Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Stochastic Observer-based Guaranteed Cost Control for Networked Control Systems with Packet Dropouts

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Abstract: This paper is concerned with an observer-based guaranteed cost control (GCC) problem for networked control systems (NCSs) with random data packet dropouts. Both the sensor-to-controller (S/C) and controller-to-actuator (C/A) packet dropouts are modeled by two mutually independent stochastic variables satisfying Bernoulli binary distribution. The resultant observer-based controller guarantees that the closed-loop system is stochastically exponentially mean-square stable and the cost function value is not more than a specified upper bound. The controller design problem is transformed to a convex optimization problem, which can be solved by a linear matrix inequality (LMI) approach. A numerical example is given to illustrate the effectiveness of the proposed design method.

1. INTRODUCTION

Networked control system (NCS) is a type of distributed control system whose feedback control loop is based on a communication network. Comparing with the conventional point-to-point control systems, the NCSs show some nice features, such as, flexibility of operation, ease of diagnosis and maintenance, small volume of wiring, low cost, etc. However, the insertion of communication networks in feedback control loops complicates the analysis and synthesis of NCSs, because the network-induced data packet transmission delays and dropouts will inevitably degrade the control performance of the NCSs, or even cause the systems instable. This has motivated increasing research interests in the study of the analysis and synthesis of NCSs in the last decade (Nilsson, Bernhardsson, & Wittenmark, 1998; Zhang, Branicky, & Phillips, 2001; Walsh, Ye, & Bushnell, 2002; Hu, & Zhu, 2003; Yue, Han, & Peng, 2004; Yang et al., 2006; Hu et al., 2007).

In NCSs, data packets through networks suffer not only transmission delays, but also, possibly, transmission packet dropouts. The latter is a potential source of instability and poor performance in NCSs because of the critical real-time requirement in control systems. Therefore, the effect of packet dropouts is also an important aspect in the analysis and synthesis of NCSs and this issue has been received widely attentions recently (Yu et al., 2004; Ling & Lemmon, 2004; Huo & Fang, 2007; Wu & Chen; 2007). There are two typical ways to model packet dropouts in previous literatures. The first approach assumes that the packet dropouts follow certain probability distributions and describes NCSs with packet dropouts via stochastic models, such as Markov jump systems. For example, the work of Wu & Chen (2007) models the packet dropouts' history behaviors as two independent Markov chains. Based on this model, the deferent model of NCSs with single-and multiple-packet transmission are also investigated in Wu & Chen (2007). The second approach is deterministic, and models the NCSs with packet dropouts as switched linear systems or asynchronous dynamical system (ADS). An iterative approach is proposed to model NCSs with arbitrary but finite data packet dropouts as switched linear systems in Yu et al., (2004). Using ADS methods, the integrity design problems for a class of NCSs with sensors, actuators failures and network-induced packet dropouts are considered in Huo & Fang (2007). Because of the complicated NCS modeling, the effect of controller-to-actuator (C/A) packet dropouts is neglected and only the packet dropouts existing in the sensor-to-controller (S/C) side is considered in the work of Yu et al., (2004) and Huo & Fang (2007).

Since data packet dropouts might be potential sources to instability and poor performance of NCSs, the main objective of this paper is to design guaranteed cost controllers for a class of NCSs with random packet dropouts. Both the S/C and C/A packet dropouts are considered which modeled by two mutually independent stochastic variables satisfying Bernoulli binary distribution. An observer-based control scheme is proposed such that the closed-loop NCS is stochastically exponentially mean-square stable and the specified GCC performance is achieved. A linear matrix inequality (LMI) approach is developed to tackle the addressed problem, which can be solved conveniently by Matlab LMI toolbox.

The paper is organized as follows. Section 2 provides preliminaries and the formulation of the problem. Section 3 investigates the stability conditions for the closed-loop NCS. Section 4 presents the design methods of the observe-based guaranteed cost controller. A numerical example is given to illustrate the proposed design methods in Section 5, followed by conclusions in Section 6.

2. PRELIMINARIES AND PROBLEM STATEMENT

If Consider the NCS with random data packet dropouts in Fig.1, where sensor, controller, and actuator are clock-driven.

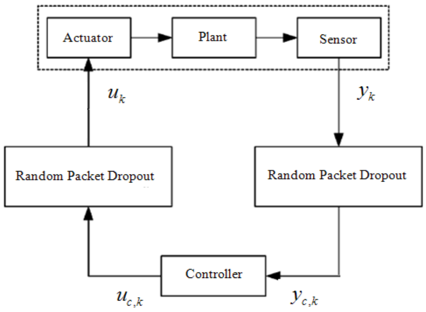

Fig. 1. Structure of an NCS with packet dropouts

The plant is assumed to be of the form

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input vector, $y_k \in \mathbb{R}^p$ is the measurement output vector. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ are known constant matrices.

Suppose the controller has a buffer to hold the most recent packets as the new sensor output when the packet dropout happens. For example, when a sensor data y_k is lost, the controller will read out the most recent data $y_{c,k-1}$ from the buffer and utilize it as $y_{c,k}$ to calculate the new controller output $u_{c,k}$, which will be sent to the plant; otherwise, the new sensor data y_k will be saved to the buffer and used by the controller as $y_{c,k}$. And on the actuator side, the latest control input is kept when a packet is lost. Thus, for Fig.1, we have

$$y_{c,k} = \begin{cases} y_k & \text{if transmitted successfully} \\ y_{c,k-1} & \text{otherwise} \end{cases} \tag{2}$$

$$u_k = \begin{cases} u_{c,k} & \text{if transmitted successfully} \\ u_{k-1} & \text{otherwise} \end{cases} \tag{3}$$

Assume that the probability of a packet dropout in the network satisfy the Bernoulli random binary distribution. Thus, the measurement output and control input can be described by

$$y_{c,k} = (1-\alpha_k)y_k + \alpha_k y_{c,k-1} \tag{4}$$

$$u_k = (1-\beta_k)u_{c,k} + \beta_k u_{k-1} \tag{5}$$

where the stochastic variables α_k and β_k are the mutually independent Bernoulli binary distributed white sequences taking values on 0 and 1 with

$$\begin{cases} \text{Prob}\{\alpha = 1\} = E\{\alpha\} := \bar{\alpha} \\ \text{Prob}\{\alpha = 0\} = 1 - E\{\alpha\} := 1 - \bar{\alpha} \end{cases} \tag{6}$$

$$\begin{cases} \text{Prob}\{\beta = 1\} = E\{\beta\} := \bar{\beta} \\ \text{Prob}\{\beta = 0\} = 1 - E\{\alpha\} := 1 - \bar{\beta} \end{cases} \tag{7}$$

From (1), (4) and (5), the NCS model can be described as

$$\begin{cases} z_{k+1} = \Phi_k z_k + \Gamma_k u_{c,k} \\ y_{c,k} = \mathrm{H}_k z_k + \alpha_k y_{c,k-1} \end{cases} \tag{8}$$

where

$$z_k = \begin{bmatrix} x_k^T & u_{k-1}^T \end{bmatrix}^T, \quad \Phi_k = \begin{bmatrix} A & \beta_k B \\ 0 & \beta_k I \end{bmatrix},$$

$$\Gamma_k = (1-\beta_k)\begin{bmatrix} B \\ I \end{bmatrix}, \quad \mathrm{H}_k = (1-\alpha_k)\begin{bmatrix} C & 0 \end{bmatrix} \tag{9}$$

In this paper, we propose an observer-based control scheme for (8) described by

$$\text{Observer:} \begin{cases} \hat{z}_{k+1} = \Phi_k \hat{z}_k + \Gamma_k u_{c,k} + L(y_{c,k} - \hat{y}_{c,k}) \\ \hat{y}_{c,k} = \mathrm{H}_k \hat{z}_k + \alpha_k y_{c,k-1} \end{cases} \tag{10}$$

$$\text{Controller: } u_{c,k} = K\hat{z}_k = K_1\hat{x}_k + K_2\hat{u}_{k-1} \tag{11}$$

where $\hat{z}_k = \begin{bmatrix} \hat{x}_k^T & \hat{u}_{k-1}^T \end{bmatrix}^T \in \mathbb{R}^{n+m}$ is the state estimate of system (8), $\hat{y}_{c,k} \in \mathbb{R}^p$ is the observer output, $L \in \mathbb{R}^{(n+m)\times p}$ and $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \in \mathbb{R}^{m\times(n+m)}$ are the observer gain and controller gain, respectively. In the control scheme (10) and (11), it is assumed that the information of the past packet transmission from controller to actuator is available to the controller, that is, at the time $k+1$, the value of β_k (0 or 1) is available to the controller. One way to achieve this is by transmitting this information with high priority when a packet is lost. And at the time k , the packet dropout information (α_k) is available to the controller, which can be achieved by comparing the value of $y_{c,k}$ and $y_{c,k-1}$.

Remark 1: The packet dropout model (4) and (5) is an extended version of the delay model of Yang et al., (2006), in which the y_{k-1} and $u_{c,k-1}$ are substituted by $y_{c,k-1}$ and u_{k-1} , respectively. But the model of Yang et al., (2006) can merely describe the network induced delay shorter than a sampling period. In the proposed control scheme (10) and (11), the plant output y_k produced at a time k is sent to the observer via a communication network at the time k if there is no packet is dropped, i.e., $y_{c,k} = y_k$, $\alpha_k = 0$ (the influence of the delay is neglected in our model). If the date packet of y_k is dropped, the controller will read out the most recent data $y_{c,k-1}$ from the buffer and utilize it as $y_{c,k}$, i.e., $y_{c,k} = y_{c,k-1}$, $\alpha_k = 1$. From (10) and (11), it can be easily see that the values of $\hat{z}_k$ and $u_{c,k}$ can be calculated by the observer and controller at the time k , if the value of β_{k-1} and α_k is available to the controller.

Defining the estimation error by

$$e_k := z_k - \hat{z}_k \tag{12}$$

the closed-loop system can be described as

$$\begin{cases} z_{k+1} = [\bar{A} + (1-\bar{\beta})\bar{B}K]z_k - (1-\bar{\beta})\bar{B}Ke_k \\ \quad + (\beta_k - \bar{\beta})\left[(\tilde{A} - \bar{B}K)z_k + \bar{B}Ke_k\right] \\ e_{k+1} = \left[\bar{A} - (1-\bar{\alpha})L\bar{C}\right]e_k \\ \quad + (\beta_k - \bar{\beta})\tilde{A}e_k + (\alpha_k - \bar{\alpha})L\bar{C}e_k \end{cases} \tag{13}$$

where

$$\bar{A} = \begin{bmatrix} A & \bar{\beta}B \\ 0 & \bar{\beta}I \end{bmatrix},\ \bar{B} = \begin{bmatrix} B \\ I \end{bmatrix},\ \tilde{A} = \begin{bmatrix} 0 & B \\ 0 & I \end{bmatrix},\ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{14}$$

Since the closed-loop system (13) contains both stochastic quantities α_k and β_k, it is actually a stochastic parameter system, and we need to introduce the notion of stochastic stability in the mean-square sense for the problem formulation.

Definition 1 (Yang et al., 2006): The closed-loop system (13) is said to be exponentially mean-square stable if there exist constants $\gamma > 0$ and $\tau \in (0 \quad 1)$ such that

$$E\{\| \eta_k \|^2\} \le \gamma \tau^k E\{\| \eta_0 \|^2\} \text{ for all } \eta_0 \in \mathbb{R}^n, k \in \mathbb{I}^+ \tag{15}$$

where $\eta_k = \begin{bmatrix} z_k^T & e_k^T \end{bmatrix}^T$.

With this definition, our objective is to design the observer (10) and controller (11) for the system (1) such that the closed-loop system (13) is exponentially mean-square stable, and the GCC performance constraint is satisfied. In other words, we aim to design a controller such that the closed-loop system satisfies the following requirements Q1) and Q2) simultaneously.

Q1) The closed-loop system (13) is exponentially mean-square stable.

Q2) The cost function associated with the closed-loop system

$$J = E\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k\right\} \tag{16}$$

is not more than a specified upper bound J^*, where $Q > 0$ and $R > 0$ are given weighting matrices.

3. STABILITY ANALYSIS

In this section, we will investigate the stability conditions for the closed-loop system (13). The following lemma will be needed in our derivation.

Lemma 1 (Yang et al., 2006): Let $V(\eta_k)$ be a Lyapunov functional. If there exist real scalars $\lambda \ge 0$, $\mu > 0, \nu > 0$, and $0 < \varphi < 1$ such that

$$\mu \| \eta_k \|^2 \le V(\eta_k) \le \upsilon \| \eta_k \|^2 \tag{17}$$

$$E\{V(\eta_{k+1}) \mid \eta_k\} - V(\eta_k) \le \lambda - \varphi V(\eta_k) \tag{18}$$

then the sequence η_k satisfies

$$E\{\| \eta_k \|^2\} \le \frac{\upsilon}{\mu} \| \eta_0 \|^2 (1-\varphi)^k + \frac{\lambda}{\mu\varphi} \tag{19}$$

The following theorem presents sufficient conditions for the existence of the controller such that the closed-loop system (13) is exponentially mean-square stable.

Theorem 1: Given the controller gain matrix K and the observer gain matrix L. The closed-loop system (13) is exponentially mean-square stable, if there exit positive definite matrices P and S satisfying the following matrix inequality

$$\begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & -S & * & * & * & * & * \\ \bar{A} + (1-\bar{\beta})\bar{B}K & -(1-\bar{\beta})\bar{B}K & -P^{-1} & * & * & * & * \\ \tilde{A} - \bar{B}K & \bar{B}K & 0 & -\varepsilon_2^{-1}P^{-1} & * & * & * \\ 0 & \bar{A} - (1-\bar{\alpha})L\bar{C} & 0 & 0 & -S^{-1} & * & * \\ 0 & \tilde{A} & 0 & 0 & 0 & -\varepsilon_2^{-1}S^{-1} & * \\ 0 & L\bar{C} & 0 & 0 & 0 & 0 & -\varepsilon_1^{-1}S^{-1} \end{bmatrix} < 0 \tag{20}$$

where $\varepsilon_1 = \bar{\alpha}(1-\bar{\alpha})$, $\varepsilon_2 = \bar{\beta}(1-\bar{\beta})$.

Proof: Define a Lyapunov function

$$V(\eta_k) = z_k^T P z_k + e_k^T S e_k \tag{21}$$

From (13), and in terms of

$$E\{(a_k - \bar{\alpha})(B_k - \bar{\beta})\} = 0,\ E\{a_k - \bar{\alpha}\} = E\{B_k - \bar{\beta}\} = 0,$$

it can be obtained that

$$\begin{aligned} & E\{V(\eta_{k+1}) \mid \eta_k\} - V(\eta_k) \\ & = E\{z_{k+1}^T P z_{k+1} + e_{k+1}^T S e_{k+1}\} - z_k^T P z_k - e_k^T S e_k \\ & = \{[\bar{A} + (1-\bar{\beta})\bar{B}K]z_k - (1-\bar{\beta})\bar{B}Ke_k\}^T P \{[\bar{A} + (1-\bar{\beta})\bar{B}K]z_k - (1-\bar{\beta})\bar{B}Ke_k\} \\ & + \{[\bar{A} - (1-\bar{\alpha})L\bar{C}]e_k\}^T S \{[\bar{A} - (1-\bar{\alpha})L\bar{C}]e_k\} \\ & + E\{(\beta_k - \bar{\beta})^2\}\left[(\tilde{A} - \bar{B}K)z_k + \bar{B}Ke_k\right]^T P \left[(\tilde{A} - \bar{B}K)z_k + \bar{B}Ke_k\right] \\ & + E\{(\beta_k - \bar{\beta})^2\}(\tilde{A}e_k)^T S (\tilde{A}e_k) \\ & + E\{(a_k - \bar{\alpha})^2\}(L\bar{C}e_k)^T S (L\bar{C}e_k) - z_k^T P z_k - e_k^T S e_k \end{aligned} \tag{22}$$

In light of

$$E\{(a_k - \bar{\alpha})^2\} = (1-\bar{\alpha})\bar{\alpha}$$

$$E\{(\beta_k - \bar{\beta})^2\} = (1-\bar{\beta})\bar{\beta},$$

the formula (22) results in

$$
\begin{aligned}
&E\left\{V\left(\eta_{k+1}\right) \mid \eta_k\right\}-V\left(\eta_k\right) \\
&=\left\{[\bar{A}+\left(1-\bar{\beta}\right)\bar{B}K]z_k-\left(1-\bar{\beta}\right)\bar{B}Ke_k\right\}^T \\
&P\left\{[\bar{A}+\left(1-\bar{\beta}\right)\bar{B}K]z_k-\left(1-\bar{\beta}\right)\bar{B}Ke_k\right\} \\
&+\left\{\left[\bar{A}-\left(1-\bar{\alpha}\right)L\bar{C}\right]e_k\right\}^T S\left\{\left[\bar{A}-\left(1-\bar{\alpha}\right)L\bar{C}\right]e_k\right\} \\
&+\left(1-\bar{\beta}\right)\bar{\beta}\left[\left(\tilde{A}-\bar{B}K\right)z_k+\bar{B}Ke_k\right]^T P\left[\left(\tilde{A}-\bar{B}K\right)z_k+\bar{B}Ke_k\right] \\
&+\left(1-\bar{\alpha}\right)\bar{\alpha}\left(L\bar{C}e_k\right)^T S\left(L\bar{C}e_k\right) \\
&+\left(1-\bar{\beta}\right)\bar{\beta}\left(\tilde{A}e_k\right)^T S\left(\tilde{A}e_k\right)-z_k^T P z_k-e_k^T S e_k \\
&=\eta_k^T \Lambda \eta_k
\end{aligned} \tag{23}
$$

where

$$
\Lambda=\begin{bmatrix} \bar{A}+\left(1-\bar{\beta}\right)\bar{B}K & -\left(1-\bar{\beta}\right)\bar{B}K \\ 0 & \bar{A}-\left(1-\bar{\alpha}\right)L\bar{C} \\ \tilde{A}-\bar{B}K & \bar{B}K \\ 0 & L\bar{C} \end{bmatrix}^T \begin{bmatrix} P & 0 & 0 & 0 \\ 0 & S & 0 & 0 \\ 0 & 0 & \left(1-\bar{\beta}\right)\bar{\beta}P & 0 \\ 0 & 0 & 0 & \left(1-\bar{\alpha}\right)\bar{\alpha}S \end{bmatrix}
$$

$$
\begin{bmatrix} \bar{A}+\left(1-\bar{\beta}\right)\bar{B}K & -\left(1-\bar{\beta}\right)\bar{B}K \\ 0 & \bar{A}-\left(1-\bar{\alpha}\right)L\bar{C} \\ \tilde{A}-\bar{B}K & \bar{B}K \\ 0 & L\bar{C} \end{bmatrix}+\begin{bmatrix} -P & 0 \\ 0 & -S+\left(1-\bar{\beta}\right)\bar{\beta}\tilde{A}^T S\tilde{A} \end{bmatrix} \tag{24}
$$

By Schur complement, it can be easily obtained that (20) is equivalent to $\Lambda<0$.

Define real scalars σ and θ satisfying

$$\sigma=\max\left\{\lambda_{\max}\left(P\right),\ \lambda_{\max}\left(S\right)\right\} \tag{25}$$

$$0<\theta<\min\left\{\lambda_{\min}\left(-\Lambda\right),\ \sigma\right\} \tag{26}$$

From $\Lambda<0$ and (26) we have

$$E\left\{V\left(\eta_{k+1}\right) \mid \eta_k\right\}-V\left(\eta_k\right)=\eta_k^T\Lambda\eta_k \le -\lambda_{\min}\left(-\Lambda\right)\eta_k^T\eta_k<-\theta\eta_k^T\eta_k \tag{27}$$

By (25) and (27) yields

$$
\begin{aligned}
&\theta \| \eta_k \|^2 \le V\left(\eta_k\right) \le \sigma \| \eta_k \|^2, \\
&E\left\{V\left(\eta_{k+1}\right) \mid \eta_k\right\}-V\left(\eta_k\right)<-\theta\eta_k^T\eta_k \le -\frac{\theta}{\sigma}V\left(\eta_k\right)
\end{aligned} \tag{28}
$$

Using Lemma 1, (28) imply that

$$E\left\{\| \eta_k \|^2\right\} \le \frac{\theta}{\sigma}\left(1-\frac{\theta}{\sigma}\right)^k \| \eta_0 \|^2, \quad 0<\frac{\theta}{\sigma}<1 \tag{29}$$

Therefore, by Definition 1, it can be verified that the closed-loop system (13) is exponentially mean-square stable.□

4. OBSERVER-BASED GCC DESIGN

In this section, a design method of the observe-based guaranteed cost controller is presented and the controller design problem is transformed to a convex optimization problem, which can be solved by a linear matrix inequality (LMI) approach.

Theorem 2: Consider system (1) with cost function defined by (16). If there exist positive-definite matrices P, S, and matrices X, Y and $\hat{P}$, such that the following matrix inequalities

$$
\begin{bmatrix}
-P+Q' & * & * & * & * & * & * \\
0 & -S & * & * & * & * & * \\
P\bar{A}+\left(1-\bar{\beta}\right)\bar{B}X & -\left(1-\bar{\beta}\right)\bar{B}X & -P & * & * & * & * \\
P\tilde{A}-\bar{B}X & \bar{B}X & 0 & -\varepsilon_2^{-1}P & * & * & * \\
0 & \bar{A}-\left(1-\bar{\alpha}\right)Y\bar{C} & 0 & 0 & -S & * & * \\
0 & S\tilde{A} & 0 & 0 & 0 & -\varepsilon_2^{-1}S & * \\
0 & Y\bar{C} & 0 & 0 & 0 & 0 & -\varepsilon_1^{-1}S
\end{bmatrix}<0 \tag{30}
$$

and matrix equation

$$P\bar{B}=\bar{B}\hat{P} \tag{31}$$

hold, where

$$Q'=\begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \tag{32}$$

then the system (10) and (11) construct an observe-based controller which guarantees that the closed-loop system (13) is exponentially mean-square stable and the corresponding value of the cost function satisfies

$$J<J^*=z_0^T P z_0+e_0^T S e_0 \tag{33}$$

where $z_0=\begin{bmatrix} x_0^T & 0 \end{bmatrix}^T$, $e_0=\begin{bmatrix} \left(x_0-\hat{x}_0\right)^T & 0 \end{bmatrix}^T$. Furthermore, the gain K and L can be given as

$$K=\hat{P}^{-1}X, \quad L=S^{-1}Y \tag{34}$$

Proof: Denoting

$$X=\hat{P}K, \quad Y=SL \tag{35}$$

Pre- and post-multiplying both sides of (30) with $diag\{I,I,P^{-1},P^{-1},S^{-1},S^{-1},S^{-1}\}$ and substituting (31) into (30), it can be obtained

$$
\begin{bmatrix}
-P+Q' & * & * & * & * & * & * \\
0 & -S & * & * & * & * & * \\
\bar{A}+\left(1-\bar{\beta}\right)\bar{B}K & -\left(1-\bar{\beta}\right)\bar{B}K & -P^{-1} & * & * & * & * \\
\tilde{A}-\bar{B}K & \bar{B}K & 0 & -\varepsilon_2^{-1}P^{-1} & * & * & * \\
0 & \bar{A}-\left(1-\bar{\alpha}\right)L\bar{C} & 0 & 0 & -S^{-1} & * & * \\
0 & \tilde{A} & 0 & 0 & 0 & -\varepsilon_2^{-1}S^{-1} & * \\
0 & L\bar{C} & 0 & 0 & 0 & 0 & -\varepsilon_1^{-1}S^{-1}
\end{bmatrix}<0 \tag{36}
$$

It can be easily obtained that (36) implies (20), hence it follows from Theorem 1 that the closed-loop system (13) is exponentially mean-square stable.

Denoting new vectors

$$u_{-1} = 0, \text{ and } \hat{u}_{-1} = 0 \tag{37}$$

the cost function (16) can be rewritten as

$$J = E\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k\right\} = E\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + u_{k-1}^T R u_{k-1}\right\} \tag{38}$$

From (23), we have

$$\begin{aligned} & E\{V(\eta_{k+1}) \mid \eta_k\} - V(\eta_k) + x_k^T Q x_k + u_{k-1}^T R u_{k-1} \\ & = \eta_k^T \Lambda \eta_k + x_k^T Q x_k + u_{k-1}^T R u_{k-1} \\ & = \eta_k^T \Lambda \eta_k + z_k^T Q' z_k \\ & = \eta_k^T \left(\Lambda + \begin{bmatrix} Q' & 0 \\ 0 & 0 \end{bmatrix}\right) \eta_k \end{aligned} \tag{39}$$

By Schur complement, it can be easily obtained that (36) is equivalent to

$$\left(\Lambda + \begin{bmatrix} Q' & 0 \\ 0 & 0 \end{bmatrix}\right) < 0 \tag{40}$$

Thus, we have

$$E\{V(\eta_{k+1}) \mid \eta_k\} - V(\eta_k) + x_k^T Q x_k + u_{k-1}^T R u_{k-1} < 0 \tag{41}$$

Summing up (41) from 0 to ∞ with respect to k yields

$$\begin{aligned} & E\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k\right\} \\ & = E\left\{\sum_{k=0}^{\infty} x_k^T Q x_k + u_{k-1}^T R u_{k-1}\right\} \\ & < E\{V(\eta_0)\} - E\{V(\eta_\infty)\} \end{aligned} \tag{42}$$

Since the closed-loop system is exponentially mean-square stable, and in light of (37), we have

$$J < E\{V(\eta_0)\} - E\{V(\eta_\infty)\} = E\{V(\eta_0)\} = J^* \tag{43}$$

where $\eta_0 = \begin{bmatrix} x_0^T & u_{-1}^T & (x_0 - \hat{x}_0)^T & (u_{-1} - \hat{u}_{-1})^T \end{bmatrix}^T$.

On the other hand, since the matrix $\bar{B} = \begin{bmatrix} B^T & I \end{bmatrix}^T \in \mathbb{R}^{(n+m)\times m}$ is of full column rank, hence, if there exists the matrix $\hat{P}$ satisfying (31) with $P > 0$, we have

$$rank(\hat{P}) \geq rank(\bar{B}\hat{P}) = rank(P\bar{B}) \geq rank(\bar{B}) = m$$

which implies that the matrix $\hat{P} \in \mathbb{R}^{m\times m}$ must be nonsingular. Therefore, (34) can be obtained from (35). □

Remark 2: The condition of Theorem 2 is an LMI (30) with matrix equation constraint (31). For the existence of the equation constraint (31), (30) cannot be solved by standard LMI methods. In the work of Yang et al., (2006), a singular value decomposition (SVD) method is first proposed to solve this type of problem. In the following, we will convert the conditions (30) and (31) into a strict LMI form using the similar method which is proposed in Yang et al., (2006).

For the matrix $\bar{B} = \begin{bmatrix} B^T & I \end{bmatrix}^T \in \mathbb{R}^{(n+m)\times m}$ is of full column rank, there exists the SVD as

$$\bar{B} = U \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V \tag{44}$$

where $U \in \mathbb{R}^{(n+m)\times(n+m)}$ and $V \in \mathbb{R}^{m\times m}$ are orthogonal matrices and $\Sigma \in \mathbb{R}^{m\times m}$ is a diagonal matrix with positive diagonal elements in decreasing order, $U_1 \in \mathbb{R}^{(n+m)\times m}$, $U_2 \in \mathbb{R}^{(n+m)\times n}$ are the matrices satisfying $U = \begin{bmatrix} U_1 & U_2 \end{bmatrix}$.

Theorem 3. The system (11) with (10) is an observe-based controller which guarantees that the closed-loop system (13) is exponentially mean-square stable and the corresponding value of the cost function satisfies (33), if there exist positive-definite matrices P_1, P_2, S, and matrices X, Y, such that the LMI (30) hold, where

$$P = U \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} U^T = U_1 P_1 U_1^T + U_2 P_2 U_2^T \tag{45}$$

where matrix U_1 and U_2 are defined in (44). Furthermore the gain K and L can be given as

$$L = S^{-1} Y \tag{46}$$

$$K = V^T \Sigma^{-1} P_1^{-1} \Sigma V X \tag{47}$$

Proof. Suppose

$$P = U \begin{bmatrix} P_1' & P_{12} \\ P_{21} & P_2' \end{bmatrix} U^T \tag{48}$$

where $P_1' \in \mathbb{R}^{m\times m}$, $P_2' \in \mathbb{R}^{n\times n}$ and $P_{12} = P_{21}^T \in \mathbb{R}^{m\times n}$, then the matrix equation constraint (31) can be rewritten as

$$U \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V \hat{P} = U \begin{bmatrix} P_1' & P_{12} \\ P_{21} & P_2' \end{bmatrix} U^T U \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V \tag{49}$$

Above formula is equivalent to

$$\begin{bmatrix} \Sigma V \hat{P} \\ 0 \end{bmatrix} = \begin{bmatrix} P_1' \Sigma V \\ P_{21} \Sigma V \end{bmatrix} \tag{50}$$

Above formula is solvable on $\hat{P}$ if and only if $P_{21}\Sigma V = 0$, that is $P_{21} = P_{12} = 0$. Hence, (48) can be rewritten as

$$P = U \begin{bmatrix} P_1' & 0 \\ 0 & P_2' \end{bmatrix} U^T \tag{51}$$

Comparing with (45), it is obviously to see that $P_1' = P_1$, $P_2' = P_2$. This implies that there exists a non-singular matrix $\hat{P}$ satisfying (31) if and only if there exit $P_1 > 0$, $P_2 > 0$ such that (45) holds. Thus we can compute the matrix $\hat{P}$ from (50) which is equivalent to

$$\Sigma V \hat{P} = P_1 \Sigma V \tag{52}$$

which implies that

$$\hat{P} = V^T \Sigma^{-1} P_1 \Sigma V \tag{53}$$

Thus (47) can be obtained from (35) and (53). For the rest of the proof, please refer to the proof of Theorem 2. □

5. A NUMERICAL EXAMPLE

In this section, a numerical example will be presented to demonstrate the effectiveness of the proposed approach. Consider the linear discrete-time system (1) with

$$A=\begin{bmatrix}0.9 & 0\\ 2 & 0.1\end{bmatrix},\quad B=\begin{bmatrix}1\\ 1\end{bmatrix},\ C=\begin{bmatrix}1 & 0\end{bmatrix},$$

and the weighting matrices of the cost function (16) and the packet dropout stochastic variable expectation are given with

$$Q=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ R=0.1,\ \bar{\alpha}=\bar{\beta}=0.1,$$

then the closed-loop system (13) can be given with

$$\bar{A}=\begin{bmatrix}0.9 & 0 & 0.1\\ 2 & 0.1 & 0.1\\ 0 & 0 & 0.1\end{bmatrix},\ \tilde{A}=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix},\ \bar{B}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix},\bar{C}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

By Theorem 3, the observer gain L and control gain K can be obtained as follows

$$K=\begin{bmatrix}-0.6778 & -0.1785 & -0.1784\end{bmatrix}$$

$$L=\begin{bmatrix}0.5957 & 1.6830 & 0.1972\end{bmatrix}^T$$

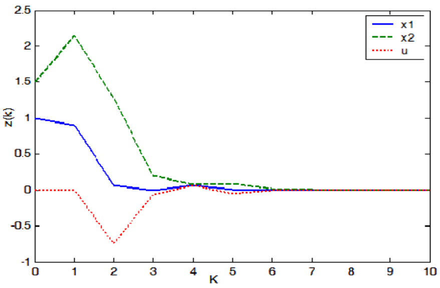

Fig. 2 The trajectories of the state variables of the closed-loop NCS

Choose the initial conditions as

$$z_0=\begin{bmatrix}x_0^T & 0\end{bmatrix}^T=\begin{bmatrix}1 & 1.5 & 0\end{bmatrix}^T,\ \hat{z}_0=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$$

the corresponding upper bound of performance index function can be obtained as $J^*=179.4352$. The simulation results of the state responses are given in Fig. 2. From the Fig. 2, it can be seen that the system can work well via the proposed observer-based control scheme.

6. CONCLUSIONS

In this paper, an observer-based GCC problem is presented to solve the effect of random packet dropouts for a class of NCSs in which sensor, controller, and actuator are all clock-driven. Both the S/C and C/A packet dropouts are modeled by two mutually independent stochastic variables satisfying Bernoulli binary distribution. An observer-based controller designed such that the closed-loop NCS is stochastically exponentially mean-square stable and the specified GCC performance is achieved. A linear matrix inequality (LMI) approach is developed to tackle the addressed problem, which can be solved conveniently by Matlab LMI toolbox. A numerical example has shown that the present design approach is both simple and effective.

ACKNOWLEDGEMENT

This work was supported by National 863 Plan of China (No. 2007AA041403) and Shanghai Rising-Star Program(No. 07QA14030).

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