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A New Objective Function for Controller Tuning

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Abstract: In this paper, a new objective function to tune the PID/PI-PD controller is proposed by modifying the integral error squared (ISE) criterion. A new entrant to the family of evolutionary algorithms namely bacterial foraging is used to find the controller parameters by minimizing the objective function. The controller gives a smooth process output with almost zero percent overshoot as desired in the control industry. Robustness is also ensured as the value of the maximum sensitivity lies between 1.3 and 2. A comparative study of the achievable performances of the PID/PI-PD controller is also done and the proposed tuning method is tested on non-minimum, integrating and higher order plant models to illustrate the effectiveness of the new objective function.

1. INTRODUCTION

In spite of the continual advances in control theory, PID controller is the most common form of feedback used in the process industry. This is because of its simple structure and ability to meet most of the control objectives for a wide range of industrial plants. Their widespread acceptance has motivated the research community towards developing new and better methods for designing PID controller. Some of the methods of PID tuning proposed in the literature are given in O'Dwyer (2006).

Recently, there has been an increased interest towards the robust tuning of PI / PID controller. This is because most of the real plants operate in a wide range of operating conditions and the controller should be able to stabilize the system for all operating conditions. Tavakoli *et al.* (2005) have proposed tuning formulas for PI controller for first order plus dead time process models based on optimal load disturbance rejection with constraint on the maximum sensitivity. An iterative method for the design of PID controller with specifications on the modulus and complementary modulus margins as well as the crossover frequency is proposed in Garcia *et al.* (2007). Chen and Moore (2005) have proposed a new method for tuning the PID controller for a class of unknown stable and minimum phase plants by making the phase Bode plot flat at the tangent frequency, so as to ensure a constant phase margin for gain variations of the plant. However, their method gives poor set point response with large overshoot and long settling time.

Åström *et al.* (1998) has presented a method for designing set point-weighted PI controller by minimizing the integral of error of the control system subjected to a step load disturbance input while satisfying constraints on maximum sensitivity and/or complementary sensitivity. The method is further extended for the design of a two degree of freedom PID controller in Panagopoulos *et al.* (2002). Åström *et al.* (1998) and Panagopoulos *et al.* (2002) have shown that the step responses corresponding to $M_s = 1.4$ gives little or no overshoot as desirable in the control industry and hence $M_s = 1.4$ was recommended as a sufficient condition for controller design. Hwang *et al.* (2002) have reported that it is difficult to obtain the true solution of the non-convex optimization problem arising in Åström *et al.* (1998) by the classical gradient based search algorithm and they have proposed a numerical solution to this non convex problem. The method proposed by Panagopoulos *et al.* (2002) has several limitations. First of all, it is an iterative method and secondly, a PI controller has to be first designed to provide the good initial conditions and a suitable search interval.

In this paper, a new objective function is proposed by modifying the integral error squared (ISE) criterion and simulation results show that the controller parameters obtained by minimizing this objective function gives better performance for both set point tracking and disturbance rejection as compared to Panagopoulos *et al.* (2002) method. Robustness is ensured as the value of maximum sensitivity (M_s) lies between 1.3 and 2. It is further shown that a single degree of freedom PID controller gives satisfactory performance for most of the plants. However, improved performance can be achieved by using the PI-PD controller proposed by Majhi and Atherton (1999) when the value of M_s is less than 1.3.

Furthermore, a new evolutionary algorithm proposed by Passino (2002) namely bacterial foraging which mimics the foraging behaviour of *E. coli* bacteria is used to minimize the proposed objective function. The bacteria undergo several stages such as chemotaxis, swarming, reproduction and elimination and dispersal in its lifetime. In the chemotaxis stage, it can tumble followed by a tumble or a tumble followed by a run whereas in swarming, each bacterium signals other via attractants to swarm together. The healthiest bacteria split into two at the same location and the least healthy bacteria die out in the reproduction stage. Finally in the elimination and dispersal stage, some of the bacteria are dispersed to a random location in the optimization domain. This reduces the probability of getting trapped in the local minima. The bacterial foraging strategy has been used to get the optimal controller parameters in (Ali and Majhi, 2006). The use of an evolutionary algorithm for minimizing makes the proposed method independent of the initial guess and gives the controller parameters in one run.

The paper is organized as follows. Section 2 presents the problem formulation for the tuning of PID controller. The simulation results are discussed in section 3 followed by conclusions in section 4.

2. FORMULATION OF THE DESIGN PROBLEM

There are many versions of a PID controller. In this paper, the general parallel form of a PID controller is considered which is described by

$$G_c(s) = \frac{U(s)}{E(s)} = K_p\left(1 + \frac{sT_d}{sT_f + 1} + \frac{1}{sT_i}\right) \quad (1)$$

where K_p, T_i and T_d are the proportional gain, integral time constant and derivative time constant, respectively. To limit the high frequency gain, the derivative term of the controller is filtered with a low pass filter. The filter time constant T_f is usually considered as a fraction of the derivative time constant T_d i.e. $T_f = \alpha T_d$. Generally, a small value of the derivative filter constant (α) is considered in the literature. In this work α is set at 0.1. Further, *e(t)* and *u(t)* are the input and output signal of the PID controller, respectively. Apart from the parallel PID controller, the PI-PD controller proposed in (Majhi and Atherton, 1999) is also considered for a comparative study. The PI and PD controller in the forward and inner feedback paths in the PI-PD configuration are:

$$G_{c1}(s) = \frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{sT_i}\right) \quad (2)$$

$$G_{c2} = K_{pb}\left(1 + \frac{sT_{db}}{0.1sT_{db} + 1}\right) \quad (3)$$

2.1 Control Requirements

The design problem is basically finding the controller parameters so that the system behaves well with respect to changes in the load disturbance, the process model and the set point. These are discussed below in brief

A. Load Disturbance Attenuation

The most common disturbances in process control that drives the process variables away from their desired values are load disturbances. These are low frequency signals and usually enter the system at the process input. Good rejection of such signals is the first design goal for regulation problems where the processes are running in steady state with constant set point for a long time.

B. Robustness Against Model Uncertainties

An efficient design method should give a controller which performs satisfactorily under inevitable practical conditions of varying process dynamics and plant-model mismatch. The sensitivity to modelling errors can be expressed as the largest value of the sensitivity function

$$M_s = \max_w \left| \frac{1}{1 + G_p(jw)G_c(jw)} \right| \quad (4)$$

where $G_p(s)$ is the plant transfer function. As M_s is the inverse of the shortest distance of the critical point (-1,0) from the Nyquist curve of the loop transfer function, it can be used as a reliable measure of robustness. Reasonable values of M_s are in the range from 1.2 to 2 (Åström *et al.*, 1998).

C. Set point Regulation

Even though the primary design goal is rejection of load disturbances; the controller should give satisfactory response to set point changes.

2.2 Design Procedure

A controller is designed so as to minimize the error $e(t)$ between the reference and the controlled variable. Hence, a criterion worth characterizing the time response of a system is usually given as an integral function of the error or its weighted products. Graham and Lathrop (1953) suggested two of the most frequently used criteria: the integral squared error (ISE) and the integral absolute error. The ISE criterion weights all errors equally and thus results in a system with a relatively oscillatory step response. In this paper, therefore the standard ISE criterion is modified as

$$J(\eta) = \int_0^\infty (e(\eta,t)^2 + t^4 \dot{u}(\eta,t)^2)dt \quad (5)$$

where η denotes the set of controller parameters which are chosen to minimize (5). $\dot{u}(\cdot,t)$ represents the derivative of the output of the parallel PID or PI for the PI-PD controller respectively. The above integral can be evaluated very efficiently using the s-domain formulation. In the frequency domain, the proposed objective function $J(\eta)$ becomes

$$J(\eta) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \{E(s)E(-s) + H(s)H(-s)\}ds \quad (6)$$

where $H(s) = \mathcal{L}\left[t^2 \dot{u}(\eta,t)\right] = \frac{d^2}{ds^2}\dot{U}(s)$. The criterion (6) is to be minimized for a satisfactory closed loop performance of the system. The integral can be optimized for a set of η values with the help of recursive algorithm (Åström, 1970) and the bacterial foraging optimization technique.

3. SIMULATION STUDY

The various plant models considered are

$$G_1(s) = \frac{1}{s(s+1)^3} \tag{7}$$

$$G_2(s) = \frac{e^{-5s}}{(s+1)^3} \tag{8}$$

$$G_3(s) = \frac{1}{(s+1)(1+0.2s)(1+0.04s)(1+0.008s)} \tag{9}$$

$$G_n(s) = \frac{1}{(s+1)^n} \qquad n = 4 \text{ to } 7 \tag{10}$$

$$G_8(s) = \frac{1-2s}{(s+1)^3} \tag{11}$$

The above transfer functions capture the dynamics of the commonly encountered processes in the control industry and are therefore used to illustrate the effectiveness of the proposed method. An integrating process is represented by G_1 whereas G_2 models a process with a long dead time. A non-minimum phase plant (11) is also included to demonstrate the wide applicability of the design procedure.

The parameters of the PID and PI-PD controller obtained by minimizing (6) and the corresponding M_s are given in Table 1 and 2 respectively. The responses to a unit change in the set point and load disturbance and the corresponding control variables are shown in the Fig. 1. One important point worth noticing is that even though the various plant models considered represents processes with large variations in plant dynamics, the resulting set point responses are very much similar with zero percent overshoot as desirable in the control industry.

As is evident from Fig. 1 that the PID controller gives better performance than PI-PD controller for the considered plants except G_1 and G_3. The PID controller is unable to reject the load disturbances for G_1 and hence a PI-PD configuration is recommended for the integrating plant considered in (7). Even though the set point responses corresponding to PID and PI-PD are almost similar for G_3, the PI-PD has better load disturbance rejection capability as is evident from the system plots. Table 1 shows that the value of the maximum sensitivity (M_s) for various plants lies between 1.3 to 1.8 except G_3 for which it is 1.2. Also, it can be observed that the proposed PID / PI-PD performs better than Åström's two degree of freedom PID controller. It can be thus concluded that improved performance can be achieved by PI-PD controller for integrating plants as well as those plants for which the PID controller gives a value of maximum sensitivity less than 1.3.

4. CONCLUSIONS

In this paper, the parameters of PID and PI-PD controller are obtained by minimizing a new objective function by the bacterial foraging optimization strategy. Simulation results show that the proposed objective function gives good performance for a wide variety of plants. Also, it is observed that a single degree of freedom PID controller is sufficient for most of the plants. However, improved performance can be achieved by a PI-PD controller for plants which give a value of maximum sensitivity less than 1.3 with the PID controller.

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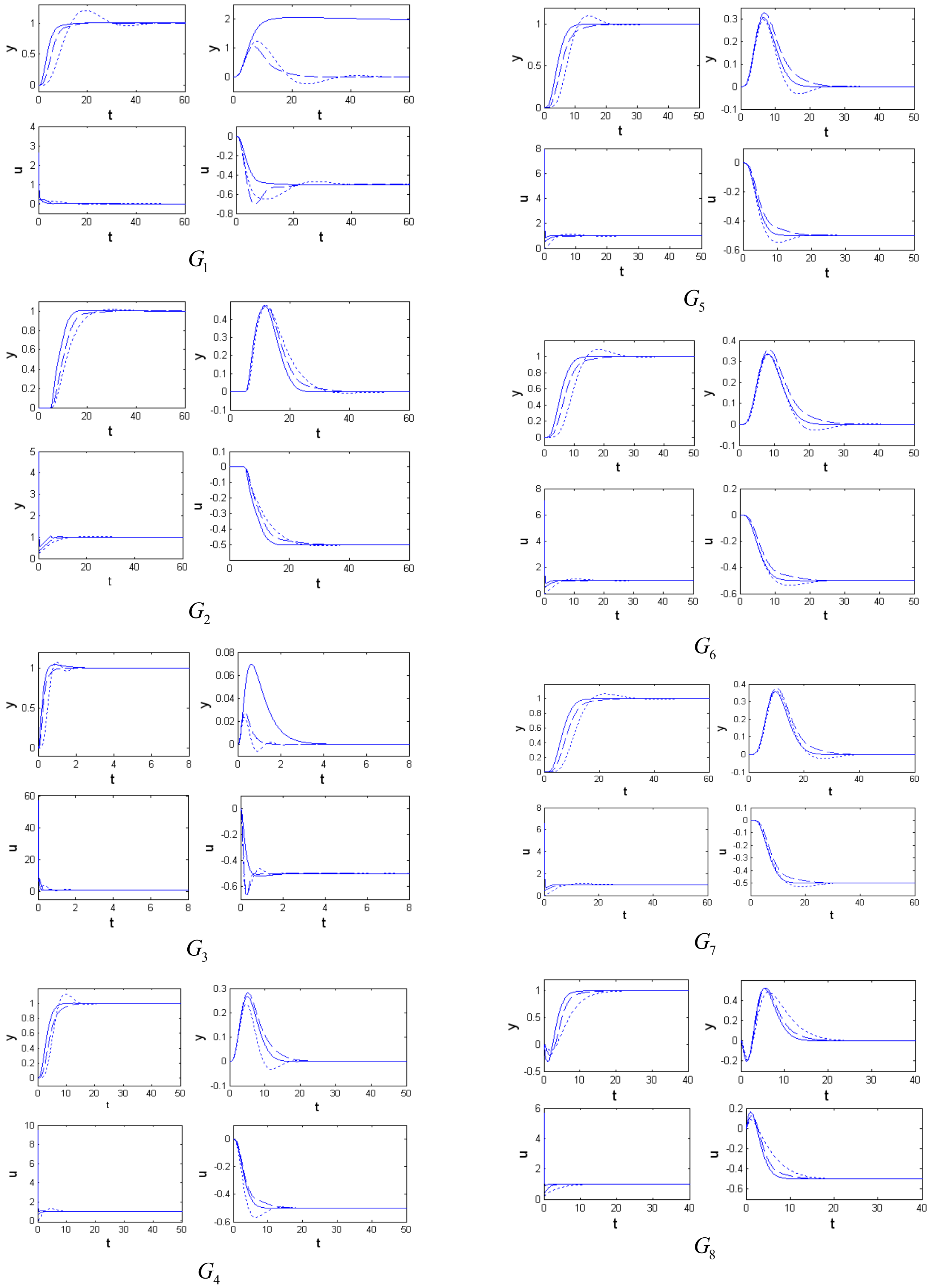

Fig. 1. PID (solid line), PI-PD (dashed line), Åström's set point weighted PID (dotted line)

Table 1.

	K_p	T_i	T_d	M_s
$G_1(s)$	0.24	881.25	1.86	1.3
$G_2(s)$	0.46	3.97	1.27	1.7
$G_3(s)$	5.19	0.88	0.15	1.2
$G_4(s)$	0.86	2.78	0.81	1.3
$G_5(s)$	0.72	3.21	0.94	1.4
$G_6(s)$	0.64	3.65	1.09	1.4
$G_7(s)$	0.60	4.10	1.25	1.5
$G_8(s)$	0.52	2.47	0.68	1.8

Table 2.

	K_p	T_i	K_{pb}	T_{db}	M_s
$G_1(s)$	0.18	3.56	0.30	2.37	1.4
$G_2(s)$	0.33	3.42	0.03	11.77	1.7
$G_3(s)$	6.49	0.19	9.57	0.18	1.4
$G_4(s)$	0.62	2.50	0.14	4.56	1.4
$G_5(s)$	0.51	2.88	0.08	6.72	1.4
$G_6(s)$	0.46	3.24	0.06	8.53	1.4
$G_7(s)$	0.43	3.61	0.05	10.23	1.5
$G_8(s)$	0.37	2.09	0.06	4.20	1.8