

Causality assignment and model approximation for quantitative hybrid bond graph-based fault diagnosis^{*}

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Abstract: Bond graph (BG) is an effective tool for modeling complex systems and it has been proven to be useful for fault detection and isolation (FDI) purposes for large continuous systems. BG provides causality between system's variables which allows FDI algorithms to be developed systematically from the graph. Similarly, Hybrid bond graph (HBG) is a bond graph-based modeling approach which provides an avenue to model complex hybrid systems; however, due to the lack of understanding, HBG has not been well-utilized for fault diagnosis. This is the first of a two-part paper that investigates the feasibility of utilizing HBG for quantitative FDI applications for hybrid systems. In this first paper, we present an analysis on the causality properties of the HBG where useful properties and insights associated with FDI applications are gained. Based on these properties, a causality assignment procedure and modeling approximation techniques are developed to achieve a HBG with a causality that facilitates efficient and effective FDI design for hybrid systems.

1. INTRODUCTION

A complex engineering system consists of many subsystems where operational safety depends on system's fault detection and isolation ability. A fault detection and isolation (FDI) system detects a fault or failure so that necessary safety is maintained and replacement of essential components can be performed effectively to ensure daily operations. Many existing FDI methods can be generally classified into *model-based* and *data-driven* approaches and good account on these methods can be found in survey papers [2, 3, 4].

A model-based FDI method requires a model and its performance depends on the availability and the quality of the system's model. Modelling of a physical system is an important and demanding step in the FDI design and the difficulty increases with the complexity of the system. Bond graph modelling provides an approach to deal with a large complex system which possesses large amount of equations describing its behavior [5]. More importantly, it has proved itself as an useful tool to develop FDI algorithms for continuous systems [6, 7, 8, 9, 10].

A hybrid system is a dynamic system whose behavior evolution combines discrete and continuous changes. Therefore, BG modeling cannot be applied to such systems. Fortunately, an extended bond graph-based modeling approach called the Hybrid bond graph (HBG) modeling has

been proposed by [12] to extend the benefits of BG to hybrid systems.

Recently, HBG has been used to develop a qualitative monitoring system for hybrid systems [11]. Unfortunately, due to the lack of insights on the causality properties of HBG, the HBG has not utilized effectively in the FDI design. First, the algorithm only considers abrupt parametric fault and the algorithm depends solely on system's transient response to isolate the fault. Second, the FDI algorithms are designed for each operating mode individually instead of utilizing the unify description of the HBG. In other words, the design of the FDI algorithms will be cumbersome, and real-time implementation would require vast resources. Moreover, the framework of the FDI obscures useful properties such as components' monitoring ability which a BG can provide. These information is useful to evaluate the performance of a monitoring system.

In this paper, we analyze the causal properties of HBG from FDI perspectives. These insights suggest methods to achieve suitable causality forms for a HBG so that effective FDI algorithms can be developed from the HBG.

2. HYBRID BOND GRAPH (HBG) MODELING

BG modelling is a systematic modelling framework that builds models across multiple domains for continuous systems [5]. It makes use of generic-components such as capacitance (C), inertias (I), and resistances (R); and the connection between the components called the *bond* to model the behavior of the system. Every bond contains an

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effort and a flow variables $\{e, f\}$ which correspond to the system's variables that belong to the components of the physical system. Hybrid bond graph (HBG) extends BG modelling by incorporating *controlled junction* to model the mode change of hybrid systems [12]. In this paper, the HBG is studied from FDI perspectives.

There are 0-Type and 1-Type controlled junctions in HBG modeling paradigm. A controlled junction has two possible states, ON or Off, which corresponds to an active junction and an inactive junction. When a controlled junction is ON, the junction behaves like a regular BG junction. On the other hand, an OFF controlled junction deactivates all the bonds and the components which are adjacently connected to the junction. This means that the components connected to the junction are inactive. An inactive 1-Type controlled junction indicates every flow variable of the bonds connected to the junction is zero. Similarly, an inactive 0-Type controlled junction indicates all every effort variable of the bonds connected to the junction is zero.

From the HBG modeling paradigm, we can see that the system mode of the hybrid system is determined by the states of the controlled junctions. Readers may refer to [12] for further details.

3. CAUSALITY PROPERTIES OF HYBRID BOND GRAPH

BG-based FDI methods depend heavily on the concept of causality to systematically solve an unknown variable to derive constraint equations for monitoring applications. Unfortunately, HBG generally experiences changes in causality structure which poses difficulties in FDI design using HBG. In this section, we study the causality properties of HBG to gain some insights on this issue.

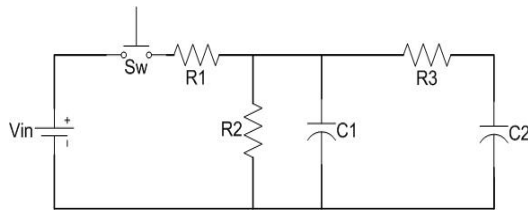


Fig. 1. A simple hybrid system: an electric circuit

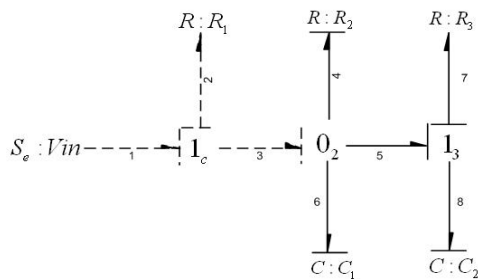


Fig. 2. HBG with a causality assignment

Figure 1 depicts a simple hybrid system in electrical domain whose HBG is depicted in Figure 2. For reference purposes, all the bonds and BG components of the HBG are enumerated. The HBG has a 1-type controlled junction denoted by 1_c that corresponds to the physical switch of the electrical system. The active bonds of the graph are represented by solid lines in the graph where dotted-lines represent inactive bonds. Figure 2 depicts three inactive bonds $\{1, 2, 3\}$ and inactive components $\{Se, R_1\}$ due to OFF state of controlled junction 1_c . We notice that the inactive bonds cause invalid causality to junction 0_2 ; hence the causality of the HBG at mode OFF is invalid. This invalid causality of junction 0_2 obstructs users from exploiting the causality assigned to the graph for generating constraint equations at OFF mode. Moreover, it is clear that this problem is significant when the hybrid system is large where the HBG has many controlled junctions.

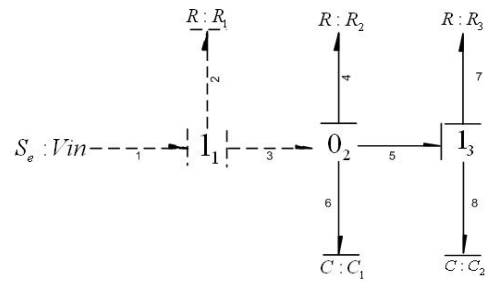


Fig. 3. HBG with an alternate causality assignment

Now, we consider the same HBG but with a different causality assignment. Figure 3 depicts the HBG with an alternate causality assignment. In this case, the inactive bonds due to the OFF state of controlled junction 1_1 does not pose any invalid causal form to the active BG components. The junction 0_2 has a valid causal form even though bond 3 is deactivated by controlled junction 1_1 . This observation suggests that it is possible to describe the behavior of a hybrid system at all modes using one unified causality assignment where all active components' causal forms are valid at all modes. The illustration motivates us to study the causality properties of the HBG in general case.

To gain better insights, we consider the following general case where a controlled junction can be connected to a 1-port $\{I, C, R\}$, a 2-ports $\{TF, GY\}$ or a multiple-ports element $\{0, 1\}$.

Figure 4 depicts a 1-Type controlled junction (1_c) which is connected to a 0-Type junction with the causality assignment shown in the figure. The causality stroke of bond 3 indicates that effort (e_3) is the output of 1_c and input of the 0-Type junction; and f_3 is the output of the 0-Type junction which is also an input to 1_c . By inspection, the OFF state of the controlled junction poses an invalid causal form to the 0-junction. The causality interpretation of this behavior is that the deactivation of bond 3 due to OFF state of 1_c causes the output of the 0-Type junction to be undefined. Similarly, this behavior also applies to 0-Type controlled junction as shown in Figure 5.

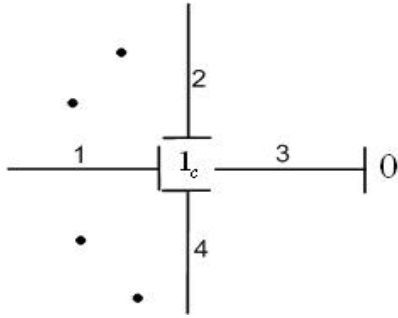


Fig. 4. 1-Type controlled junction

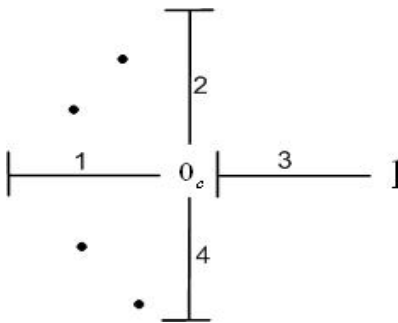


Fig. 5. 0-Type controlled junction

In HBG modeling, all 1-port elements that are connected to a controlled junction will be inactivated when the controlled junction is OFF, therefore we need not have to consider the case for 1-port element. As for 2-ports elements, we have the following property.

Property 1. If a controlled junction is adjacently connected to a 2-ports element, then the controlled junction poses an invalid causal form to the two-ports element when the controlled junction is OFF.

Proof. The proof is straightforward. By assumption, there is a 2-ports element connected adjacently to the controlled junction, the OFF state of controlled junction causes the bond between the controlled junction and the 2-ports element to be inactivated. Since a 2-ports element requires two bonds for a valid causality description, the deactivated bond results an invalid causal form to the 2-ports element. And this completes the proof.

The implication of property 1 is clear. One necessary condition for a HBG to have valid causality at all modes is that the graph must not have a 2-ports element adjacently connected to any controlled junction. Hence, we assume the following condition for the remainder of this paper.

Assumption 1. Every 2-ports element of the HBG is not adjacently connected to any controlled junction.

Property 2. If the output of a controlled junction is an input of any 1-port component, then the OFF state of the controlled junction poses no invalid causal form to any active bond graph components.

Proof. Due to assumption 1, there is no 2-ports BG element connected to the controlled junction. Hence, we only need to consider the case where there are multiple-ports components $\{0, 1\}$ connected to the controlled junction. We first consider the case of a 0-Type controlled junction. In BG modeling language, any two adjacently connected junctions of the same Type can be merged into one junction; therefore we can always assume any multiple-ports element which is connected to a 0-Type controlled junction is a 1-Type junction. From the rules of causality, we know that if the output of the controlled junction is an input of a 1-port component, then the output of the 1-Type junction is not a variable of the bonds that connects between the two junctions. Therefore, the output of the 1-Type junction is always well-defined, i.e., the 1-Type junction always have a valid causal form even when the 0-Type controlled junction is OFF. The same argument applies to the case of 1-Type controlled junction and this completes the proof.

Property 2 states a sufficient condition on the causality of a controlled junction so that its adjacently components will not have any invalid causality when the junction is OFF. This property suggests that if the causalities of all controlled junctions of a HBG are assigned in such a way that all their outputs are inputs to some 1-port components, then all active the components of the HBG would have valid causal forms for all operating modes. We say a controlled junction is in *preferred causality* if the junction's output is an input of a 1-port component connected to the controlled junction. Most importantly, these properties suggests that for a given acausal HBG (HBG without causality assignment), we would like to assign a causality to the HBG such that all controlled junctions are in their preferred causality. And since our focus is on FDI applications, we would also like all storage elements (C, I) of the HBG to be in derivative causality to avoid the need of unknown initial conditions in the computation. We say a HBG with such causality assignments the *Diagnostic Hybrid Bond Graph* (DHBG). In short, an DHBG is a HBG with a causality assignment that all its controlled junction and storage elements are in preferred causality. From FDI perspectives, we would like to achieve DHBG since the consistent valid causal form of every active component provides a convenient way to describe the behavior of the hybrid system at all modes. This description allows efficient and effective FDI designs based on HBG.

Based on the properties gained, we develop an orderly causality assignment procedure, called the Sequential Hybrid Causality Assignment procedure (SHCAP) to achieve an DHBG from an acausal HBG (HBG without causality assignment).

The SHCAP is listed as follows.

- (1) Choose any controlled junction that has no source element adjacently connected to it and assigned its preferred causality.
- (2) Repeat step 1 until all the controlled junctions that has no source element connected to them have been assigned.
- (3) Choose any remaining controlled junction that has a source element connected to the junction. For the

- pair $\{S_e, 1\}$ or $\{S_f, 0\}$, the output of the controlled junction must not be an input of $\{0, 1\}$. For the pair $\{S_f, 1\}$ or $\{S_e, 0\}$, the output of the controlled junction must be the input of the connected source element.
- (4) Repeat step 3 until all remaining controlled junctions have been assigned. After this step, all controlled junctions are treated as ordinary $(0, 1)$ junctions in the following procedures.
 - (5) Choose any remaining source element (S_e, S_f) and assigned its causality. Immediately extend the causal implications through the HBG as far as possible using the constraint elements $(0, 1, GY, TF)$.
 - (6) Repeat step 5 until all sources have been assigned.
 - (7) Choose any storage element $(C, \text{ or } I)$, and assign it with a derivative causality. Immediately extend the causal implications through the HBG as far as possible using the constraint elements $(0, 1, GY, TF)$.
 - (8) Repeat step 7 until all storage elements have been assigned with a causality.
 - (9) Choose any unassigned R element and assign a causality to it. Immediately extend the causal implications through the HBG as far as possible using the constraint elements $(0, 1, GY, TF)$.
 - (10) Repeat step 9 until all remaining bonds have been assigned.

The main purpose of SHCAP is to derive DHBG which has desirable causal properties that facilitates analysis and FDI designs. We observe that the additional preferred causality of the controlled junctions imposes additional restriction in the causality assignment of the HBG. In BG modeling, R elements have no preferred causality. This suggests that a R element could facilitate causality assignment for a HBG. Unfortunately, the number of R element is fixed in a physical system. In the next section, we present a model approximation technique that aims to facilitate the application of SHCAP algorithm.

4. MODEL APPROXIMATION FOR CAUSALITY ASSIGNMENT

As we have mentioned in the preceding section, a R element could ease the causality assignment constraints of the system's bond graph. This additional freedom of causality assignment facilitates the application of SHCAP procedure to obtain an DHBG for FDI purposes.

In electronics systems, stray components, e.g., stray capacitor is often neglected from the circuit's schematic for analysis purposes. This modelling approximation captures the essence of the physical system's behavior while eliminating the insignificant components from the model that have minimum influence to the physical system. Here in contrast, we would approximate the physical system by adding stray resistors (R elements) to the model that has minimum influence to the physical system. This approximation allows R elements to be added to the BG model of a physical system.

Unfortunately, an addition of R element adds unknown variables to the model. These additional unknown variables require constraints to solve, and hence they reduce the analytical redundancy of the model which undermines the monitoring ability of the system. Moreover, how this

concept can be applied effectively and systematically to the HBG remains unclear. Here, we propose a method to add R elements to the graph without degrading the diagnostic properties of the system. Additionally, a procedure is developed to combine the modeling approximation method with the SHCAP algorithm to yield the DHBG systemically.

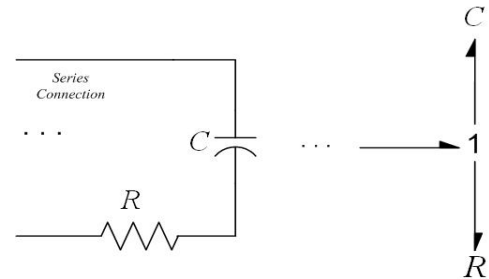


Fig. 6. Series connection

Without loss of generality, we consider a series connection (1-Type junction) of an electric circuit (see Figure 6). For this part of electric circuit, we can approximate it by assuming a stray resistor $R_s \approx 0$ in series with the circuit. In this case, the physical system is approximated with an additional R_s component in the series loop. The same approximation concept also applies to parallel connection of a circuit (see Figure 8). For parallel connection, we approximate the original physical system by assuming a high resistance R element, $R_s \approx \infty$ in the parallel connection. The approximated HBG is shown in Figure 9. These stray resistors serve as guidelines in assigning the causality as of a HBG to attain DHBG.

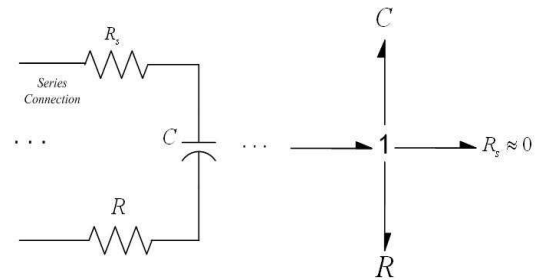


Fig. 7. Series connection with R_s component

From the BG viewpoint, we know that each additional R element adds a pair of variables $\{e, f\}$ to the bond connected to the R element. If these variables are unknown, then the approximation will degrade the system's monitoring ability. This degradation is due to the reduction of constraint equations in the approximated HBG model which are used to solve the additional unknown variables.

Fortunately, this problem can be avoided if the approximation is done at suitable parts of the system. To gain better insights on how this problem can be avoided. We consider the case of series connection (see Figure 7). We let $\{e_s, f_s\}$ be the effort and flow variables associated to the $R_s \approx 0$. Since R_s is assumed to be small, we can

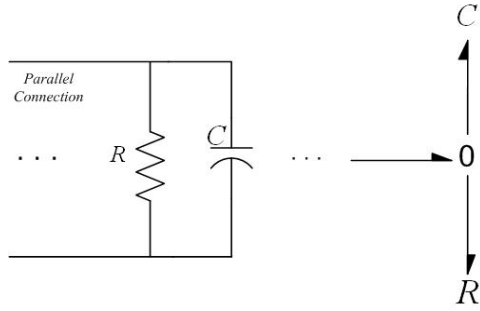


Fig. 8. Parallel connection

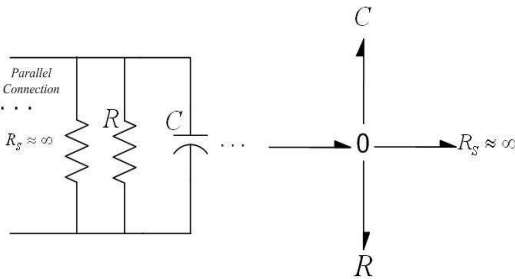


Fig. 9. Parallel connection with R_s component,

let $e_s = 0$. If f_s is measurable by a flow sensor D_f that is attached to the series connection, then f_s variable is known. This observation suggests that if a R_s is added only at a series connection that has a flow sensor, then we can avoid adding unknown variables to the model. Likewise for parallel connection, if the $R_s \approx \infty$ is added to parallel junction that has an effort sensor, then we can let $f_s = 0$ since the R_s has high-resistance, and e_s be the measured value provided by the effort sensor. We can easily identify the location where this modeling approximation can be done since the location of the physical sensors are explicitly shown in a BG. This explicit description of BG allows the idea of approximation to be implemented easily.

The properties gained from the preceding studies suggest that we can combine the SHCAP algorithm with the modeling approximation method via the follows steps. For a given acausal HBG, we

- (1) Step 1: Add a R_s element to every *sensor junction*.
- (2) Step 2: Apply SHCAP algorithm to assign the causality.
- (3) Step 3: Remove all R_s that have *indifferent causal form*.

Note that a *sensor junction* is a BG junction that has a sensor attached to it. And for a given HBG with causality assignment, we say a R_s is in *indifferent causal form* if the elimination of the R_s does not pose an invalid causal form to the junction where the R_s is connected, i.e., the R_s is redundant from the perspective of causality.

The procedure combines the model approximation technique with the SHCAP algorithm effectively to derive the DHBG. With an DHBG, all active components of the HBG

will have valid causal forms at all modes. As a result, we are able to derive an unified set of constraints equations to describe the behavior of the hybrid system. This unified equations lay a crucial foundation to develop efficient and effective FDI algorithms for hybrid systems.

5. CASE STUDY

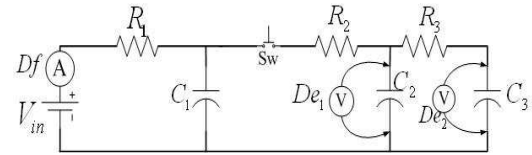


Fig. 10. A electrical hybrid system

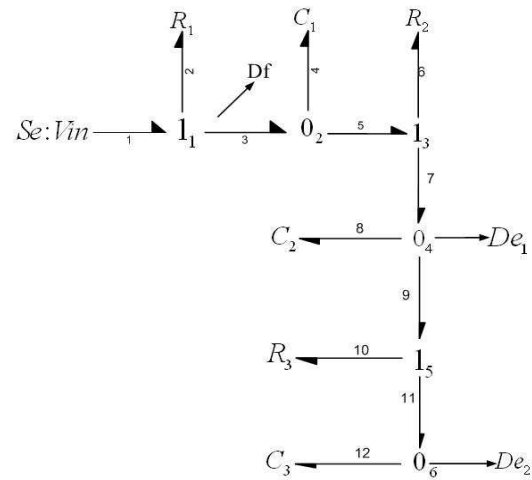


Fig. 11. System's acausal HBG

For illustration purposes, we consider a 2-mode electrical system. Figure 10 shows the electrical system that has a switch (Sw), 7 components, and 3 sensors. D_f denotes the current (flow) sensor, $\{De_1, De_2\}$ represent the voltage (effort) sensors that measure the respective nodes shown in the figure. The corresponding acausal HBG (without causality) of the hybrid system is shown in Figure 11. The graph is enumerated for reference purposes. Controlled junction 1_3 models the switching behavior due to the switch of the physical system. It is clear that this system has 2 modes which corresponds to the ON and OFF state of the switch. In the graph, the sensor junctions are the $\{1_1, 0_4, 0_6\}$ and the system's sensors are represented by $\{D_f, De_1, De_2\}$ which are connected to the graph's respective junctions, and the known input is represented by a source element Se .

To derive an DHBG from the HBG, we apply the modeling approximation by adding 3 stray resistors $\{R_{s1}, R_{s2}, R_{s3}\}$ to the sensor junctions to approximate the electrical system. After the model approximation, we apply the SHCAP algorithm to assign causality strokes to each bond of the approximated HBG. Finally, all R_s elements that have

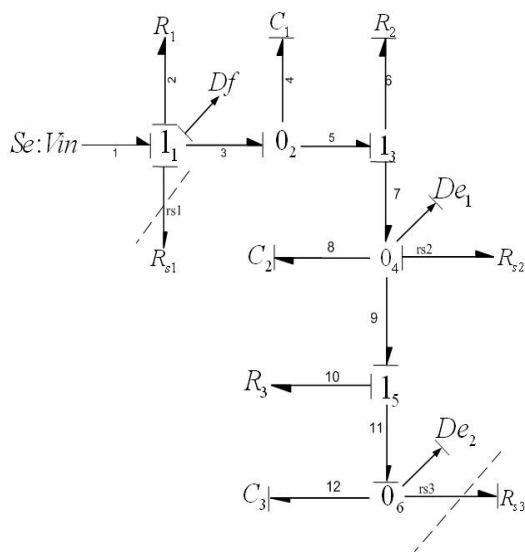


Fig. 12. Derivation of DHBG

indifference causal forms are eliminated from the graph. In this case, R_{s1} and R_{s3} have indifferent causal forms which are eliminated from the graph. These steps are illustrated in Figure 12.

6. CONCLUSIONS

Hybrid bond graph (HBG) is an useful bond graph-based modeling technique for hybrid systems. In this paper, we analyzed the causality properties of the HBG from the perspectives of fault diagnosis and isolation (FDI). Many useful insights and properties have been gained from the analysis. Based on these results, we proposed the concept of Diagnostic Hybrid Bond Graph (DHBG). DHBG is a HBG which describes the behavior of a hybrid system at all modes based on one unified set of causality assignment. Two systematic algorithms called the Sequential Hybrid Causality Assignment Procedure (SHCAP) and Modeling Approximation (MA) technique are developed and integrated in this paper to derive an DHBG from a HBG. The consistent and unified causality description of the DHBG suggests that we are able to govern the hybrid system's behavior based on one unified set of constraint equations. With these equations, efficient and effective quantitative bond graph-based FDI algorithms can be developed easily for large complex hybrid systems based on HBG. In fact, we have already developed preliminary results on these algorithms which are illustrated separately in [1].

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