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Synchronization in a Network of Chaotic Solid-State Nd:YAG Lasers ⋆

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Abstract: In this work, compex dynamical networks of chaotic solid-state Nd:YAG lasers (used as nodes) are arranged in coupled star arrays and identical synchronization is achieved. We consider two cases of interest: ***i)*** synchronization without master Nd:YAG laser (where the collective behavior is a new chaotic state) and ***ii)*** with master Nd:YAG laser (where the collective behavior is imposed by the dynamics of the master node to multiple slave nodes). Synchronization in complex networks is achieved by appealing to complex systems theory. Synchronization of chaotic Nd:YAG lasers in the complex network is shown in the amplitude of the electronic field of each laser.

1. INTRODUCTION

Chaos synchronization has received a special attention during the last years, see e.g. (Special Issue [2000], Wu and Chua [1993], Cruz-Hernández and Nijmeijer [2000], Sira Ramírez and Cruz-Hernández [2001], Cruz-Hernández [2004], López-Mancilla and Cruz-Hernández [2006], Aguilar-Bustos and Cruz-Hernández [2006], Pecora and Carroll [1990], Feldmann et al. [1996], Nijmeijer and Mareels [1997], López-Mancilla and Cruz-Hernández [2005], and Posadas-Castillo et al. [2006]). This property that originally was searched and used for two coupled chaotic oscillators in different applications (Pikovsky et al. [2003] and Boccaleti et al. [2002]), at the present time, synchronization is required in complex dynamical networks (with many coupled oscillators) see e.g. (Special Issue [2007], Bar-Yam [1997], Pogromsky and Nijmeijer [2001], and Posadas-Castillo et al. [2007b]), this topic has received a great interest from the scientific community. Particularly interesting is the case where the connected oscillators (nodes) have *chaotic* or *hyperchaotic* dynamics. Synchronization in complex dynamical networks is supposed to have direct applications in different fields, see e.g. (Strogatz [1993], Blasius [1999], Gamarra et al. [2001], Posadas-Castillo et al. [2008], and Yamapi et al. [2007]).

A specific array of nodes, where a single node (called common or central) is interconnected with multiple nodes is recognized as an essential configurations in different fields of the science and technology, many practical systems - in different areas of the science- have been observed and characterized by this type of structures, for example in: computer sciences, economy, engineering, biology, social sciences, etc. In particular, some examples are seen in communication systems (communication among networks with multiple users) (Chow et al. [2001]), in a network of computers (server connected to a set of terminal computers), in manufacture cells (arm robots working like team), multi-robot systems, etc., where the information from a central node is used by a group of terminal nodes, and the simultaneous communication from a single node to multiple nodes is required. This particular coupling configuration among nodes constitutes a kind of complex dynamical networks, the so-called *star coupled networks*. Therefore, there exists a strong motivation in trying new coupling topologies that allow to achieve synchronization for many coupled chaotic nodes.

There exist two principal groups in complex dynamical networks (according to the form in that the nodes are coupled or connected): *i) regular complex networks* (which follow a pattern defined in the form of being connected), for example: globally coupled networks, nearest-neighbor coupled networks, and star coupled networks (Wang [2002]), and *ii) irregular complex networks* (without a pattern

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defined in the form of being connected) (Wang [2002b]), examples are: random networks, small-world, scale-free networks, etc.

In this work, solid-state Nd:YAG (*Neodymium doped: Yttrium Aluminium Garnet*) lasers (Terry [2002]) will be used like chaotic nodes, to construct the arrangements in star coupled networks to be synchronized. Synchronization means that the irregular time evolution of one laser **-**either in the optical power**-,** can be exactly reproduced by other lasers that conform the complex dynamical networks. In particular, five chaotic Nd:YAG lasers are arranged in coupled star arrays and identical synchronization is achieved in two cases: with and without master chaotic Nd:YAG laser. Synchronization in the dynamical networks is achieved by appealing to results from complex systems theory. Synchronization in the dynamical networks of chaotic Nd:YAG lasers is shown in the amplitude of the electronic field of each laser.

The rest of this work is organized as follows: In Section 2, we describe a mathematical model for a single Nd:YAG laser, to be used as fundamental node to construct dynamical networks. In Section 3, we give a brief review on complex dynamical systems and their synchronization. In Section 4, we show synchronization in dynamical networks of five chaotic Nd:YAG lasers, by using the star coupled configuration, we show two cases (with and without master Nd:YAG laser). Finally, some conclusions are given in Section 5.

2. DYNAMICS OF A ND:YAG LASER

In this section, we present the chaos generator (Nd:YAG laser) used as fundamental node in the dynamical networks to be synchronized. As in (Posadas-Castillo et al. [2008]), we take a modification of the equations suggested in (Terry [2002]) for a single solid-state Nd:YAG laser with a sinusoidally modulated loss, described by the following state equations

$$\begin{aligned} \dot{X} &= \left(F - \left(\alpha_0 + \alpha_1 \cos\left(\omega t\right)\right)\right) X, \\ \dot{F} &= \gamma \left(A_0 - F - F X^2\right), \end{aligned} \tag{1}$$

where $X(t)$ and $F(t)$ constitute the states, physically represent the amplitude of the electronic field of the laser and its gain, respectively. The parameters α_0 and A_0 denote the rates of intra cavity loss and pump strength, respectively. While, α_1 represents the strength of modulation of the intra cavity loss at a frequency ω, and γ is a ratio of the time scale of light in the laser cavity, and the upper level spontaneous emission lifetime of the lasing media.

The Nd:YAG laser is modulated with a depth α_1 relative to its mean losses α_0. In absence of modulation, the Nd:YAG laser is stable and exhibits damped oscillations to their fixed-point values. The laser under consideration is of class B, where only the electronic field and gain variables need be considered. The laser is subjected to identical periodic modulations of the loss and may become chaotic in certain parameter values.

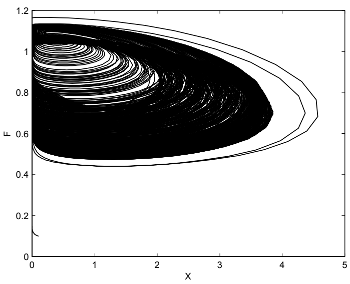

Fig. 1. Projection of chaotic attractor of Nd:YAG laser on the (X, F)-plane.

We performed our simulations using $\gamma = 10^{-2}$ to avoid stiffness problems that arise with smaller values of γ. It is known that for suitable values of parameters α_0 and α_1, the Nd:YAG laser (1) exhibits *chaotic oscillations.* In Figure 1 is shown the projection of chaotic attractor on the (X, F)-plane; where we have taken the following set of parameter values: $\alpha_0 = 0.9$, $\alpha_1 = 0.2$, $A_0 = 1.2$, $w = 0.045t$, and $\gamma = 0.01$.

3. REVIEW ON SYNCHRONIZATION OF COMPLEX DYNAMICAL NETWORKS

3.1 Complex dynamical networks

A set interconnected of nodes can be defined as a *complex dynamical network*, each node is considered like basic element with behavior depending of the nature of the network. As in (Wang [2002] and Wang [2002b]), we consider a complex dynamical network composes of N identical nodes, linearly and diffusively coupled through the first state of each node. Each node constitutes a n-*dimensional dynamical system*, described as follows

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + u_i, \qquad i = 1, 2, \ldots, N, \tag{2}$$

where $\mathbf{x}_i = (x_{i1},\ x_{i2},\ ...,\ x_{in})^T \in \mathbb{R}^n$ is the *state vector* of the node i, $u_i = u_{i1} \in \mathbb{R}$ is the *input* signal of the node i, and is defined by

$$u_{i1} = c \sum_{j=1}^{N} a_{ij} \Gamma \mathbf{x}_j, \qquad i = 1, 2, \ldots, N, \tag{3}$$

the constant $c > 0$ represents the *coupling strength*, and $\Gamma \in \mathbb{R}^{n \times n}$ is a constant 0-1 matrix linking coupled states. Assume that $\Gamma = \mathrm{diag}\left(r_1,\ r_2, \ldots, r_n\right)$ is a diagonal matrix with $r_i = 1$ for a particular i and $r_j = 0$ for $j \neq i$. This means that two coupled nodes are linked through their $i - th$ state. Whereas, $\mathbf{A} = (a_{ij}) \in \mathbb{R}^{n \times n}$ is the *coupling matrix*, which represents the coupling configuration in (2)-(3). If there is a connection between node i and node j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$ for $i \neq j$. The diagonal elements of $\mathbf{A}$ are defined as

$$a_{ii} = -\sum_{j=1,\ j \neq i}^{N} a_{ij} = -\sum_{j=1,\ j \neq i}^{N} a_{ji}, \qquad i = 1, 2, \ldots, N. \tag{4}$$

Suppose that the dynamical network (2)-(3) is connected in the sense that there are no isolated clusters. Then, $\mathbf{A}$ is a symmetric irreducible matrix. In this case, zero is an eigenvalue of $\mathbf{A}$ with multiplicity 1 and all the other eigenvalues are strictly negatives (Wang [2002] and Wang [2002b]). Synchronization state in complex dynamical networks (2)-(3), can be characterized by the nonzero eigenvalues of $\mathbf{A}$. The complex dynamical network (2)-(3) is said to achieve (asymptotically) *synchronization*, if (Wang [2002b]):

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \ \dots \ = \mathbf{x}_N(t), \quad \text{as} \ \ t \to \infty. \tag{5}$$

The diffusive coupling condition (4) guarantees that the synchronization state is a solution, $\mathbf{s}(t) \in \mathbb{R}^n$, of an *isolated* node, that is

$$\dot{\mathbf{s}}(t) = f(\mathbf{s}(t)), \tag{6}$$

where $\mathbf{s}(t)$ can be an equilibrium point, a periodic orbit, or a chaotic attractor. Thus, *stability* of the synchronization state,

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \ \dots \ = \mathbf{x}_N(t) = \mathbf{s}(t), \tag{7}$$

of complex dynamical network (2)-(3) is determined by the dynamics of an isolated node, i.e. the nonlinear function f (and its solution $\mathbf{s}(t)$), the coupling strength c, the inner linking matrix Γ, and the coupling matrix $\mathbf{A}$.

3.2 Synchronization conditions

Theorem 1 (Wang [2002] and Wang [2002b]) *Consider the dynamical network (2)-(3). Let*

$$0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N \tag{8}$$

be the eigenvalues of its coupling matrix $\mathbf{A}$. *Suppose that there exists a* $n \times n$ *diagonal matrix* $\mathbf{D} > 0$ *and two constants* $\bar{d} < 0$ *and* $\tau > 0$, *such that*

$$[Df(\mathbf{s}(t)) + d\Gamma]^T \mathbf{D} + \mathbf{D}[Df(\mathbf{s}(t)) + d\Gamma] \leq -\tau \mathbf{I}_n \tag{9}$$

for all $d \leq \bar{d}$, *where* $\mathbf{I}_n \in \mathbb{R}^{n \times n}$ *is an unit matrix. If, moreover,*

$$c\lambda_2 \leq \bar{d}, \tag{10}$$

then, the synchronization state (7) of dynamical network (2)-(3) is exponentially stable.

Since $\lambda_2 < 0$ and $\bar{d} < 0$, inequality (10) is equivalent to

$$c \geq \left| \frac{\bar{d}}{\lambda_2} \right|. \tag{11}$$

Synchronizability of dynamical network (2)-(3) with respect to a specific coupling configuration can be characterized by the second-largest eigenvalue (λ_2) of $\mathbf{A}$.

3.3 Star coupled networks

The coupling configurations commonly studied in synchronization of complex dynamical networks are the so-called: globally coupled networks, nearest-neighbor coupled networks, and star coupled networks. In this work, we concentrate on the *synchronization in star coupled networks with identical nodes (chaotic Nd:YAG lasers)*. In the sequel, we will show the particular arrangement of the coupling matrix for this class of complex dynamical networks.

Let $G = (V, E)$ be a *graph*, consisting of $N = |V|$ nodes, with $V = V(G) = \{v_1, v_2, \dots, v_N\}$ the *node set* and $M = |E|$ edges between nodes, where $E = E(G) = \{e_1, e_2, \dots, e_M\}$ denotes the *link set*. We consider complex dynamical networks where all nodes are connected to a same node (i.e. the networks have a common or central node). Such regular coupling networks are reported in the current literature as star coupled networks (Wang [2002] and Wang [2002b]). In addition, we assume that all the nodes are connected and are finite, without self-loops, and without multiple edges between two nodes. Under the mentioned assumptions, there are two principal associated matrices of interest with a graph G (Merris [1994] and Diestel [2000]): *i)* The familiar (0,1) *Adjacency matrix* $A(G)$: $N \times N$ matrix whose entries a_{ij} are given by

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E(G), \\ 0 & \text{otherwise,} \end{cases}$$

where $(i,j) \in E(G)$ means that node i connects with node j, i.e. are adjacent. For a single graph without self-loops, the adjacency matrix must have 0's on the diagonal. And *ii)* the diagonal matrix, *Degree matrix* $D(G)$: $N \times N$ matrix whose entries d_{ij} are given by

$$d_{ij} = \begin{cases} d_i & \text{if } i = j, \\ 0 & \text{otherwise,} \end{cases}$$

where d_i is the degree of the node i, and given that in this topology, each node i is connected with $N-1$ nodes, then we have $d_1 = d_2 = \cdots = d_N = N-1$.

The *Laplacian matrix* of a graph $L(G)$ with N nodes is a $N \times N$ matrix $L(G) = D(G) - A(G)$, with entries l_{ij} expressed as

$$l_{ij} = \begin{cases} -1 & \text{if } (i,j) \in E(G), \\ d_i & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Let us now compute the Laplacian matrix $L(G) = D(G) - A(G)$ for the mentioned networks (star coupled networks), which corresponds to the coupling matrix $\mathbf{A}_{sc}$, that is

$$\mathbf{A}_{sc} = L(G) = \begin{bmatrix} N-1 & 0 & 0 \cdots & 0 \\ 0 & N-1 & 0 \cdots & 0 \\ \vdots & \ddots & \ddots \ddots & \vdots \\ 0 & 0 & 0 \cdots & 0 \\ 0 & 0 & 0 \cdots & N-1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \cdots & 1 \\ 1 & 0 & 0 \cdots & 0 \\ \vdots & \ddots & \ddots \ddots & \vdots \\ 1 & 0 & 0 \cdots & 0 \\ 1 & 0 & 0 \cdots & 0 \end{bmatrix},$$

thus, the coupling matrix for star coupled networks is given by

$$\mathbf{A}_{sc} = \begin{bmatrix} N-1 & -1 & -1 \cdots & -1 \\ -1 & 0 & 0 \cdots & 0 \\ \vdots & \ddots & \ddots \ddots & \vdots \\ -1 & 0 & 0 \cdots & 0 \\ -1 & 0 & 0 \cdots & 0 \end{bmatrix}. \tag{12}$$

Note that the Laplacian matrix have rows with sum equal to zero. The eigenvalues of $\mathbf{A}_{sc}$ are $(0, -1, -1, \cdots, -N)$. Therefore, the second largest eigenvalue of $\mathbf{A}_{sc}$ is $\lambda_{2sc} = -1$, which is unrelated with the size of the network.

For the star coupled networks, there exists a critical coupling strength $\left(c \geq \bar{d}\right)$, so that the complex dynamical networks (2)-(3) can synchronize.

The star coupled configuration is shown for N nodes in Figure 2, with the common or central node **1**.

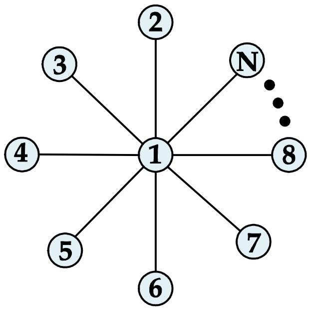

Fig. 2. Star coupled configuration with N nodes.

4. DYNAMICAL NETWORKS OF CHAOTIC ND:YAG LASERS

We show in this section, synchronization in dynamical networks, constitute with five coupled chaotic Nd:YAG lasers by using star configuration.

Based on the mathematical model of Nd:YAG laser (1) as fundamental node, we construct the complex dynamical networks with N chaotic Nd:YAG, described by

$$\begin{bmatrix} \dot{X}_{i1} \\ \dot{F}_{i2} \end{bmatrix} = \begin{bmatrix} (F_{i2}-(\alpha_0+\alpha_1 \cos(\omega t)))X_{i1}+u_{i1} \\ \gamma\left(A_0 - F_{i2} - F_{i2}X_{i1}^2\right) \end{bmatrix}, \quad (13)$$

$$u_{i1} = c\sum_{j=1}^{N} a_{ij}X_{j1}, \qquad i = 1, 2, ..., N.$$

In particular, we consider $N = 5$, and $\Gamma = \mathrm{diag}\,(1,\ 0)$, i.e. we have five chaotic Nd:YAG lasers constituting the dynamical networks to be synchronized in star topologies.

On the other hand, to satisfy the condition (9), we need a positive constant d such that zero is an exponentially stable equilibrium point of the $n-$dimensional system

$$Df(\mathbf{s}(t)) + d\Gamma, \qquad (14)$$

which is equivalent to have a single node with self-feedback, where the positive constant d is such that the self-feedback term $-dz_1$ could stabilize the following single node

$$\begin{aligned} \dot{z}_1 &= f_1(\mathbf{z}) - dz_1, \\ \dot{z}_2 &= f_2(\mathbf{z}), \\ &\vdots \\ \dot{z}_n &= f_n(\mathbf{z}). \end{aligned} \qquad (15)$$

With the particular value of $d = 0.3$ we can stabilize to zero the state (X_{11}) of the single chaotic solid-state Nd:YAG laser (15), this is illustrated in Fig. 3. With these values the Theorem 1 guarantees synchronization in the dynamical networks with five chaotic Nd:YAG lasers.

The specific arrangement for the dynamical networks with five chaotic Nd:YAG lasers; is defined as follows, for the chaotic node **1**,

$$\begin{bmatrix} \dot{X}_{11} \\ \dot{F}_{12} \end{bmatrix} = \begin{bmatrix} (F_{12}-(\alpha_0+\alpha_1 \cos(\omega t)))X_{11}+u_{11} \\ \gamma\left(A_0 - F_{12} - F_{12}X_{11}^2\right) \end{bmatrix}, \quad (16)$$

$$u_{11} = c\begin{pmatrix} a_{11}X_{11} + a_{12}X_{21} + \\ a_{13}X_{31} + a_{14}X_{41} + a_{15}X_{51} \end{pmatrix}, \qquad (17)$$

while the chaotic node **2** is designed as

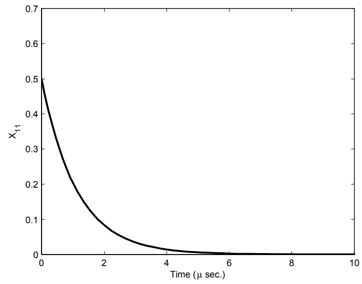

Fig. 3. For $d = 0.3$, the state X_{11} of a single chaotic Nd:YAG laser can be stabilized to zero.

$$\begin{bmatrix} \dot{X}_{21} \\ \dot{F}_{22} \end{bmatrix} = \begin{bmatrix} (F_{22}-(\alpha_0+\alpha_1 \cos(\omega t)))X_{21}+u_{21} \\ \gamma\left(A_0 - F_{22} - F_{22}X_{21}^2\right) \end{bmatrix}, \quad (18)$$

$$u_{21} = c\begin{pmatrix} a_{21}X_{11} + a_{22}X_{21} + \\ a_{23}X_{31} + a_{24}X_{41} + a_{25}X_{51} \end{pmatrix}, \qquad (19)$$

the chaotic node **3** is described as

$$\begin{bmatrix} \dot{X}_{31} \\ \dot{F}_{32} \end{bmatrix} = \begin{bmatrix} (F_{32}-(\alpha_0+\alpha_1 \cos(\omega t)))X_{31}+u_{31} \\ \gamma\left(A_0 - F_{32} - F_{32}X_{31}^2\right) \end{bmatrix}, \quad (20)$$

$$u_{31} = c\begin{pmatrix} a_{31}X_{11} + a_{32}X_{21} + \\ a_{33}X_{31} + a_{34}X_{41} + a_{35}X_{51} \end{pmatrix}, \qquad (21)$$

the chaotic node **4** is defined as

$$\begin{bmatrix} \dot{X}_{41} \\ \dot{F}_{42} \end{bmatrix} = \begin{bmatrix} (F_{42}-(\alpha_0+\alpha_1 \cos(\omega t)))X_{41}+u_{41} \\ \gamma\left(A_0 - F_{42} - F_{42}X_{41}^2\right) \end{bmatrix}, \quad (22)$$

$$u_{41} = c\begin{pmatrix} a_{41}X_{11} + a_{42}X_{21} + \\ a_{43}X_{31} + a_{44}X_{41} + a_{45}X_{51} \end{pmatrix}, \qquad (23)$$

and the chaotic node **5** is

$$\begin{bmatrix} \dot{X}_{51} \\ \dot{F}_{52} \end{bmatrix} = \begin{bmatrix} (F_{52}-(\alpha_0+\alpha_1 \cos(\omega t)))X_{51}+u_{51} \\ \gamma\left(A_0 - F_{52} - F_{52}X_{51}^2\right) \end{bmatrix}, \quad (24)$$

$$u_{51} = c\begin{pmatrix} a_{51}X_{11} + a_{52}X_{21} + \\ a_{53}X_{31} + a_{54}X_{41} + a_{55}X_{51} \end{pmatrix}. \qquad (25)$$

4.1 Some star connection topologies with chaotic Nd:YAG lasers

In the sequel, we present some star connection topologies with five chaotic Nd: YAG lasers (chaotic nodes) to be synchronized.

Case 1 *(Network without master node): Five uncoupled chaotic nodes (1) to be synchronized in a dynamical network in star configuration without master node.*

A diagram of this topology for synchronization of this array of lasers is shown in Figure 4, where the coupling signal $X_{i1}(t)$ is purely via overlap of the electric field (Uchida et al. [1999]).

The coupling matrix (12) is given by

$$\mathbf{A}_{sc} = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix},$$

with eigenvalues: $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = \lambda_4 = -1$, and $\lambda_5 = -5$, with a coupling value $c = 1$. For this case,

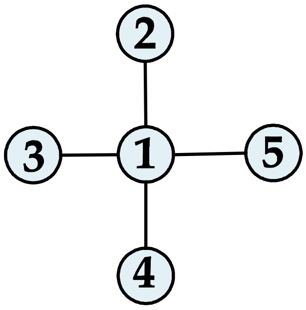

Fig. 4. Star coupled configuration without master node.

we have used to construct the arrangement, the coupling signals X_{i1}, $i = 1, 2, ..., 5$; designing in this way, the input signals $u_{i1} = g\left(X_{i1}; c\right)$, $i = 1, 2, ..., 5$, given explicitly by:

$$u_{11} = -4X_{11} + X_{21} + X_{31} + X_{41} + X_{51}, \qquad (26)$$

$$u_{21} = X_{11} - X_{21}, \qquad (27)$$

$$u_{31} = X_{11} - X_{31}, \qquad (28)$$

$$u_{41} = X_{11} - X_{41}, \qquad (29)$$

$$u_{51} = X_{11} - X_{51}. \qquad (30)$$

To construct the star coupled network without master node shown in Figure 4, we use the Eqs. (16), (18), (20), (22), and (24) with the input signals (26)-(30).

We take the initial conditions:

$$X_{11}(0) = 0.1, \quad F_{12}(0) = 0.1;$$
$$X_{21}(0) = 0.6, \quad F_{22}(0) = 0.6;$$
$$X_{31}(0) = 0.7, \quad F_{32}(0) = 0.7;$$
$$X_{41}(0) = 0.4, \quad F_{42}(0) = 0.4;$$
$$X_{51}(0) = 0.55, \quad F_{52}(0) = 0.55.$$

Figure 5 shows synchronization in the first state (X_{11} vs X_{21}, X_{11} vs X_{31}, X_{11} vs X_{41}, X_{11} vs X_{51}, X_{21} vs X_{31}, X_{21} vs X_{41}, X_{21} vs X_{51}, X_{31} vs X_{41}, X_{31} vs X_{51}, and X_{41} vs X_{51}) of five Nd:YAG lasers (synchronization is achieved in 5 sec. approximately) and the chaotic attractor of the collective behavior (a new chaotic state) in the network.

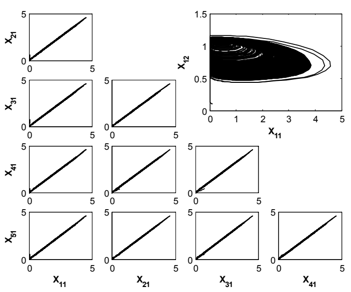

Fig. 5. Synchronization in the first state (X_{i1}, $i = 1, 2, ..., 5$) of five chaotic Nd:YAG lasers in star configuration without master node, and the new chaotic attractor of the collective behavior in the network, projected onto the (X_{11}, F_{12})-plane.

Case 2 *(Network with master node): Five uncoupled chaotic nodes (1) to be synchronized in a dynamical network in star configuration with master node* 1*.*

A diagram of this topology for synchronization of this array of nodes is shown in Figure 6 with common or central node **1** like master node. The coupling signal $X_{i1}(t)$ is purely via overlap of the electric field.

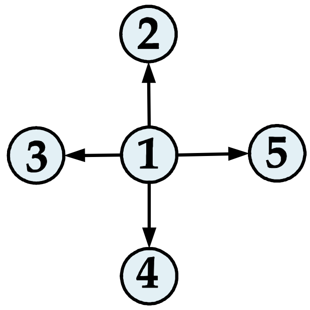

Fig. 6. Star coupled configuration with chaotic Nd:YAG laser **1** like master.

The coupling matrix (12) is given by

$$\mathbf{A}_{sc} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix},$$

with eigenvalues $\lambda_1 = 0$, and $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = -1$, and coupling value $c = 1$. To construct the corresponding arrangement, we have used the coupling signal X_{11} only for the design of the input signals $u_{i1} = g\left(X_{i1}; c\right)$, $i = 2, ..., 5$. We use the same Eqs. (16), (18), (20), (22), and (24), with the input signals (27)-(30) and for $u_{11} \equiv 0$ in (26). We have taken the same initial conditions as the previous case. Figure 7 shows synchronization in the first state of five Nd:YAG lasers: X_{11} vs X_{21}, X_{11} vs X_{31}, X_{11} vs X_{41}, X_{11} vs X_{51}, X_{21} vs X_{31}, X_{21} vs X_{41}, X_{21} vs X_{51}, X_{31} vs X_{41}, X_{31} vs X_{51}, and X_{41} vs X_{51}. In this case synchronization is achieved in 5 sec. approximately, and the collective behavior in the dynamical network, is imposed by the chaotic Nd:YAG laser 1 (see, Figure 1).

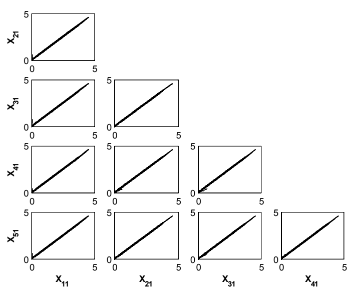

Fig. 7. Synchronization in the first state of five chaotic Nd:YAG lasers with master node **1**.

5. CONCLUSIONS

In this paper, we have presented multiple synchronization of coupled chaotic Nd:YAG lasers, in particular by using star coupled networks. We have achieve synchronization of five chaotic Nd:YAG lasers (used as fundamental node) in star complex networks for two coupling scenarios: with and without chaotic master Nd:YAG laser. This result is particularly interesting given its possible application in communication networks, where is required that a single sender transmits simultaneously information to many receivers.

In addition, the approach can be implemented on experimental setup, and shows great potential for actual optical communication systems in which the encoding is required to be secure.

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