

# Computationally Efficient Suboptimal Mid Course Guidance Using Model Predictive Static Programming (MPSP)

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**Abstract:** For a homing interceptor, suitable initial condition must be achieved by mid course guidance scheme for its maximum effectiveness. To achieve desired end goal of any mid course guidance scheme, two point boundary value problem must be solved online with all realistic constrain. A Newly developed computationally efficient technique named as MPSP (Model Predictive Static Programming) is utilized in this paper for obtaining suboptimal solution of optimal mid course guidance. Time to go uncertainty is avoided in this formulation by making use of desired position where midcourse guidance terminate and terminal guidance takes over. A suitable approach angle towards desired point also can be specified in this guidance law formulation. This feature makes this law particularly attractive because warhead effectiveness issue can be indirectly solved in mid course phase.

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## 1. INTRODUCTION

Mid course guidance is the longest phase of an interceptor missile. Midcourse guidance strategy differs for engagement of air-breathing target to engagement of high speed targets in the form of reentry vehicle. For air-breathing target, the strategy would be to maximize interceptor velocity at interception, where as for interception of high velocity target (i.e target velocity being larger than missile velocity), midcourse guidance must allow interceptor to arrive at a particular point with a specific approach angle. This requirement specifically arises from (i) minimum trajectory correction requirement during homing phase, (ii) requirement of low value of seeker look angle. Again missile velocity need not be wasted unnecessarily while achieving these objectives. Hence design of midcourse guidance needs special care while addressing a particular class of target. Numerous midcourse guidance schemes are developed for missiles. To name a few, one can find general energy management steering based guidance [1], singular perturbation theory based guidance [2], optimal control theory based guidance ([3]-[4]) etc. in the literature. For engaging high speed targets, PN law with a shaping term [7] can also achieve desired performance, although the solution is not general enough. Singular perturbation theory based guidance is well known energy efficient technique. But it does not address the problem of achieving desired flight path angle and heading angle.

The idea of using optimal control techniques for the guidance of flight vehicles is not new ([3]-[6]). However, such a formulation leads to two point boundary value problems (TPBVP)[4]. The solution approach of the TPBVP problem requires numerical techniques like shooting method [7], gradient method [8] etc. Basically, they are iterative processes and require a lot of storage and computation

time. In addition, it leads to an open-loop solution of the control variable. An alternative approach, namely the Hamilton-Jacobi-Bellman (HJB) approach [4], attempts to get solution in the state feedback form. However, this approach either leads to complicated nonlinear partial differential equations which are impossible to solve or it leads to the curse-of-dimensionality issue, leading to huge (infeasible) computational and storage space requirements. Because of this reason, the optimal control theory based formulations are seldom used in online applications. Even though attempts are being made to overcome this computational difficulty using optimal critic techniques ([10]-[13]), these techniques still require high off-line computations, and hence, do not lead to closed form solutions for the control variable in true sense (mainly because in off-line computations all possible operating scenarios like the adaptive critic technique cannot be accounted for).

In this paper, a new mid course guidance scheme for missile based on nonlinear model predictive static programming (MPSP) technique is described based on the philosophy, which is quite similar to the optimal control theory based approach([13]-[15]). One of the major advantages of this new technique is that it leads to a closed form solution for the control history (sequence) update. Moreover, the computational requirements are substantially less and the algorithm can be implemented online.

## 2. INTERCEPTOR MATHEMATICAL MODEL AND OBJECTIVE OF MID COURSE GUIDANCE

For successful interception of high speed target, interceptor must have sufficient capability to fulfil terminal guidance requirement and proper initial condition for terminal guidance phase. So it is quite important for mid course guidance to provide proper initial condition to terminal

guidance phase. As interceptor spends most of its time during mid course guidance phase, it is quite imperative that this phase should be energy efficient while simultaneously achieving its primary performance related objectives. Hence an optimal Mid course guidance must enable the interceptor to reach a particular point at a particular range to go with desired velocity vector and also with minimum effort. Moreover, missile must reach desired location with proper flight path angle and heading angle while retaining sufficient velocity to satisfy terminal guidance requirement due to handover errors and subsequent requirement due to target maneuver.

Objective of mid course guidance is the following : Interceptor has to reach desired point  $(x_d, y_d, z_d)$  with desired heading angle  $(\phi_d)$  and flight path angle  $(\gamma_d)$  using minimum acceleration  $\eta_\phi$  and  $\eta_\gamma$ . The axis used in guidance is given in fig 1.

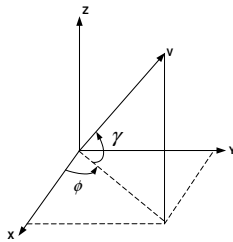


Fig. 1. Axis used in guidance

The following point mass model is being used for this purpose

$$\begin{aligned} \dot{x} &= V \cos \gamma \cos \phi \\ \dot{y} &= V \cos \gamma \sin \phi \\ \dot{z} &= V \sin \gamma \\ \dot{V} &= \frac{T - D}{m} \\ \dot{\phi} &= \frac{g \eta_\phi}{V \cos \gamma} \\ \dot{\gamma} &= \frac{g(\eta_\gamma - \cos \gamma)}{V} \end{aligned} \quad (1)$$

where  $x, y$  and  $z$  are the missile position in launcher fixed frame.  $V$  is the velocity of missile.  $T$  is thrust and  $D$  is aerodynamic drag of missile.

### 3. MODEL PREDICTIVE STATIC PROGRAMMING (MPSP) DESIGN

In this section, we combined the philosophies of Model Predictive Control (MPC) and Approximate Dynamic Programming (ADP) to propose an innovative technique for a class of finite horizon optimal control problems. Here we present the mathematical details of the new *Model Predictive Static Programming (MPSP)* design taking into account the discrete nonlinear model of dynamic system. The state and output dynamics of which are given by

$$X_{k+1} = F_k(X_k, U_k) \quad (2)$$

$$Y_k = h(X_k) \quad (3)$$

where  $X \in \mathbb{R}^n$ ,  $U \in \mathbb{R}^m$ ,  $Y \in \mathbb{R}^p$  and  $k = 1, 2, \dots, N$  are the time steps. The primary objective is to come up with a suitable control history  $U_k$ ,  $k = 1, 2, \dots, N - 1$ , so

that the output at the final time step  $Y_N$  goes to a desired value  $Y_N^*$ , i.e.  $Y_N \rightarrow Y_N^*$ . In addition, we aim to achieve this task with minimum control effort (which is described later in this section).

For the technique presented here, one needs to start from a "guess history" of the control solution. With the application of such a guess history, obviously the objective is not expected to be met, and hence, there is a need to improve this solution. In this section, we present a way to compute an error history of the control variable, which needs to be subtracted from the previous history to get an improved control history. This iteration continues until the objective is met i.e. until  $Y_N \rightarrow Y_N^*$ . Note that the technique presented here comes up with a control update history in closed form, and hence, the computational requirement is substantially lesser. Hence, algorithm can be implemented online. Next, we present the mathematical details of the MPSP design.

Expanding  $Y_N$  about  $Y_N^*$  using Taylor series expansion

$$Y_N = Y_N^* + \left[ \frac{\partial Y_N}{\partial X_N} \right] dX_N + HOT \quad (4)$$

where  $HOT$  contains the higher order terms. From (4) we can write the error in the output as

$$Y_N - Y_N^* = \left[ \frac{\partial Y_N}{\partial X_N} \right] dX_N + HOT \quad (5)$$

Using small error approximation, we write

$$\Delta Y_N \cong dY_N = \left[ \frac{\partial Y_N}{\partial X_N} \right] dX_N \quad (6)$$

However from (2), we can write the error in state at time step  $(k + 1)$  as

$$dX_{k+1} = \left[ \frac{\partial F_k}{\partial X_k} \right] dX_k + \left[ \frac{\partial F_k}{\partial U_k} \right] dU_k \quad (7)$$

where  $dX_k$  and  $dU_k$  are the error of state and control at time step  $k$  respectively.

Expanding  $dX_N$  as in (7) (for  $k = N - 1$ ) and substituting it in (6), we obtain,

$$dY_N = \frac{\partial Y_N}{\partial X_N} \left( \left[ \frac{\partial F_{N-1}}{\partial X_{N-1}} \right] dX_{N-1} + \left[ \frac{\partial F_{N-1}}{\partial U_{N-1}} \right] dU_{N-1} \right) \quad (8)$$

Similarly the error in state at time step  $(N - 1)$ ,  $dX_{N-1}$ , can be expanded in terms of the errors in state and control at time step  $(N - 2)$  and (8) can be re-written as:

$$\begin{aligned} dY_N &= \left[ \frac{\partial Y_N}{\partial X_N} \right] \left[ \frac{\partial F_{N-1}}{\partial X_{N-1}} \right] \left( \left[ \frac{\partial F_{N-2}}{\partial X_{N-2}} \right] dX_{N-2} \right. \\ &\quad \left. + \left[ \frac{\partial F_{N-2}}{\partial U_{N-2}} \right] dU_{N-2} \right) + \left[ \frac{\partial Y_N}{\partial X_N} \right] \left[ \frac{\partial F_{N-1}}{\partial U_{N-1}} \right] dU_{N-1} \end{aligned}$$

Next,  $dX_{N-2}$  can be expanded in terms of  $dX_{N-3}$  and  $dU_{N-3}$  and so on. Continuing the process initial  $k = 1$ , we can write

$dY_N = AdX_1 + B_1dU_1 + B_2dU_2 + \dots + B_{N-1}dU_{N-1}$ (9)  
 where

$$A = \begin{bmatrix} \frac{\partial Y_N}{\partial X_N} \end{bmatrix} \begin{bmatrix} \frac{\partial F_{N-1}}{\partial X_{N-1}} \end{bmatrix} \dots \begin{bmatrix} \frac{\partial F_1}{\partial X_1} \end{bmatrix}$$

$$B_k = \begin{bmatrix} \frac{\partial Y_N}{\partial X_N} \end{bmatrix} \begin{bmatrix} \frac{\partial F_{N-1}}{\partial X_{N-1}} \end{bmatrix} \dots \begin{bmatrix} \frac{\partial F_{k+1}}{\partial X_{k+1}} \end{bmatrix} \begin{bmatrix} \frac{\partial F_k}{\partial U_k} \end{bmatrix},$$

$$k = 1, \dots, N - 1$$

Since the initial condition is specified, there is no error in the first term; which means  $dX_1 = 0$ . With this (9) reduces to

$$dY_N = B_1dU_1 + B_2dU_2 + \dots + B_{N-1}dU_{N-1}$$

$$= \sum_{k=1}^{N-1} B_k dU_k \quad (10)$$

Note that while deriving (10), we have assumed that the control variable at each time steps to be independent of the previous values of states and control input. Intuitive justification of this assumption comes from the fact it is a decision variable, and hence, independent decision can taken at any point of time.

At this point, we would like to point out that if one evaluates each of the  $B_k$ ,  $k = 1, \dots, (N - 1)$  as in (10), it will be a computationally intensive tasks (especially when  $N$  is high). However, it is possible to compute them recursively. For doing this, first we define  $B_{N-1}^0$  as follows

$$B_{N-1}^0 = \begin{bmatrix} \frac{\partial Y_N}{\partial X_N} \end{bmatrix} \quad (11)$$

Next we compute  $B_k^0$ ,  $k = (N - 2), (N - 3), \dots, 1$  as

$$B_k^0 = B_{k+1}^0 \begin{bmatrix} \frac{\partial F_{k+1}}{\partial X_{k+1}} \end{bmatrix} \quad (12)$$

Finally,  $B_k$ ,  $k = (N - 2), (N - 3), \dots, 1$  can be computed as:

$$B_k = B_k^0 \begin{bmatrix} \frac{\partial F_k}{\partial U_k} \end{bmatrix} \quad (13)$$

Equation (11)-(13) provides a *recursive way* of computing  $B_k^0$ ,  $k = (N - 1), (N - 2), \dots, 1$ , which leads to saving of computational time enormously.

In equation (10), we have  $(N-1)m$  unknowns and  $p$  equations. Usually  $p < (N - 1)m$ , and hence, it is an under-constrained system of equations. Hence there is a scope for meeting additional objectives. We take advantage of this opportunity and aim to minimize the following objective (cost) function

$$J = \frac{1}{2} \sum_{k=1}^{N-1} (U_k^0 - dU_k)^T R_k (U_k^0 - dU_k) \quad (14)$$

where  $U_k^0$ ,  $k = 1, \dots, (N - 1)$  is the previous control history solution and  $dU_k$  is the corresponding error in

the control history. The cost function in (14) needs to be minimized subjected to the constraint in (10), where  $R_k > 0$  (a positive definite matrix) is the weighting matrix, which needs to be chosen judiciously by the control designer. The selection of such a cost function is motivated from the fact that we are interested in finding a  $l_2$ -norm minimizing control history, since  $(U_k^0 - dU_k)$  is the updated control value at  $k$  (see (22)).

Equations (10) and (14) formulate an appropriate constrained static optimization problem. Hence, using optimization theory, the augmented cost function is given by

$$\bar{J} = \frac{1}{2} \sum_{k=1}^{N-1} (U_k^0 - dU_k)^T R_k (U_k^0 - dU_k)$$

$$+ \lambda^T (dY_N - \sum_{k=1}^{N-1} B_k dU_k) \quad (15)$$

Then the necessary conditions of optimality are given by

$$\frac{\partial \bar{J}_k}{\partial dU_k} = -R_k (U_k^0 - dU_k) - B_k^T \lambda = 0 \quad (16)$$

$$\frac{\partial \bar{J}_k}{\partial \lambda} = dY_N - \sum_{k=1}^{N-1} B_k dU_k = 0 \quad (17)$$

Solving for  $dU_k$  from (16), we get

$$dU_k = R_k^{-1} B_k^T \lambda + U_k^0 \quad (18)$$

Substituting for  $dU_k$  from (18) into (17), it leads to

$$-A_\lambda \lambda + b_\lambda = dY_N \quad (19)$$

where

$$A_\lambda = \left[ - \sum_{k=1}^{N-1} B_k R_k^{-1} B_k^T \right], \quad b_\lambda = \left[ \sum_{k=1}^{N-1} B_k U_k^0 \right]$$

Note that  $A_\lambda$  is a  $p \times p$  matrix and  $b_\lambda$  is a  $p \times 1$  vector. Assuming  $A_\lambda$  to be nonsingular, the solution for  $\lambda$  from (19) is given by

$$\lambda = -A_\lambda^{-1} (dY_N - b_\lambda) \quad (20)$$

Using (20) in (18), it leads to

$$dU_k = -R_k^{-1} B_k^T A_\lambda^{-1} (dY_N - b_\lambda) + U_k^0 \quad (21)$$

Hence, the updated control at time step  $k = 1, 2, \dots, (N - 1)$  is given by

$$U_k = U_k^0 - dU_k = R_k^{-1} B_k^T A_\lambda^{-1} (dY_N - b_\lambda) \quad (22)$$

It is clear from (22) that the updated control history solution in (22) is a *closed form solution*, and hence, control solution can be updated with very minimal computational requirement. We also mention that the relative magnitude of the control input at various time steps can be adjusted by properly adjusting the weight matrixes  $R_k$ ,  $k = 1, \dots, (N - 1)$  associated with the cost function.

At this point, we would like to point out that we have used ‘‘small error approximation’’ in deriving the closed form control update. This approximation may not hold good in general. Hence the process needs to be repeated

in an iterative manner before one arrives at the converged (optimal) solution, which is define as the solution when  $Y_N \rightarrow Y_N^*$ . Note that to minimize computational time, one iteration may be carried out at each instant of time.

From the discussion in this section one may observe that even though we have used the philosophies of MPC and ADP, the new formulation outlined here does not fall into either of these. This is because we have considered a static costate variable  $\lambda$  and have formulated the problem as a static optimization problem (rather than a dynamic one). Hence, it is named as *Model Predictive Static Programming* (MPSP) technique. Because a state space model is considered here, we call it as MPSP-state space formulation.

#### 4. MID COURSE GUIDANCE WITH MPSP

In state equation of the interceptor as described in equation 1, time is used as an independent variable. Hence if we want to propagate state, we must have knowledge of final time which is quite difficult to predict accurately before hand. So instead of time,  $x$  can be used as independent variable as final position is known (because Missile has to reach at a desired point after mid course). For this purpose missile model can be modified as

$$\begin{aligned} t' &= \frac{1}{V \cos \gamma \cos \phi} \\ y' &= \tan \phi \\ z' &= \frac{\tan \gamma}{\cos \phi} \\ V' &= \frac{1}{mV \cos \gamma \cos \phi} \\ \phi' &= \frac{g \eta_\phi}{V^2 \cos^2 \gamma \cos \phi} \\ \gamma' &= \frac{g(\eta_\gamma - \cos \gamma)}{V^2 \cos \gamma \cos \phi} \end{aligned} \quad (23)$$

where  $X'$  represent the derivative of state with respect to position  $x$ . For MPSP design, state model has to be in discreet form as

$$X_{k+1} = F_k(X_k, U_k) \quad (24)$$

Where states  $X_k$  and  $U_k$  are defined as:

$$X_k = [t_k \ y_k \ z_k \ V_k \ \phi_k \ \gamma_k]^T \quad (25)$$

$$U_k = [\eta_{\phi_k} \ \eta_{\gamma_k}]^T \quad (26)$$

Following are the step for designing mid course guidance based on MPSP technique:

- Initialize  $\eta_{\phi_k}$  and  $\eta_{\gamma_k}$  with some guess.
- Define present state as  $k = 1$  and desired state as  $k = N$  and calculate increment of  $x$  (h) as

$$h = \frac{x_d - x_1}{N - 1} \quad (27)$$

- Propagate point mass model of missile using  $\eta_{\phi_k}$  and  $\eta_{\gamma_k}$  till  $x_d$  to get final state of missile( $X_f$ ) and find  $dY_N$  as

$$dY_N = \begin{bmatrix} y_f - y_d \\ z_f - z_d \\ \phi_f - \phi_d \\ \gamma_f - \gamma_d \end{bmatrix} \quad (28)$$

- If either element of  $dY_N$  is not within desired limit, the correction has to be made for  $\eta_{\phi_k}$  and  $\eta_{\gamma_k}$ . For this,  $B_{N-1}^0$  is calculated as

$$B_{N-1}^0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

$B_{N-1}$  can be calculated as:

$$B_{N-1} = B_{N-1}^0 \left[ \frac{\partial F_{N-1}}{\partial U_{N-1}} \right] \quad (30)$$

where  $\frac{\partial F_{N-1}}{\partial U_{N-1}}$  is obtained from (24) at  $N - 1$  th step.

- For  $k = 1$  to  $N - 2$   $B_k$  can be calculated as

$$B_k = B_k^0 \left[ \frac{\partial F_k}{\partial U_k} \right] \quad (31)$$

where

$$B_k^0 = B_{k+1}^0 \frac{\partial F_{k+1}}{\partial X_{k+1}} \quad (32)$$

$\frac{\partial F_{k+1}}{\partial U_{k+1}}$  and  $\frac{\partial F_{k+1}}{\partial X_{k+1}}$  can be calculate from eq 24 at  $k + 1$  th step.

- Once  $B_k$  is calculated  $B_\lambda$  and  $A_\lambda$  can be calculated from (19) and  $\eta_{\phi_k}$  and  $\eta_{\gamma_k}$  can be written as :

$$\begin{bmatrix} \eta_{\phi_k} \\ \eta_{\gamma_k} \end{bmatrix} = B_k^T A_\lambda^{-1} [B_\lambda - dY_N] \quad (33)$$

Here  $R_k$  is chosen as unit matrix. Note that  $\eta_{\phi_k}$  and  $\eta_{\gamma_k}$ , will be used as the guidance command .

#### 5. RESULT AND DISCUSSION

The initial position (x ,y and z coordinates ) of missile for simulation is chosen as 0, 0 and 6500 m and velocity is chosen as 1400m/s . Initial heading angle and flight path angle is taken as 45 deg.

The maximum acceleration of the interceptor has a specific value. To show the capability of guidance, different cases for final condition have been chosen(see Table 1).

Table 1. final condition of missile

Case	Position X(m)	Position Y(m)	Position Z(m)	Vel (m/s)	$\phi$ (deg)	$\gamma$ (deg)
1	6600	6200	15000	-	0	45
2	6600	6200	15000	-	70	45

In case 1, missile has to reach at desired position with  $\phi = 0$ . Figure 2 shows  $x$  vs  $y$  position of missile. Straight Blue lines is the path corresponding to first guess of acceleration. Red line is the final path achieved by missile and rest are the intermediate path which are converging to red line after few iteration (here it is taking 5 iteration). Green Star point is the desired point . The similar phenomena can be observed in figure 3 which shows  $x$  vs  $z$  position of interceptor. Figures 4 and 5 show the heading angle

and flight path angle demanded (Green Star point) and achieved by missile (red line). Rest of the lines are the intermediate heading angle and flight path angle. Figure 6 and 7 show the acceleration required (red line) to achieve desired goal. The remaining lines are the intermediate history of acceleration.

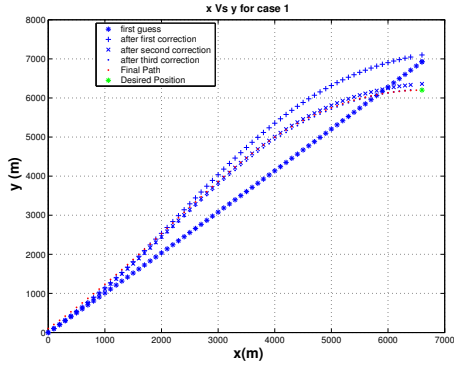


Fig. 2.  $x$  position vs  $y$  position for case 1

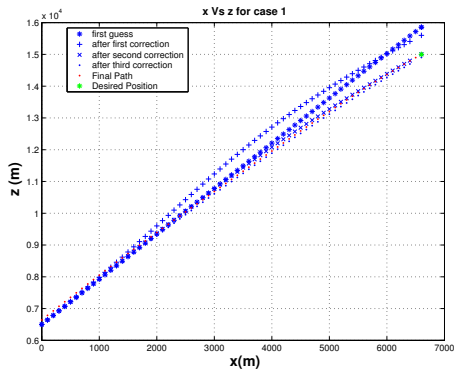


Fig. 3.  $x$  position vs  $z$  position for case 1

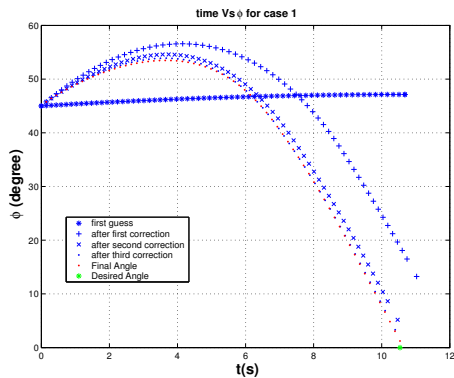


Fig. 4. Heading angle ( $\phi$ ) vs Time for case 1

In case 2, missile has to reach desired position with  $\phi = 70$  deg. Figure 8 shows  $x$  vs  $y$  position of interceptor. Here it can be observed that straight blue line is similar to case 1 because first guess is the same for both case. Here it takes 7 iterations to reach desired path. Figure 9 shows  $x$  vs  $z$  position of interceptor. It is similar to case 1 because desired condition is same. Figure 10 and 11 show the acceleration required (red line) to achieve desired goal for case 2.

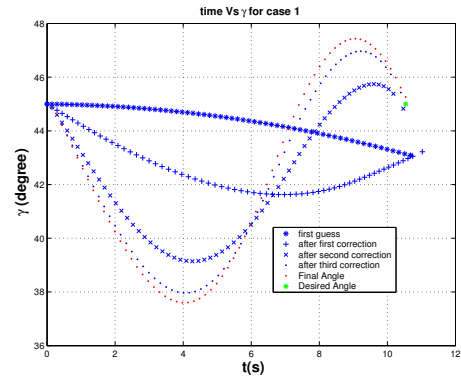


Fig. 5. flight path angle ( $\gamma$ ) vs Time for case 1

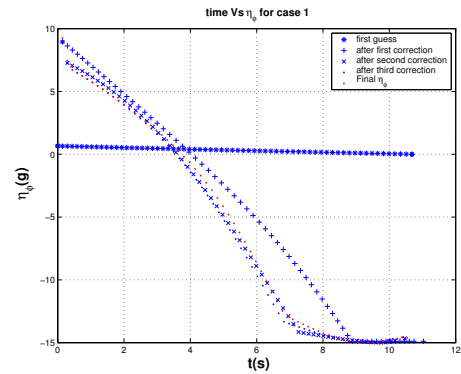


Fig. 6. Acceleration in  $xy$ -plane vs Time for case 1

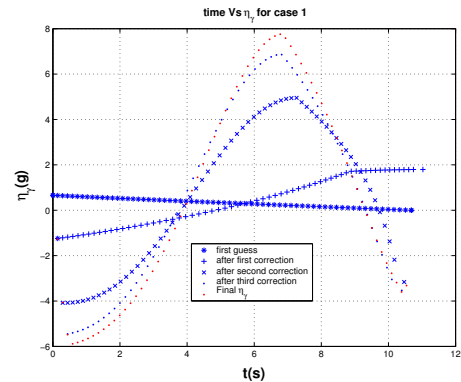


Fig. 7. Acceleration in  $xz$ -plane vs Time for case 1

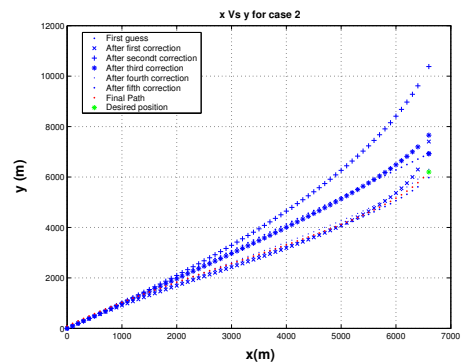


Fig. 8.  $x$  position vs  $y$  position for case 2

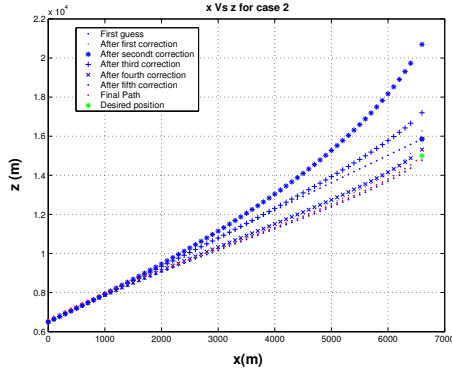


Fig. 9.  $x$  position vs  $z$  position for case 2

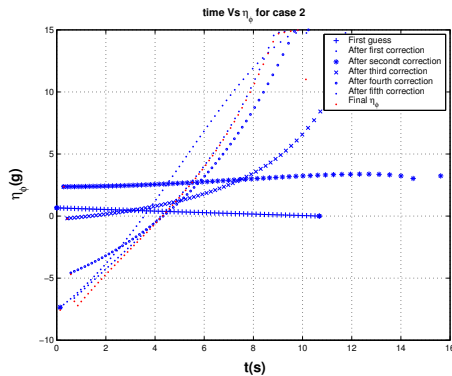


Fig. 10. Acceleration in  $xy$ -plane vs Time for case 2

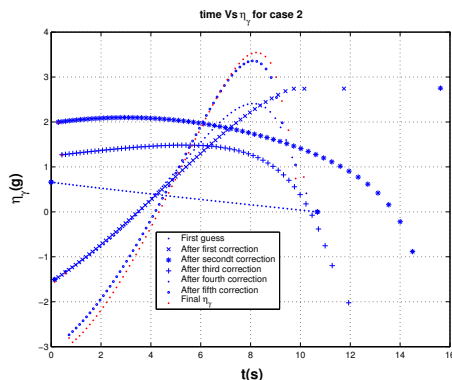


Fig. 11. Acceleration in  $xz$ -plane vs Time for case 2

## 6. CONCLUSION

In this paper, a newly developed MPSP (Model Predictive Static Programming) is utilized to solve optimal mid-course guidance problem for a homing interceptor. Here the acceleration demand has been minimized for reaching desired position with desired velocity vector. This technique is computationally efficient and can be applied online for getting closed form suboptimal solution of mid course guidance problem. Time to go uncertainty is avoided in this formulation by making use of desired position to go, which is quite accurate, as missile position is accurately known from INS (inertial navigation systems) and desired position is known from pre launched computation. In this formulation, closed form control history solution is suitably updated at every control interval, which minimizes jump in control variable at two successive instant.

Moreover output of the guidance scheme are acceleration commands in two perpendicular planes, which are robust control variables for the plant.

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