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Controller design for Unstable FOPTD Plants based on Sensitivity

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Abstract: This paper presents a new method for designing PID controller for unstable first order plus time delay (FOPTD) plant models. The controller design problem is solved by pole zero cancellation and keeping the minimum distance of the Nyquist curve of the open loop transfer function from the critical point to a specified value. Analytical expressions correlating the controller parameters and the plant model parameters are also provided for ease of use. Simulation results are given to show the performance that can be achieved.

1. INTRODUCTION

The proportional-integral-derivative (PID) controllers are widely used in the control industry because of its simple structure and satisfactory performance for a wide range of operating conditions. A number of methods of tuning PID controller proposed in the literature have been compiled in O' Dwyer (2006).

One of the methods of tuning PID controller is the gain and phase margin method as these are well known measures of robustness. Also, the phase margin is related to the damping of the system and can therefore serve as a performance measure. Ho *et al.* (1995) have developed simple analytical formulas to tune PI and PID controllers for first order and second order plus dead time plant models so as to meet user defined gain margin and phase margin. The method is further extended for unstable plants in Ho *et al.* (1998). However the gain and phase margin specifications can sometimes give poor results because each of these criteria measures the closeness of the loop transfer function to the (-1,0) point at only one particular frequency. Also, the gain and phase margin specifications may fail to give reasonable bounds on the sensitivity functions. This motivated the researchers to use constraints on sensitivity functions for controller design. The maximum sensitivity measures the closeness of the loop transfer function to the point (-1,0) at all frequencies, not just the two frequencies as associated with gain and phase margins respectively and hence can therefore serve as a better performance measure.

Several controller design methods based on the sensitivity have been proposed in the literature. Ogawa (1995) used the QFT-framework to design a PI controller that satisfies a bound on the sensitivity for an uncertain plant. Kristiansson and Lennartson (2002) have proposed an optimisation based approach to tune PI/PID controllers with low pass filters on the derivative gain to reject the load disturbances while optimizing the control effort and bounding the sensitivity functions. A method to tune the PI, PID and a PID augmented by a filter on the D element that stabilizes a given set of plants and satisfies both gain margin constraints and a bound on the complementary sensitivity is proposed in Yaniv *et al.* (2004). Ma and Zhu (2006) have used the maximum sensitivity and the frequency corresponding to the maximum sensitivity as the design parameters and the parameters of a PI controller are obtained by making the Nyquist curve touch the sensitivity circle at the specified frequency.

Recently, Visioli (2001) has proposed tuning methods for integrating and unstable plants by minimizing ISE, ISTE and ITSE. The optimization is carried out using a genetic algorithm and the results are fitted by simple equations. Åström *et al.* (1998) and Panagopoulos *et al.* (2002) have obtained the parameters of the set-point weighted PI/ PID controller by a numerical method based on optimization of load disturbance rejection with constraints on maximum sensitivity (M_s) and / or complementary sensitivity. The responses corresponding to two values of the maximum sensitivity (1.4 and 2.0) are investigated in detail and it is observed that $M_s = 1.4$ gives the output step response with little or no overshoot whereas $M_s = 2.0$ gives a faster response with better disturbance rejection. However, faster responses are oscillatory with large overshoot and hence $M_s = 1.4$ was recommended as a sufficient condition to tune the controller. The method proposed by Panagopoulos *et al* (2002) has several limitations. First of all, it is an iterative method and secondly, a PI controller has to be first designed to provide the good initial conditions and a suitable search interval. Also, unstable plant has not been considered in Panagopoulos *et al*, (2002).

The controller tuning based on specifications on the maximum sensitivity is extended for unstable FOPTD plant models in this work. After pole-zero cancellation and a suitable approximation of the plant delay, explicit expressions for controller parameters are obtained in terms of plant parameters so that the minimum distance of the Nyquist curve of the open loop transfer function from the critical point achieves a user defined value. Guidelines are also provided regarding the selection of this user defined parameter. The performance of the proposed controller is compared with some of the existing methods and it is found that the proposed controller gives good results for both set point tracking and disturbance rejection thereby justifying the sufficiency of the maximum sensitivity as a tuning parameter. The paper is organized as follows: the controller settings for unstable FOPTD plant are derived in section 2. The simulation results are discussed in section 3 followed by conclusions in section 4.

2. CONTROLLER DESIGN

Analytical expressions for the controller parameters are derived in this section. The unstable first order plus time delay (FOPTD) model considered is

$$G(s) = \frac{K}{Ts-1} e^{-\theta s} \tag{1}$$

The general series form of a PID controller is given by

$$G_c(s) = K_p \left(1 + \frac{1}{sT_i}\right)\left(\frac{1+sT_d}{1+s\alpha T_d}\right) \tag{2}$$

where K_p, T_i and T_d are the proportional gain, the integral time constant and the derivative time constant respectively. A small value of the derivative filter constant (α) is considered in the literature. In this work, α is assumed as 0.1. The derivative filter term in (2) is neglected in the following for ease in analysis. Using 1/1 Padé approximation for the delay, and assuming $T_d = \frac{\theta}{2}$, the loop transfer function becomes

$$L(s) = \frac{KK_p(1-\theta s/2)(1+sT_i)}{sT_i(Ts-1)} \tag{3}$$

The expression for the sensitivity function is obtained by using the relation $S(s) = 1/(1+L(s))$ and is given by

$$S(jw) = \frac{-w^2T_iT - jwT_i}{\left(\frac{KK_p\theta w^2 T_i}{2} + KK_p - w^2T_iT\right) + jw\left(KK_p\left(T_i - \frac{\theta}{2}\right) - T_i\right)} \tag{4}$$

The following equation is obtained by squaring the magnitude of both sides of (4).

$$w^4T_i^2q^2 + w^2\left(2KK_pT_ib + c^2T_i^2 + K^2K_p{}^2\left(T_i - \frac{\theta}{2}\right)^2 - 2KK_pT_i^2 + KK_p\theta T_i\right) + K^2K_p{}^2 = 0 \tag{5}$$

$$\text{where, } b = KK_p\theta/2 - T \tag{6}$$

$$q^2 = b^2 - y^2T^2 \tag{7}$$

$$c^2 = 1 - y^2 \tag{8}$$

$y = \frac{1}{|S(jw)|}$ is the user specified sensitivity value. Using the condition that w^2 should have repeated roots (5) gives

$$T_i^2(K^2K_p{}^2 + c^2 - 2KK_p) - T_iKK_p(2q - 2b - \theta + \theta KK_p) + K^2K_p{}^2\frac{\theta^2}{4} = 0 \tag{9}$$

Applying Routh-Hurwitz stability criterion, we get the following condition.

$$\frac{T_i}{T_i - \theta/2} < KK_p < \frac{2T}{\theta} \tag{10}$$

KK_p is therefore selected as $\sqrt{2T/\theta}$ and (9) is solved for T_i with $y = 1.2$. If T_i is negative, (9) is solved again with y incremented by 0.1. The value of y is increased in steps of 0.1 until a positive value of T_i is obtained. The results obtained are given in Table 1.

Table 1.

$\frac{\theta}{T}$	KK_p	T_i	M_s	M_s^*
0.1	4.47	0.82	1.47	1.3
0.2	3.16	1.56	1.84	1.5
0.3	2.58	2.42	2.27	1.7
0.4	2.24	3.42	2.86	1.9
0.5	2.00	3.91	3.72	2.0
0.6	1.82	6.00	4.89	2.3
0.7	1.69	7.68	6.90	2.5
0.8	1.58	10.15	10.72	2.8
0.9	1.49	13.17	20.62	3.1

M_s^* represents the user specified value of the maximum sensitivity for which (9) is solved to get the integral time constant whereas M_s is the true value of the maximum sensitivity calculated using the robust control toolbox of MATLAB corresponding to the obtained controller parameters. The difference between the true and the specified values is because of the approximations of the delay term. From Table 1, it is observed that for unstable plants the value of M_s which gives satisfactory performance increases as $\frac{\theta}{T}$ increases. It is also observed that for $\frac{\theta}{T}$ greater than 0.5, the M_s value increases drastically and the step response becomes quite oscillatory.

Table 2.

$\frac{\theta}{T}$	KK_p	T_i	M_s
0.6	1.62	6.00	3.46
0.7	1.52	7.68	4.43
0.8	1.44	10.15	5.92
0.9	1.38	13.17	8.75
1.0	1.32	17.43	13.90

Therefore, for $0.5 < \frac{\theta}{T} \le 1$, the gain product is reduced to $KK_p = (2T/\theta)^{0.4}$. The integral time constant remains the same as given in Table 1. The new controller settings and the corresponding M_s are given in Table 2. The equation governing the relationship between T_i and $\frac{\theta}{T}$ is obtained using the curve fitting toolbox of MATLAB and is given by

$$T_i = 20.86(\frac{\theta}{T})^3 - 14.17(\frac{\theta}{T})^2 + 10.83(\frac{\theta}{T}) - 0.1557 \quad (11)$$

The proposed controller settings can be therefore summarized as follows:

1) For $\frac{\theta}{T}$ less than equal to 0.5, $KK_p = (2T/\theta)^{0.5}$.

2) For $0.5 < \frac{\theta}{T} \le 1$, $KK_p = (2T/\theta)^{0.4}$.

3) The derivative time constant (T_d) is set equal to half of the plant delay.

4) The value of the integral time constant is obtained from (11).

3. SIMULATION STUDY

In this section, two typical examples from the literature are considered to illustrate the usefulness of the proposed method. The simulation results show that the proposed control scheme gives satisfactory performance both for set-point tracking and disturbance rejection.

Example1

The unstable FOPTD process with transfer function $G(s) = \frac{e^{-0.7s}}{s-1}$ is considered in this example.

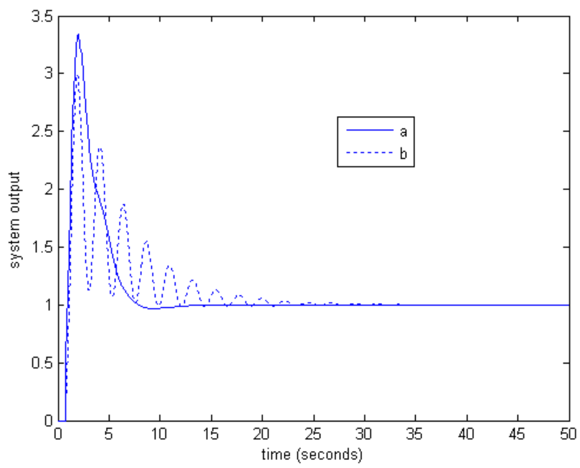

Fig. 1. Set point responses of example 1: a) proposed PID b) Padma Sree's PID

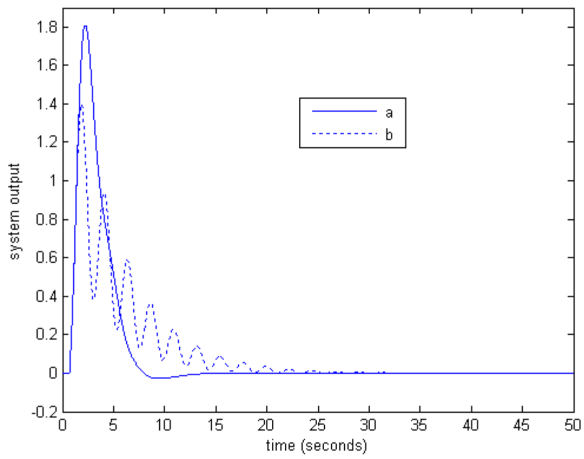

Fig. 2. Load disturbance responses of example 1: a) proposed PID b) Padma Sree's PID

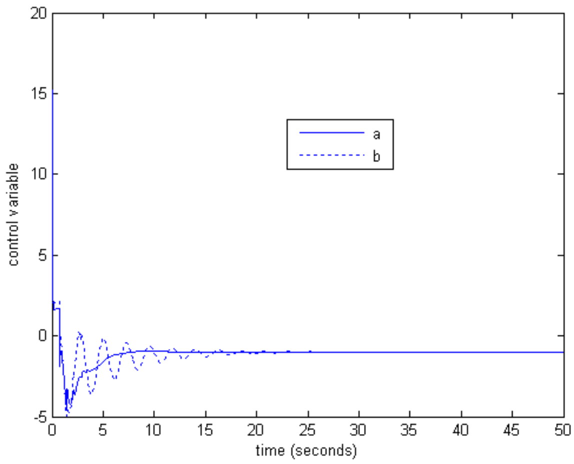

Fig. 3. Control variables for set point responses of example 1: a) proposed PID b) Padma Sree's PID

The PID controller settings recommended in Padma Sree *et al.* are $K_p = 2.018$, $T_i = 11.15$ and $T_d = 0.3427$. The proposed controller settings are obtained from Table 2. The plots of the system output for a unit change in the set point and load disturbance are shown in Figs. 1 and 2 whereas the corresponding control variables are shown in Figs. 3 and 4 respectively. As is evident from the system plots that the overshoot for the proposed method is more for both set point and load disturbance but the settling time for the proposed method is less as compared to Padma Sree's PID. Also the controller proposed by Padma Sree *et al.* gives oscillatory response which is not desirable from the actuator point of view.

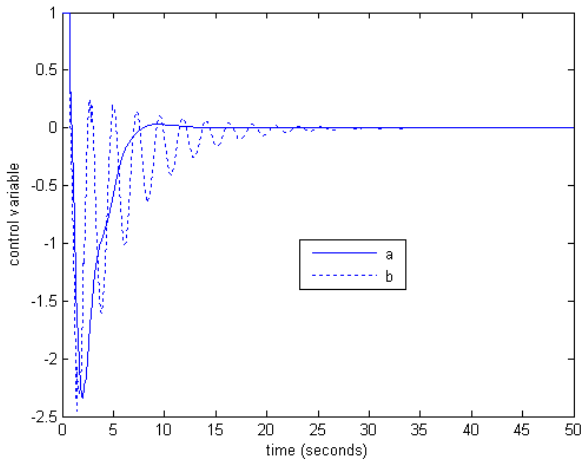

Fig. 4. Control variables for load disturbance responses of example 1: a) proposed PID b) Padma Sree's PID

Example 2

The unstable FOPTD plant considered by Visioli is $G(s)=\frac{e^{-0.2s}}{s-1}$. The controller parameters recommended by Visioli for set point tracking and disturbance rejection are: $K_p = 6.23$, $T_i = 0.73$, $T_d = 0.09$ and $K_p = 6.85$, $T_i = 0.41$, $T_d = 0.10$ respectively. The proposed controller parameters are $K_p = 3.16$, $T_i = 1.56$ and $T_d = 0.1$. (The ISTE controller settings proposed by Visioli are considered because the author claims it to be the best). The system outputs with the considered tuning rules are plotted in Figs. 5 and 6 whilst the corresponding control variables are shown in Figs. 7 and 8 respectively.

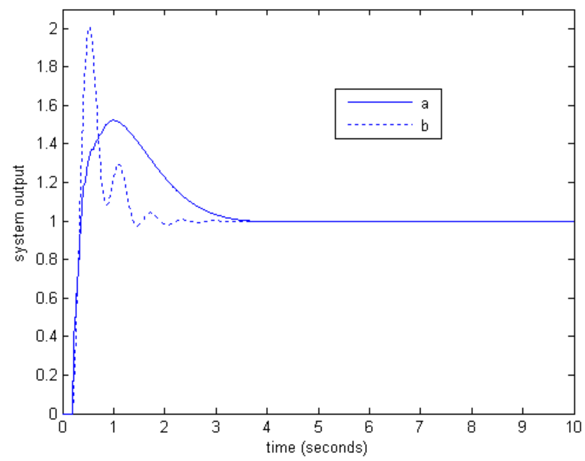

Fig. 5. Set point responses of example 2: a) proposed PID b) Visioli's PID

It can be observed from Fig. 5 that both the methods give the same speed of response but the proposed method gives less overshoot even though the settling time is slightly more. The plot of the load disturbance responses shows that Visioli's method performs better but the proposed method is also able to reject the load disturbances successfully. One point to be stressed is that two different controller settings one for set point tracking and one for disturbance rejection is proposed by Visioli whereas only one set of tuning rules are given by the proposed method. Also, it can be observed that the control signal corresponding to Visioli's tuning rules exhibits significant oscillations before reaching the steady state values which is not desirable from the actuator point of view.

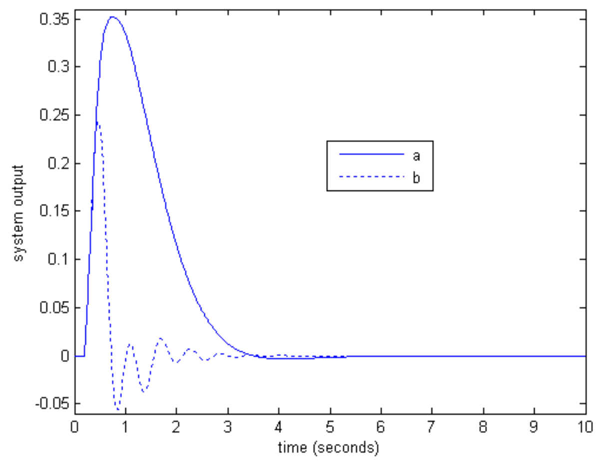

Fig. 6. Load disturbance responses of example 2: a) proposed PID b) Visioli's PID

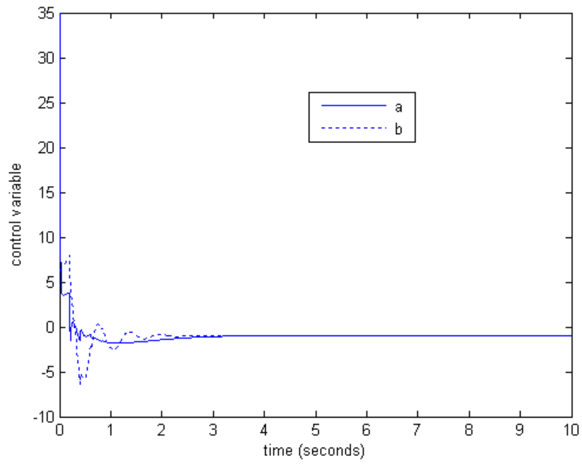

Fig. 7. Control variables for set point responses of example 2 : a) proposed PID b) Visioli's PID

4. CONCLUSIONS

In this paper, analytical expressions correlating the parameters of a PID controller and the plant parameters are obtained for unstable first order plus time delay (FOPTD) plant models by keeping the distance of the Nyquist curve of the open loop transfer function from the critical point to a specified value. The effectiveness of the proposed tuning rules is established by simulation results.

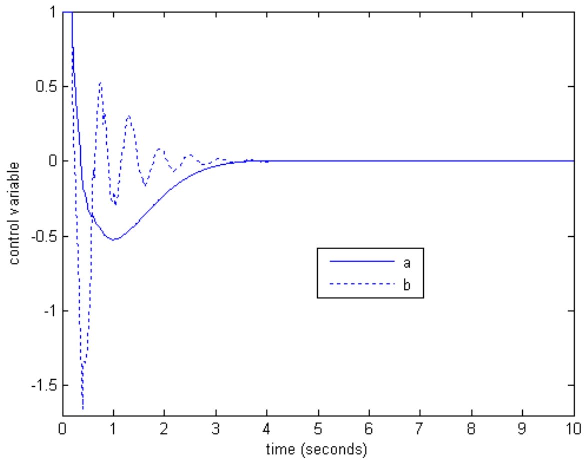

Fig. 8. Control variables for load disturbance responses of example 2: a) proposed PID b) Visioli's PID

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