Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A simple state feedback controller design method of networked control systems with time delay and packet dropout ⋆

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Abstract: This paper presents a simple yet effective method to design state feedback controller for networked control systems (NCSs). By introducing the lifting technique into NCSs and by considering the balance between effectiveness and simplicity, a novel discrete-time switch model is proposed with the consideration of time delay and packet dropout during the transmission of packets. In terms of the given model, we give sufficient conditions for the existence of state feedback controller such that the closed-loop NCSs are asymptotically stable. Based on the obtained stability conditions, a homotopy-based iterative LMI algorithm is developed to get the state feedback gain. Simulation and experimental results are given to demonstrate the effectiveness of the proposed approaches.

1. INTRODUCTION

Networked control systems (NCSs) of which communication networks are used for the connections between spatially distributed system components have recently attracted much attention from research communities. Certain issues such as network-induced delay (Nilsson et al. [1998], Hu et al. [2003], Tipsuwan et al. [2004], Zhang et al. [2005], Liu et al. [2006], Zhang et al. [2001]), packet dropout (Zhang et al. [2001], Xiong et al. [2007]), network constraints (Montestruque et al. [2003], Peter et al. [2003]), signal quantization (Li et al. [2004], Montestruque et al. [2007]), and scheduling (Zhang et al. [2005], Walsh et al. [2001]), were investigated and some useful results were reported. In addition, due to the advantages of low cost, simple installation and high reliability (Xiong et al. [2007], Yue et al. [2004]), NCSs have been finding applications in DC motors (Liu et al. [2006]), vehicles (Seiler et al. [2005]) and robots (Tipsuwan et al. [2003]), etc.

In practice, controller design for NCSs with both network-induced delay and packet dropout is a very interesting and practical problem since the packets in NCSs usually suffer network-induced delay and packet dropout simultaneously during the network transmissions. Recently, some important results are developed in this field. Based on the LMI approach, state feedback control method was investigated by Yue et al. [2004] and Yu et al. [2004] respectively. In the case of H_∞ control, Yue et al. [2005] studied the controller design for NCSs with external disturbance and parameter uncertainties. Note that those results investigated NCSs in continuous-time domain. In discrete-time domain, Yu et al. [2004] and Lin et al. [2005] studied the stabilization problem for NCSs. However, in those results, the controller and actuator are combined together. That means network exists only between sensor and controller. Xiong et al. [2007] studied the state feedback control for NCSs under a general framework. However, it assumes network-induced delay is constant, and therefore can not deal with the case when network-induced delay is random. Therefore, despite the progress made in controller design for NCSs with both network-induced delay and packet dropout, it has become evident that the state feedback control for NCSs in discrete-time domain, especially for NCSs under a general framework and with random network-induced delay, are still required.

In this paper, we focus on solving the state feedback controller design problem of NCSs in discrete-time domain and under a general framework, where random network-induced delay and arbitrary packet dropout are taken into account simultaneously. By using the lifting technique (see Li et al. [2002], Park et al. [2004]) and by considering the balance between effectiveness and efficiency, a simple mathematic model is proposed for the considered NCSs. It describes NCSs as a switch system, and therefore enables us to apply the theory from switch systems to study NCSs in discrete-time domain. In terms of the given model, we give sufficient conditions for the existence of state feedback controller such that the closed-loop NCSs are asymptotically stable. Based on the obtained stability conditions,

⋆ The research work of HongBo Li and ZengQi Sun was jointly supported by the National Science Foundation (NSF) of China (Grant No:60574035, 60674053) and the National Key Project for Basic Research of China (Grant No: 2002cb312205). The research work of Mo-Yuen Chow was partially sponsored by the NSF through Grant No.IIS-0426852 (Intelligent Human-Machine Interface and Control for Highly Automated Chemical Screening Processes).

we further investigated the corresponding state feedback controller design problem. Simulation and experimental results are given to demonstrate the effectiveness of the proposed approaches.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices respectively. $\|\cdot\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrices. The superscript "^{T}" denotes matrix transposition; and for symmetric matrices X and Y, the notation $X > Y$ means that $X - Y$ is positive definite. I is the identity matrices with appropriate dimensions, and the notation $\mathbb{Z}^+$ stands for the set of nonnegative integers. Finally, in symmetric block matrices, we use "$*$" as an ellipsis for the terms introduced by symmetry.

2. PROBLEM FORMULATION

The setup of NCSs considered in this paper is depicted in Fig.1, where networks exist between sensor and controller, and between controller and actuator. The controller has a buffer denoted as buffer A, and the actuator has a buffer denoted as buffer B. Each component is described as follows.

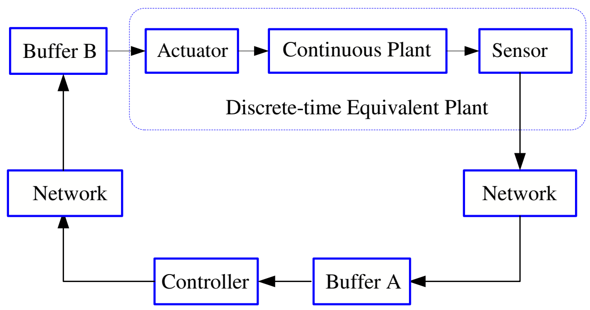

Fig. 1. The structure of the concerned networked control systems

The controlled plant is given by

$$\boldsymbol{x}(k+1) = \boldsymbol{F}\boldsymbol{x}(k) + \boldsymbol{G}\boldsymbol{u}(k) \tag{1}$$

where $k \in \mathbb{Z}^+$ is the time index, $\boldsymbol{x}(k) \in \mathbb{R}^n$ and $\boldsymbol{u}(k) \in \mathbb{R}^m$ are the plant state and control input respectively. $\boldsymbol{x}_0 := \boldsymbol{x}(0) \in \mathbb{R}^n$ is the initial plant state. $\boldsymbol{F}$ and $\boldsymbol{G}$ are known matrices with appropriate dimensions.

The sensor is clock-driven, i.e., at time instant kh, it sends the most recent plant state to the controller, where h is the step length of the plant.

The controller, as a receiver, has a buffer size of 1. The packet with the latest time stamp is used to update the buffer content. The networked controller is a memoryless state feedback controller of following form:

$$\boldsymbol{u} = \boldsymbol{K}\mathrm{buffer}(A) \tag{2}$$

where $\boldsymbol{K}$ is the feedback gain to be designed, buffer(A) is the updated content of buffer A. The controller is event-driven, i.e., whenever there is new data in buffer A, the controller starts calculating new control signal and transmits it to the actuator.

In the presence of networks, network-induced delay, packet dropout and packet out-of-order occur inevitably. Let τ_k express the RTT (Round Trip Time) delay encountered by kth packet from the sensor, i.e., the time interval from the time instant kh when the sensor samples the plant state to the time instant $kh + \tau_k$ when the control signal based on this packet reaches the actuator. Apparently, τ_k is a combination of the sensor-to-controller delay, the controller processing delay and the controller-to-actuator delay. In this paper, without loss of generality, we assume $\tau_k \in U_1 := [\tau_{\min},\ \tau_{\max}]$, where $\tau_{\min}$ and $\tau_{\max}$ are the lower and upper bounds of τ_k respectively, $\tau_{\min}$ is shorter than h, $\tau_{\max}$ can be larger then h. Note that using control signal with longer time delay to control the plant will make the NCSs model and the NCSs analysis complex. To strike the balance between effectiveness and simplicity, the control signal with τ_k shorter than $2h$ are used to control the plant in the concerned NCSs. Noting that, we introduce the following definition.

Definition 1. A packet from the sensor is called *effective packet* under the controller (2), if the RTT delay encountered by this packet is shorter than $2h$.

Since only effective packet is used to control the plant, we can consider the rest packets from the sensor as dropout packets. Let $S := \{i_1, i_2, \cdots\} \subseteq \mathbb{Z}^+$ ($i_{m+1} > i_m$, $m \in \{1, 2, \cdots\}$) denote the sequence of time index of the *effective packets* in NCSs, and $\boldsymbol{N}_{drop} := \max_{i_m \in S}\{i_{m+1} - i_m\}$ be the maximum upper bound. Then the following concept and mathematical model is used to capture the nature of packet dropout in NCSs.

Definition 2. The packet dropout process in the concerned NCSs is defined as

$$\{\eta(i_m) := i_{m+1} - i_m, \quad i_m \in S\} \tag{3}$$

which means that, from i_m to i_{m+1}, the number of packet dropout is $\eta(i_m) - 1$. By noting $i_m \in S$ and by considering the definition of $\boldsymbol{N}_{drop}$, we can conclude that $\eta(i_m)$ takes values in a finite set $\Omega := \{1, 2, \cdots, \boldsymbol{N}_{drop}\}$.

Let τ_{i_m} express the RTT delay encountered by mth effective packet. By considering the definition of *effective packet*, we have $\tau_{i_m} \in U := [\tau_{\min},\ 2h]$. From previous discussions, the time delay and packet dropout information can be embodied as

$$\{\{\eta(i_m),\ \tau_{i_m}\}:\ \ i_m \in S, \tau_{i_m} \in U, \eta(i_m) \in \Omega\} \tag{4}$$

Definition 3. (4) is said to model random time delay and arbitrary packet dropout, if τ_{i_m} takes values in U randomly, and i_m takes values in $\mathbb{Z}^+$ arbitrarily. Correspondingly, $\eta(i_m)$ takes values in Ω arbitrarily.

In the considered NCSs, the actuator also has a buffer size of 1. The control signal based on the most recent effective packet will be put into buffer B. The actuator is clock-driven. At time instant kh, it updates the actuator output based on the value read from buffer B. Thus, the last control signal is used to control the plant when the new control signal is not available.

The objective of this paper is to propose a mathematic model to describe the concerned NCSs, and construct a networked controller (2) such that the closed-loop system is asymptotically stable.

3. MODELS FOR NETWORKED CONTROL SYSTEMS.

In this section, we will present a discrete-time switch model for the concerned NCSs.

Refer to $[i_m h \ i_{m+1} h]$ as a "lifted sampling period". As shown in Fig.2, for NCSs during the "lifted sampling period", three cases may arise and are discussed as follows.

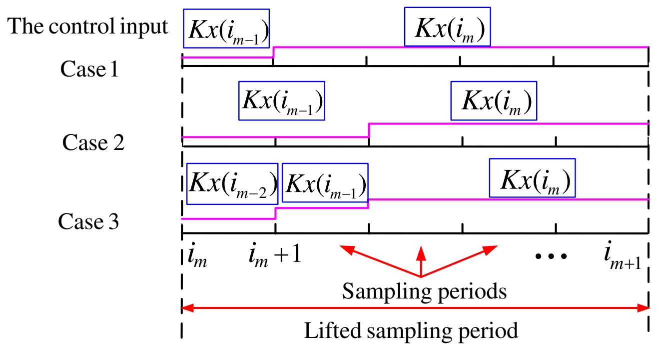

Fig. 2. A timing diagram of the concerned networked control systems.

Case 1: Because of the time delay $\tau_{i_m} \in [\tau_{\min} \ h)$, the last control signal, $\boldsymbol{Kx}(i_{m-1})$, is used to control the plant for $[\, i_m h \ (i_m+1)h\,)$, and the new control signal, $\boldsymbol{Kx}(i_m)$, is used to control the plant during $[\,(i_m+1)h \ i_{m+1}h\,)$. Then the evolution of NCS can be described as:

$$\begin{aligned} \boldsymbol{x}(i_m+1) &= \boldsymbol{Fx}(i_m) + \boldsymbol{GKx}(i_{m-1}) \\ \boldsymbol{x}(i_m+2) &= \boldsymbol{F}^2\boldsymbol{x}(i_m) + \boldsymbol{FGKx}(i_{m-1}) + \boldsymbol{GKx}(i_m) \\ &\vdots \end{aligned} \tag{5}$$

Therefore, for the time instant $i_{m+1}h$, we have

$$\begin{aligned} \boldsymbol{x}(i_{m+1}) = \boldsymbol{F}^{\eta(i_m)}\boldsymbol{x}(i_m) + \sum_{r=0}^{\eta(i_m)-2} \boldsymbol{F}^r\boldsymbol{GKx}(i_m) \\ + \boldsymbol{F}^{\eta(i_m)-1}\boldsymbol{GKx}(i_{m-1}) \end{aligned} \tag{6}$$

Case 2: In this case, we have $\tau_{i_m} \in [h \ 2h)$, the new control signal and the last control signal, i.e., $\boldsymbol{Kx}(i_m)$ and $\boldsymbol{Kx}(i_{m-1})$, are used to control the plant during $[\,(i_m+2)h \ i_{m+1}h\,)$ and $[\,i_m h \ (i_m+2)h\,)$ respectively. Similar to case 1, we can get:

$$\begin{aligned} \boldsymbol{x}(i_{m+1}) = \boldsymbol{F}^{\eta(i_m)}\boldsymbol{x}(i_m) + \sum_{r=0}^{\eta(i_m)-3} \boldsymbol{F}^r\boldsymbol{GK}x(i_m) \\ + \sum_{r=\eta(i_m)-2}^{\eta(i_m)-1} \boldsymbol{F}^r\boldsymbol{GKx}(i_{m-1}) \end{aligned} \tag{7}$$

Case 3: In this case, we have $\tau_{i_m} \in [h \ 2h)$, and the new control signal, $\boldsymbol{Kx}(i_m)$, is also used to control the plant during $[\,(i_m+2)h \ i_{m+1}h\,)$. However, the last two control signals, $\boldsymbol{Kx}(i_{m-2})$ and $\boldsymbol{Kx}(i_{m-1})$, are used to control the plant for $[\,i_m h \ (i_m+1)h\,)$ and $[\,(i_m+1)h \ (i_m+2)h\,)$ respectively. In this situation, we have

$$\begin{aligned} \boldsymbol{x}(i_{m+1}) = \boldsymbol{F}^{\eta(i_m)}\boldsymbol{x}(i_m) + \sum_{r=0}^{\eta(i_m)-3} \boldsymbol{F}^r\boldsymbol{GKx}(i_m) \\ + \boldsymbol{F}^{\eta(i_m)-2}\boldsymbol{GKx}(i_{m-1}) + \boldsymbol{F}^{\eta(i_m)-1}\boldsymbol{GKx}(i_{m-2}) \end{aligned} \tag{8}$$

To facilitate the stability analysis of NCS, introduce $\boldsymbol{z}(i_m) = \left[\boldsymbol{x}_{i_m}^T \ \boldsymbol{x}_{i_{m-1}}^T \ \boldsymbol{x}_{i_{m-2}}^T\right]^T$ into (6)-(8). Then, the NCSs can be expressed by the following switched system

$$\boldsymbol{z}(i_{m+1}) = \boldsymbol{M}_r\boldsymbol{z}(i_m) \tag{9}$$

where $\boldsymbol{M}_r$ is function of the switch $\{r\}$ which take values in a finite set $\mathbb{S} := \{1, 2, 3\}$. Note that the switch state $\{r\} = 1$, $\{r\} = 2$ and $\{r\} = 3$ are corresponding to case 1, case 2 and case 3 respectively. Then we have

$$\begin{aligned} \boldsymbol{M}_3 &= \begin{bmatrix} \boldsymbol{F}^{\eta(i_m)} + \sum_{r=0}^{\eta(i_m)-3} \boldsymbol{F}^r\boldsymbol{GK} & \boldsymbol{F}^{\eta(i_m)-2}\boldsymbol{GK} & \boldsymbol{F}^{\eta(i_m)-1}\boldsymbol{GK} \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \\ \boldsymbol{M}_2 &= \begin{bmatrix} \boldsymbol{F}^{\eta(i_m)} + \sum_{r=0}^{\eta(i_m)-3} \boldsymbol{F}^r\boldsymbol{GK} & \sum_{r=\eta(i_m)-2}^{\eta(i_m)-1} \boldsymbol{F}^r\boldsymbol{GK} & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \\ \boldsymbol{M}_1 &= \begin{bmatrix} \boldsymbol{F}^{\eta(i_m)} + \sum_{r=0}^{\eta(i_m)-2} \boldsymbol{F}^r\boldsymbol{GK} & \boldsymbol{F}^{\eta(i_m)-1}\boldsymbol{GK} & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \end{aligned} \tag{10}$$

where $\eta(i_m) \in \Omega$.

Without loss generosity, we assume that the initial control inputs are zeros, i.e., $\boldsymbol{u}(0) = 0$. Since the last control signal is used to control the plant when the new control signal is not available, we have $\boldsymbol{u}(l) = 0$ for $0 \le l \le i_1$, where i_1 is the time index of the first effective packet. With the initial plant state $\boldsymbol{x}_0 := \boldsymbol{x}(0)$, for $0 \le l \le i_1$, we have

$$\boldsymbol{x}(l) = \boldsymbol{F}^l\boldsymbol{x}_0 \tag{11}$$

Let $\boldsymbol{z}(0) = \left[\boldsymbol{x}_0^T \ 0 \ 0\right]^T$, then for $0 \le l \le i_1$, we have

$$\boldsymbol{z}(l) = \hat{\boldsymbol{M}}_l\boldsymbol{z}_0 \tag{12}$$

where $\boldsymbol{z}(l) := \left[\boldsymbol{x}_l^T \ \boldsymbol{x}_{i_{m-1}}^T \ \boldsymbol{x}_{i_{m-2}}^T\right]^T$, and

$$\hat{\boldsymbol{M}}_l = \begin{bmatrix} \boldsymbol{F}^l & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \tag{13}$$

Moreover, for $i_m < l < i_{m+1}$, the behavior of NCSs can be expressed by

$$\boldsymbol{z}(l) = \bar{\boldsymbol{M}}_r\boldsymbol{z}(i_m) \tag{14}$$

where $\boldsymbol{z}(l) := \left[\boldsymbol{x}_l^T \ \boldsymbol{x}_{i_{m-1}}^T \ \boldsymbol{x}_{i_{m-2}}^T\right]^T$, and for $i \in \mathbb{S}$, $\bar{\boldsymbol{M}}_r$ is similar to (10) but with $\eta(i_m)$ replaced by $l - i_m$.

4. STABILITY ANALYSIS OF CLOSED-LOOP NCSS.

In this section, we will give sufficient conditions for the existence of state feedback controller such that the closed-loop NCSs (9) is asymptotically stable.

Theorem 1. With a given network parameter $\boldsymbol{N}_{drop}$ and a given matrix $\boldsymbol{K}$, the NCS (9) in the presence of random time delay and arbitrary packet dropout is asymptotically stable, if for $i \in \mathbb{S}$, there exists a common positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{3n\times 3n}$ satisfying

$$\boldsymbol{M}_i^T \boldsymbol{P} \boldsymbol{M}_i - \boldsymbol{P} < 0 \tag{15}$$

where $\boldsymbol{M}_i$ is of form (10).

Proof: Construct a Lyapunov functional as:

$$V(i_m) = \boldsymbol{z}^T(i_m)\boldsymbol{P}\boldsymbol{z}(i_m) \tag{16}$$

First, we prove $\lim\limits_{l\to\infty} \boldsymbol{z}(l) = 0$ for NCS (9). Let r in (9) be i, then we have $V(i_m) = \boldsymbol{z}^T(i_m)\boldsymbol{P}\boldsymbol{z}(i_m)$ and $V(i_{m+1}) = \boldsymbol{z}^T(i_m)\boldsymbol{M}_i^T\boldsymbol{P}\boldsymbol{M}_i\boldsymbol{z}(i_m)$. From the conditions (15), one can easily show that

$$V(i_{m+1}) - V(i_m) = \boldsymbol{z}^T(i_m)(\boldsymbol{M}_i^T\boldsymbol{P}\boldsymbol{M}_i - \boldsymbol{P})\boldsymbol{z}(i_m) < 0 \tag{17}$$

for any $\boldsymbol{z}(i_m) \neq 0$. For $i_m < l < i_{m+1}$, we have $V(l) = \boldsymbol{z}^T(i_m)\bar{\boldsymbol{M}}_i^T\boldsymbol{P}\bar{\boldsymbol{M}}_i\boldsymbol{z}(i_m)$. From (14), one can see that $\bar{\boldsymbol{M}}_i \subset \boldsymbol{M}_i$, and correspondingly, $\bar{\boldsymbol{M}}_i^T\boldsymbol{P}\bar{\boldsymbol{M}}_i - \boldsymbol{P} < 0$ is a subset of (15). Thus, if the conditions (15) hold, one can easily show that:

$$V(l) - V(i_m) = \boldsymbol{z}^T(l)\left(\bar{\boldsymbol{M}}_i^T\boldsymbol{P}\bar{\boldsymbol{M}}_i - \boldsymbol{P}\right)\boldsymbol{z}(l) < 0 \tag{18}$$

for any $\boldsymbol{z}(l) \neq 0$, where $i_m < l < i_{m+1}$.

From (17), we get $V(i_{m+1}) - V(i_m) < 0$ for any $\boldsymbol{z}(i_m) \neq 0$, which implies that $\lim\limits_{i_m\to\infty} V(i_m) = 0$ for NCS (9). By considering (18), we can get $\lim\limits_{l\to\infty} V(l) = 0$ for NCS (9), where $l \neq i_m$. Summarizing the above two cases, we can conclude that $\lim\limits_{l\to\infty} V(l) = 0$ for $l \in \mathbb{Z}^+$, which implies that $\lim\limits_{l\to\infty} \boldsymbol{z}(l) = 0$ for NCS (9).

We now prove that NCS (9) is stable if the conditions (15) hold. That is, given any $\varepsilon > 0$, we can find a $\delta(\varepsilon) > 0$ such that $\|\boldsymbol{z}(0)\| < \delta(\varepsilon)$ implies $\|\boldsymbol{z}(l)\| < \varepsilon$ for $l \in \mathbb{Z}^+$. Let $\alpha_1 = \|\boldsymbol{P}\|$ and $\alpha_2 = 1/\left\|\boldsymbol{P}^{-1}\right\|$. From the definition of Lyapunov function, we get $\alpha_2\|\boldsymbol{z}(l)\|^2 \le V(l) \le \alpha_1\|\boldsymbol{z}(l)\|^2$. In NCS (9), three cases may arise and are discussed as follows.

Case 1: $0 \le l \le i_1$. From $\boldsymbol{z}_0 = \left[\boldsymbol{x}_0^T\ 0\ 0\right]^T$, we have $\|\boldsymbol{z}_0\| = \|\boldsymbol{x}_0\|$. Let $\alpha_3 := \max_{0\le l\le i_1}\left\|\hat{\boldsymbol{M}}_l\right\|$, then given any $\varepsilon > 0$, if we let $\boldsymbol{x}_0 < \min\{1/\alpha_3,\ 1\}\varepsilon$, we have $\|\boldsymbol{z}(l)\| = \left\|\hat{\boldsymbol{M}}_l\boldsymbol{z}_0\right\| \le \left\|\hat{\boldsymbol{M}}_l\right\|\|\boldsymbol{z}_0\| < \left\|\hat{\boldsymbol{M}}_l\right\|\min\{1/\alpha_3,\ 1\}\varepsilon < \varepsilon$.

Case 2: $l = i_m$. From previous discussions, we can get $V(l) < V(i_1) \le \alpha_1\|\boldsymbol{z}(i_1)\|^2 = \alpha_1\left\|\hat{\boldsymbol{M}}_{i_1}\boldsymbol{z}_0\right\|^2$ and $\|\boldsymbol{z}(l)\| \le \sqrt{V(l)/\alpha_2}$. Therefore, with any given $\varepsilon > 0$, if we let $\boldsymbol{x}_0 < (\sqrt{\alpha_2/\alpha_1}/\alpha_3)\varepsilon$, then we can get $\|\boldsymbol{z}(l)\| \le \sqrt{V(l)/\alpha_2} < \sqrt{\left\|\hat{\boldsymbol{M}}_{i_1}\right\|^2\|\boldsymbol{z}_0\|^2\alpha_1/\alpha_2} = \|\boldsymbol{z}_0\|\alpha_3\sqrt{\alpha_1/\alpha_2} < \varepsilon$.

Case 3: $i_m < l < i_{m+1}$. From (18), we have $V(l) < V(i_m)$ for this case. Let $\alpha_4 := \max_{i\in\mathbb{S}}\left\|\bar{\boldsymbol{M}}_i\right\|$. With any given $\varepsilon > 0$, if we let $\boldsymbol{x}_0 < \sqrt{\alpha_2/\alpha_1}/(\alpha_3\alpha_4)\varepsilon$, then by considering the discussions in case 2, we can get $\|\boldsymbol{z}(l)\| \le \left\|\bar{\boldsymbol{M}}_r\right\|\|\boldsymbol{z}(i_m)\| \le \alpha_4\|\boldsymbol{z}(i_m)\| < \|\boldsymbol{z}_0\|\alpha_3\alpha_4\sqrt{\alpha_1/\alpha_2} < \varepsilon$.

Based on the above analysis, if we let

$$\alpha := \min\{\min\{1/\alpha_3,\ 1\},\ \sqrt{\alpha_2/\alpha_1}/\alpha_3,\ \sqrt{\alpha_2/\alpha_1}/(\alpha_3\alpha_4)\}$$

then we can conclude that $\|\boldsymbol{z}(0)\| < \alpha\varepsilon$ implies that $\|\boldsymbol{z}(l)\| < \varepsilon$ for NCS (9), where $l \in \mathbb{Z}^+$.

According to the definition of "asymptotically stable", we can complete the proof.

5. STATE FEEDBACK CONTROLLER DESIGN

In this section, we consider the design of networked controller (2) such that the closed-loop system (9) is asymptotically stable.

By using Schur complement to (15), (15) is equivalent to

$$\Psi_i(\boldsymbol{Q},\ \boldsymbol{K}) := \begin{bmatrix} -\boldsymbol{Q} & * \\ \boldsymbol{M}_i\boldsymbol{Q} & -\boldsymbol{Q} \end{bmatrix} < 0 \tag{19}$$

where $\boldsymbol{Q} = \boldsymbol{P}^{-1}$, $\boldsymbol{M}_i$ is of form (10) with variable $\boldsymbol{K}$.

From theorem 1 we see that the feasible solutions of (19) can lead to the desired networked controllers (2). Unfortunately, (19) can not be formulated into LMIs since $\boldsymbol{M}_i\boldsymbol{Q}$ involve the products between the unknown variables $\boldsymbol{K}$ and $\boldsymbol{Q}$. However, an important feature of (19) is that if $\boldsymbol{K}$ is fixed, finding $\boldsymbol{Q}$ becomes an LMI problem and vice versa. So (19) is in fact a set of bilinear matrix inequalities (BLMI). To circumvent the synthesis problem, a homotopy-based iterative LMI algorithm is developed as follows.

Define

$$\Sigma_i(\boldsymbol{Q},\ \boldsymbol{K},\lambda) :=$$
$$\begin{bmatrix} -\boldsymbol{Q} & * \\ \boldsymbol{H}_i^1\boldsymbol{Q} + \lambda\boldsymbol{H}_i^2 diag\{\boldsymbol{K},\boldsymbol{K},\boldsymbol{K}\}\boldsymbol{Q} + (1-\lambda)\boldsymbol{H}_i^2\bar{\boldsymbol{K}} & -\boldsymbol{Q} \end{bmatrix} < 0 \tag{20}$$

where $i \in \mathbb{S}$, and

$$\boldsymbol{H}^1 = \begin{bmatrix} \boldsymbol{F}^{\eta(i_m)} & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$
$$\boldsymbol{H}_i^2 = \boldsymbol{M}_i - \boldsymbol{H}^1 \quad (i \in \mathbb{S})$$
$$\bar{\boldsymbol{K}} = diag\{\boldsymbol{K},\cdots\boldsymbol{K}\}\boldsymbol{Q} \tag{21}$$

λ is a real number varying from 0 to 1. Especially, when $\lambda = 0$ and $\lambda = 1$, we have

$$\Sigma_i(\boldsymbol{Q},\ \boldsymbol{K}) = \begin{cases} \Psi_i(\boldsymbol{Q},\ \boldsymbol{K}) & \text{if } \lambda = 1 \\ \Phi_i(\boldsymbol{Q},\ \bar{\boldsymbol{K}}) & \text{if } \lambda = 0 \end{cases} \tag{22}$$

where

$$\Phi_i(\boldsymbol{Q},\ \bar{\boldsymbol{K}}) := \begin{bmatrix} -\boldsymbol{Q} & * \\ \mathrm{H}^1\boldsymbol{Q} + \mathrm{H}_i^2\bar{\boldsymbol{K}} & -\boldsymbol{Q} \end{bmatrix} < 0 \tag{23}$$

Based on the above discussions, an iterative LMI algorithm can be summarized as follows.

Controller Design Procedure:

Step 1. Initiation: Set $k = 0$, select N, N_{max}. Solve $\Phi_i(\boldsymbol{Q},\ \bar{\boldsymbol{K}})$ to get $\boldsymbol{Q}$, $\bar{\boldsymbol{K}}$, and let $\boldsymbol{Q}(0) := \boldsymbol{Q}$, $\bar{\boldsymbol{K}}(0) := \bar{\boldsymbol{K}}$.

Step 2. Set $k = k+1$ and $\lambda_k = k/N$. Let $\boldsymbol{Q} := \boldsymbol{Q}(k-1)$, $\bar{\boldsymbol{K}} := \bar{\boldsymbol{K}}(k-1)$. If (20) upon $\boldsymbol{K}$ is feasible, then denote the feasible solution as $\boldsymbol{K}(k)$, let $\boldsymbol{Q}(k) := \boldsymbol{Q}(k-1)$, $\bar{\boldsymbol{K}}(k) := \bar{\boldsymbol{K}}(k-1)$, and go to Step 4. Otherwise, go to Step 3.

Step 3. Let $\boldsymbol{K} := \boldsymbol{K}(k-1)$ and $\bar{\boldsymbol{K}} := \bar{\boldsymbol{K}}(k-1)$. If (20) upon $\boldsymbol{Q}$ is feasible, then solve the minimization problem:

$$\begin{array}{l}\textbf{OP}: \min \operatorname{tr}\left(trace(\boldsymbol{Q})\right) \\ \text{s.t. Inequalities (20)}\end{array} \tag{24}$$

Denote the feasible solution as $\boldsymbol{Q}(k)$, let $\boldsymbol{K}(k) := \boldsymbol{K}(k-1)$ and $\bar{\boldsymbol{K}}(k) := \bar{\boldsymbol{K}}(k-1)$, then go to Step 4. Otherwise, set $N = 2N$. If $N > N_{max}$, then the algorithm fails in giving feasible solution, else set $k = 0$, go to Step 2.

Step 4. If $k < N$, go to Step 2. If $k = N$, the obtained solutions $\boldsymbol{K}(k)$ and $\boldsymbol{Q}(k)$ are a set of feasible solutions of (19).

Remark 1: Note that in the Controller Design Procedure, (20) and (23) can be easily solved by using the command "*feasp*" in MATLAB environment, and the optimization problem **OP** can be readily solved by using the command "*mincx*" in MATLAB environment. Therefore, the proposed controller design procedure can be readily solved. For more details on the commands "*feasp*" and "*mincx*", interested readers can refer to the MATLAB Help.

Remark 2: It is also worth noting that since the actuator is clock-driven, (1) can be considered as discretized from a continuous-time system given by

$$\dot{\boldsymbol{x}}_{\mathrm{p}}(t) = \boldsymbol{A}\boldsymbol{x}_{\mathrm{p}}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{25}$$

with sampling period h and

$$\boldsymbol{F} = e^{\boldsymbol{A}h}, \quad \boldsymbol{G} = \int_0^h e^{\boldsymbol{A}\tau} d\tau \boldsymbol{B}. \tag{26}$$

6. EXPERIMENTAL EXAMPLE

To validate the proposed approaches, we set up a networked DC motor control system over network. The parameters of the motor used in this paper are listed in Table I. Let $\boldsymbol{x}_p = [\theta, \, \omega]^T$, where θ and ω are the output angle and the angular speed respectively, then the DC motor dynamics can be expressed as:

$$\dot{\boldsymbol{x}}_p(t) = \begin{bmatrix} 0 & 1 \\ 1 & -217.4 \end{bmatrix} \boldsymbol{x}_p(t) + \begin{bmatrix} 0 \\ 1669.5 \end{bmatrix} \boldsymbol{u}(t) \tag{27}$$

Table 1. The paramors of the DC motor

J	Inertia	$(10.3 + 0.7) \times 10^{-7}\mathrm{kg} \bullet \mathrm{m}^2$
L	Inductance	$0.24 \times 10^{-3}\mathrm{H}$
R	Resistance	2.32Ω
K	Torque Constant	$23.4 \times 10^{-3}\mathrm{N} \bullet \mathrm{m/A}$
n	Gear reduction ratio	1/318
K_e	Back-EMF Constant	$23.4 \times 10^{-3}\mathrm{Vs/rad}$

The block diagram of the networked servo motor control system is shown in Fig.1, and the actual system setup is depicted in Fig.3. As shown in Fig.3, the experimental apparatus consists of a PC controller, a local board and a DC motor. The PC controller is used to implement the networked controller. The local board is on the plant side and is used for two tasks. The first task is to convert the control signal read from the controller into a pulse width-modulation (PWM) signal, and then send the PWM signal to drive the DC motor. The second one is to send the plant state measurements to the PC controller. To carry out the experiment, the PC controller and the motor were placed in different locations and were linked by CERNET (China Education and Research Network). To carry out experiment, we assume the controller processing delay is zero due to its negligible small value by comparing with the network delay, and the sampling period used is 0.03s. According to the practical measurements, in the used CERNET, the RTT delay ranges from 5ms to 62ms, and in three consecutive packets from sensor, there is at least one effective packet. Note that the upper bound of RTT delay is longer than $2h$, and up to 66.7% of the packets can be lost.

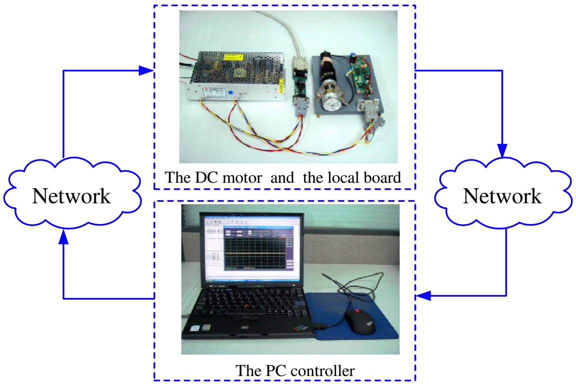

Fig. 3. The actual networked DC motor control system setup

We apply the proposed controller design method to the concerned networked motor control system, and therefore obtain $\boldsymbol{K} = [-0.0660, \, 0.0276]$. The control system is designed to drive the networked DC motor to a pre-set angle. With the initial state $\boldsymbol{x}_0 = [3.65, 0]^T$, the simulation result of this system using the designed controller is depicted in Fig.4, and the experimental result of networked motor control system is depicted in Fig.5. Apparently, the experimental result is consistent with the simulation result. As can be seen in this example, although the upper bound of RTT delay is longer than $2h$, and up to 66.7% of the packets can be lost, the proposed controller can still stabilize the networked DC motor system very well. This demonstrates the effectiveness of the proposed approaches.

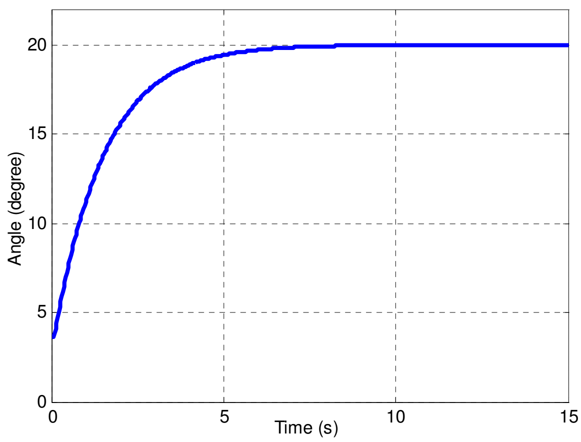

Fig. 4. Typical simulation result of networked DC motor control system under the proposed controller

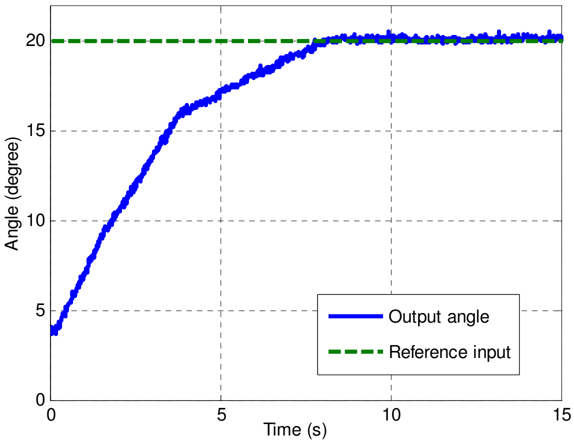

Fig. 5. Typical experimental result of networked DC motor control system under the proposed controller

7. CONCLUSIONS

This paper has investigated the state feedback controller design and stability analysis problems for NCSs under effects of network-induced delay and packet dropout. A discrete-time switch model is proposed by introducing the lifting technique into the considered NCSs, which enables us to apply the theory from switch systems to study NCSs in discrete-time domain. In the proposed framework, sufficient conditions for the existence of state feedback controller such that the closed-loop NCSs is asymptotically stable is derived and the corresponding controller design problem is also addressed. It has been shown that the obtained controller design procedure can be readily solved and the proposed controller can be easily implemented. Simulation and experimental results are given to demonstrate the effectiveness of the proposed approaches.

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