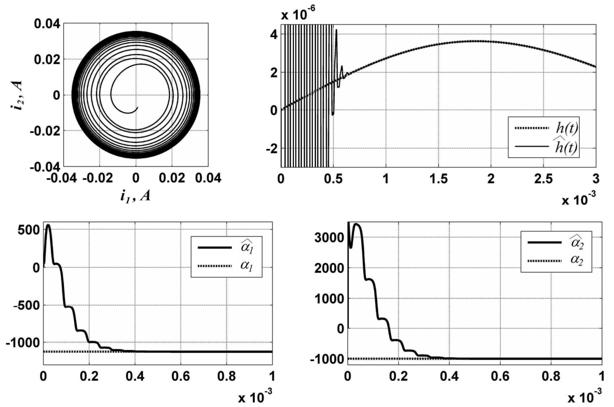


Fig. 3. Limit cycle rise and parameter estimation.

2). Let us provide comparative simulation between basic observer (25) and observer with dynamic compensator (29). Firstly, the height of profile is changed with time as repeating sequence

$$h(t) = \begin{cases} -10^{-6} + 0{,}000333\, t, & 0 < t < 0{,}006 \\ 3 \cdot 10^{-6} - 0{,}000333\, t, & 0{,}006 < t < 0{,}012 \end{cases}.$$

We can see the simulation results on the figure 4.

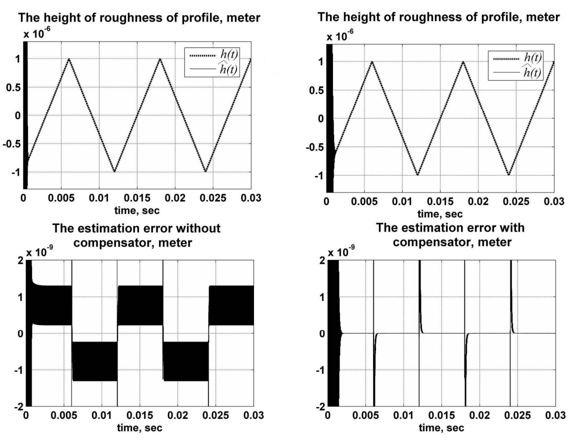


Fig. 4. The profile is changed according to linear law.

As it is seen, observer (29) provides asymptotic convergence of estimation error to zero when profile is changed according to linear law.

Secondly, $h(t)$ is equal to

$$h(t) = 10^{-6}\, sin(1000t).$$

The simulation results are presented on the figure 5.

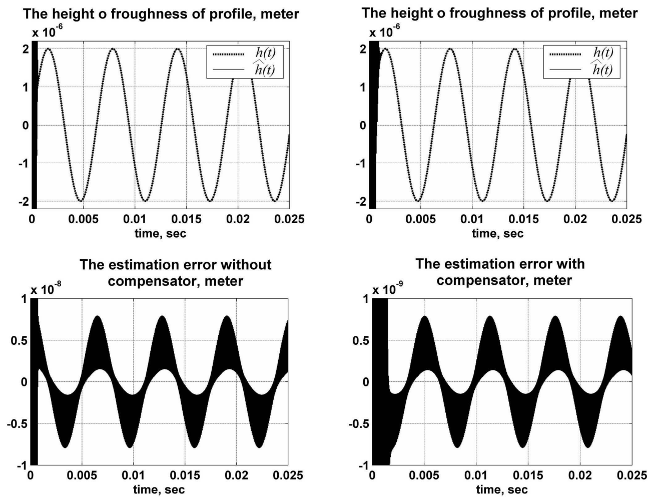


Fig. 5. The profile is changed according to sine law.

One can see that estimation error obtained with the help of observer (29) is less by order than one obtained by basic observer (25).

REFERENCES

Biryukov G.S., Serko A.L. (1987). *Geometric variable measuring and their measurement assurance,* pp. 354-357. Standard Publisher, Moscow.

Morrison E. (1996). *The development of a prototype high-speed stylus profilometer and its application to rapid 3D surface measurement,* vol. 1, p. 37-42. Nanotechnology.

Zhuravlev Yu.N (2003). *Active magnetic bearings: Theory, Design, Application,* pp. 52-56. Politehnika Publisher, Moscow.

Grebennikov E.A. (1986). *Averaging procedure in applications*. Nauka, Moscow.

Utkin V.I. (1981). *Sliding mode identification principles,* vol. 257, pp. 558-561. Doklady AN USSR, Moscow.

Utkin V.A. (2006). *The method of dynamic compensation in a problem of estimation of external disturbances,* pp. 754-771. In proc. SICPRO'06. Moscow.