

## Monitoring ability analysis and qualitative fault diagnosis using hybrid bond graph <sup>★</sup>

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**Abstract:** In part I of this work, we lay a foundation for quantitative bond graph-based FDI design of hybrid systems using hybrid bond graph (HBG). We discussed the causality properties of HBG from FDI perspectives, and proposed the concept of Diagnostic Hybrid Bond graph (DHBG) which is advantageous for efficient and effective FDI applications. Part II presents a continuation of our previous paper [1]. In this part II of our work, we exploit the DHBG to analyze the system's fault monitoring ability. Additionally, we proposed a quantitative FDI framework for effective fault diagnosis for hybrid systems.

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### 1. INTRODUCTION

HBG is a bond graph(BG)-based modelling paradigm which provides a systematic mean to construct models for complex hybrid systems. In part I of our work, several useful insights are gained from the perspectives of FDI, which leads to the developments of the Diagnostic Hybrid Bond Graph (DHBG) [1]. For details on HBG, readers may refer to [8].

In summary, DHBG is a HBG that is assigned with a suitable set of causality such that the causality of every active BG component remains valid at all operating modes. This causality feature allows consistent causal description, so that we are able to derive a set of unified equations to describe a hybrid system at all modes using HBG. This unique feature of DHBG opens many possibilities to develop efficient and effective FDI algorithms.

Part II of this work presents applications of DHBG for fault diagnosis. In this paper, we proposed a quantitative FDI framework for hybrid systems to exploit the rich information contained in the system's HBG. Based on the unique causality structure of the DHBG, we proposed the notion of an unified analytical redundancy relation, called Global Analytical Redundancy Relation (GARR), which describes the behavior of the system quantitatively at all modes. With this GARR, we developed a method to analyze the hybrid system's fault monitoring ability in an efficient manner. Additionally, we proposed a quantitative bond graph-based FDI framework for high-performance fault monitoring. Unlike the indirect qualitative approach developed in [7], linearization is not required in our framework and our quantitative approach allows both incipient and abrupt parametric faults to be detected and isolated effectively and efficiently in real-time at all modes. Moreover, our framework allows fault to be detected and

isolated when the system is operating in both transient and steady state.

### 2. ANALYTICAL REDUNDANCY RELATIONS (ARR)

A model-based FDI approach works by evaluating the system's constraint equations using known data, e.g., known inputs, sensors data, and parameter values of the monitored system. Analytical redundancy relations (ARRs) are static or dynamic equations representing compatibility constraints between different process variables. In essence, ARR is a combination of system's observables which are written only in terms of known variables, sources, and measurements. Hence, an ARR is a relationship between a set of known variables of the form  $f(K) = 0$  where  $K$  is the set of known variables.

ARR evaluation verifies the consistency of the monitored system during operation. Any inconsistency in one of these constraint equations is an indication of a fault occurred in the system. The numerical values evaluated from the ARR equations are called *residuals* which allow users to isolate a fault occurred in the system. Besides fault diagnosis, the ARR can also be used for offline analysis such as *monitoring ability analysis* to study the diagnostic properties such as fault detectability and fault isolability of each component [6].

BG modelling allows users to deal with a large amount of equations of the system by exploiting the causality information of the graph [2]. Since the residuals are essentially a combination of system variables, ARRs can also be derived using causal paths of the BG. This feature allows users to generate ARRs from the BG systematically. For continuous systems, BG has proved itself a useful tool for generating ARRs [4, 3, 5, 6] for complex systems. However, HBG has yet to be exploited for quantitative FDI for hybrid systems.

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### 3. GLOBAL ARR GENERATION FOR HYBRID SYSTEMS

In part I of this work, we illustrated the advantages of DHBG and how it can be derived from a given HBG. One unique advantage of the DHBG is that all active BG elements' causalities remain valid at all operating modes. This unique feature of DHBG allows users to derive one set of constraint equations that describes the hybrid system's behavior. This set of constraint relations function as ARR equations for hybrid systems at all operating modes. We call these constraint relations *Global ARRs* (GARRs). The *global* implies the set of constraint relations are applicable at all operating modes. Similar to continuous systems, the GARRs can also be used to deduce the monitoring ability of the hybrid systems. In this section, we present a systematic approach to derive the GARRs from a given DHBG.

#### 3.1 Global ARR generation procedure

In HBG modeling language, an OFF controlled junction enforces the respective power variables of its adjacently connected bonds to zero. This property suggests that we are able to extend the covering path method to DHBG via the following procedures.

- (1) Step 1: Define a boolean variable  $a_i$  for each controlled junction  $i = \{1, \dots, q\}$ .
- (2) Step 2: For each controlled junction  $i = 1, \dots, q$  which has  $n_i$  bonds adjacently connected to it, we substitute every flow variable  $f_i^j$  (for  $j = 1, \dots, n_i$ ) of the  $n_i$  bonds with  $a_i \cdot f_i^j$  if the considered junction is 1-Type. Similarly, we substitute every effort variable  $e_i^j$  (for  $j = 1, \dots, n_i$ ) of the  $n_i$  bonds with  $a_i \cdot e_i^j$  if the considered junction is 0-Type.
- (3) Step 3: Choose a junction. The junction can be a normal junction or a controlled junction.
- (4) Step 4: Based on the constitutive relation of the considered junction, we eliminate the unknown variables of the relation using the causal path analysis of the DHBG. If all unknown variables of the relation can be solved, i.e., expressed only in term of known variables (input, sensors variables), then a GARR is the relation with its unknown variables expressed in term of the known variables. If there exists an unsolvable unknown variable, we proceed to Step 5.
- (5) Step 5: Choose the next junction.
- (6) Repeat Step 4 until all the junctions of the DHBG are considered.

Covering path method is a classical method which utilizes sequence of linkages that have identical causal direction called *causal paths* to generate quantitative ARR equations from a BG [3]. Causal paths are connections of causal linkages of the bonds which are indicated by the causality strokes of the bonds of the graph. Here, we apply the causal path concept based on covering path method to generate GARRs for hybrid systems. Without loss of generality, each GARR equation can be written as  $F_l(\theta, \mathbf{x}, De, Df, u) = 0$  for  $l = 1, \dots, m$ .  $m$  denotes the number of GARRs derived from the DHBG,  $\theta = [\theta_1, \dots, \theta_p]^T$  represents the nominal parameters of the HBG elements which are assumed to be known during fault-free opera-

tion,  $\mathbf{x} = [x_1, \dots, x_q]^T \in X$  represents the states of the  $q$  controlled junctions,  $X$  denotes the set of all possible states of  $\mathbf{x}$ .  $u$  denotes the system's inputs,  $De$  and  $Df$  denotes the effort and flow sensors of the graph, and  $p$  represents the number of HBG parameters used to describe the hybrid system.

### 4. MONITORING ABILITY ANALYSIS

For continuous systems, a residual  $r_l$  is sensitive to a fault in those components whose parameters are in the ARR. When the system is fault-free, every computed residual will be consistent with the system's behavior. That implies that each residual is null or  $|r_l|$  is below a small threshold value  $\epsilon_l$  when this system is fault-free. To apply the set of residuals for FDI, we define a binary coherence vector  $C = [c_1 \dots c_m]$ . Each component  $c_l$  of  $C$  is obtained by the following simple decision rule:

$$c_l = \begin{cases} 1 & \text{if } |r_l| > \epsilon_l; \\ 0 & \text{otherwise} \end{cases} \quad \text{for } l = 1, \dots, m \quad (1)$$

If the system is fault-free, then the binary coherence vector  $C$  will be a zero vector. On the other hand, if the system is faulty, then the coherence vector will be a nonzero vector.

To study the fault monitoring ability, we generate a fault signature matrix (FSM) from the  $m$  ARRs of the system [6]. A typical FSM is shown in Table 1. In Table

	$r_1$	$\dots$	$r_m$	$D_b$	$I_b$
$\theta_1$	1 or 0				
$\dots$					
$\dots$					
$\dots$					
$\theta_p$					

Table 1. Fault signature matrix (FSM)

1, the column headers consist residual  $r_1, \dots, r_m$ , fault detectability ( $D_b$ ), and fault isolability ( $I_b$ ). Each entry of the table holds a boolean value. For each row, the boolean entries under the columns  $r_1, \dots, r_m$  form the fault signature of the parameter  $\theta_i$  that corresponds to a fault in the parameter  $\theta_i$ . Under a residual column, a 1 in an entry indicates that the residual is sensitive to a fault in the parameter of the matching row. On the other hand, a 0 in the entry represents the residual is insensitive to the fault in the corresponding parameter. If at least a 1 appears in the fault signature of a parameter  $\theta_i$ , then the parameter is said to be fault detectable. This ability is represented by a  $D_b = 1$  in the matrix. When the fault signature of a parameter  $\theta_i$  is unique, then a fault occurs in parameter  $\theta_i$  is said to be fault isolable. This is denoted by  $I_b = 1$ . It is beneficial to note that fault isolability is a stronger condition than fault detectability and the fault detectability is a necessary but not sufficient condition for fault isolability.

Unlike continuous systems, hybrid systems are multiple modes in nature. This suggests that we need to evaluate the system's monitoring ability in different operating modes for effective FDI analysis and designs. Fortunately, the global characteristic of the GARRs provide an efficient and effective mean to generate the FSM for each operating mode. Before we illustrate on how each FSM can be

generated from GARR, we would like to introduce some definitions that are useful to describe the different degrees of fault monitoring ability for hybrid systems.

*Definition 1.* A parameter  $\theta_i$  is said to be *all-mode detectable* if the parameter is detectable at all operating modes.

This definition refers to a parameter which is detectable at all modes. Assuming a hybrid system contains some critical components where their health status are important, then it is desirable that these components' parameters are all-mode fault detectable. Definition 1 defines a strong and desirable fault diagnosis property of a component's parameter. Some weaker definitions is stated as follows.

*Definition 2.* A parameter  $\theta_i$  is said to be *weakly detectable* if there exists a mode such that the parameter is detectable.

*Definition 3.* A parameter  $\theta_i$  is said to be *all-mode non-detectable* if the parameter is undetectable at all operating modes.

Definition 2 refers to components whose parameters are fault detectable at some modes but not all modes. On the other hand, definition 3 refers to parameters that are non-detectable for all modes. If a critical component is found to be all-mode non-detectable, then the designer may need to change to the system such that the component is weakly detectable or all-mode detectable. In the same manner, definitions for fault isolability are defined as follows.

*Definition 4.* A parameter  $\theta_i$  is said to be *all-mode isolable* if the parameter is detectable at all operating modes.

*Definition 5.* A parameter  $\theta_i$  is said to be *weakly isolable* if there exists a mode such that the parameter is isolable.

*Definition 6.* A parameter  $\theta_i$  is said to be *all-mode non-isolable* if the parameter is non-isolable at all operating modes.

With these definitions, we are able to characterize the monitoring ability of each parameter of a hybrid system using HBG. Now, we are going to exploit the unified GARRs to deduce the fault monitoring ability for each component.

Based on the structure of the HBG, we can deduce the following properties.

*Property 1.* If a component is adjacently connected to a controlled junction, then the component's parameters are not all-mode detectable and not all-mode isolable.

*Proof:* The proof is straightforward. Since the component is adjacently connected to a controlled junction, the component will be deactivated when the controlled junction is OFF. Because there is no energy flow in the component when it is deactivated, there is no way we can observe the behavior of the component using flow or effort sensors. Hence, the component's parameters are not all-mode detectable. Since fault detectability is a necessary condition for fault isolability, we can conclude that the component is not all-mode isolable and this completes the proof.

Property 1 provides a necessary condition for component's parameters to be all-mode detectable and isolability, but not weakly detectable. Based on the characteristic of the

controlled junction, another property can be deduced from property 1.

*Property 2.* If a component is adjacently connected to a controlled junction, then the component is non-detectable at operating modes where the controlled junction is OFF.

*Proof:* The proof follows the same argument as the proof of property 1.

Property 2 provides a sufficient condition to determine the non-detectable components based on the structure of the HBG without inspecting the GARRs. Based on property 2, we can conclude  $D_b = 0$  for those components at the respective operating modes. To determine other monitoring ability for each component as define in definitions 1-6, we need the GARR equations.

In HBG modeling, each operating mode  $k$  of a hybrid system is represented by its corresponding controlled junction state  $\mathbf{x}_1 \in X$  of the system's HBG. Hence without loss of generality, we can evaluate the monitoring ability of the hybrid systems at all modes by inspecting the GARR equations for  $\forall \mathbf{x} \in X$ . To derive the FSM at mode  $\mathbf{x}_1 \in X$ , we inspect the parameters  $\theta$  of the  $m$  GARR equations at  $\mathbf{x} = \mathbf{x}_1$ .

By following the same approach of generating FSM from ARR equations in continuous systems, we obtain the FSM from the  $m$  GARR equations each mode  $\mathbf{x} \in X$ . In other words, based on  $F_l(\theta, \mathbf{x}_1, De, Df, u)$  for  $l = 1, \dots, m$ , we can formulate the FSM of mode  $\mathbf{x} = \mathbf{x}_1$  (see Table 2).

mode $\mathbf{x}=\mathbf{x}_1$	$r_1$	$\dots$	$r_m$	$D_b$	$I_b$
$\theta_1$	1 or 0				
.					
.					
.					
$\theta_p$					

Table 2. Fault signature matrix (FSM) at mode= $\mathbf{x}_1$

Based on the  $D_b$  and  $I_b$  of Table 2, we are able to determine the weakly detectability and isolability of the parameters for each mode  $\mathbf{x} \in X$ . If a parameter is weakly detectable or isolable for all modes  $X$ , then the component is said to be all-mode detectable or all-mode isolable.

It is worth to note that *all-mode detectability* of a component can be directly deduced by inspecting  $F_l$  for  $l = 1, \dots, m$ . A sufficient condition for this test is stated as follows.

*Property 3.* A parameter  $\theta_i$  is all-mode detectable if for all  $\mathbf{x}_1 \in X$ , there exists a  $l \in \{1, \dots, m\}$  such that  $F_l$  is dependent on  $\theta_i$ .

*Proof:* If for all  $\mathbf{x}_1 \in X$ , there exists a  $l \in \{1, \dots, m\}$  such that  $F_l$  is dependent on  $\theta_i$ , then a change of the component's parameter  $\theta_i$  is always be detectable by some residuals. In other words, the  $D_b$  of the parameter is always 1 for every FSM and this completes the proof.

#### 4.1 A Case study

For illustration purposes, we consider a 2-mode electrical system.

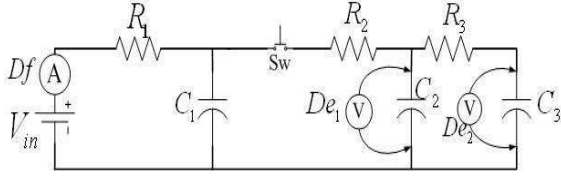


Fig. 1. An electrical hybrid system

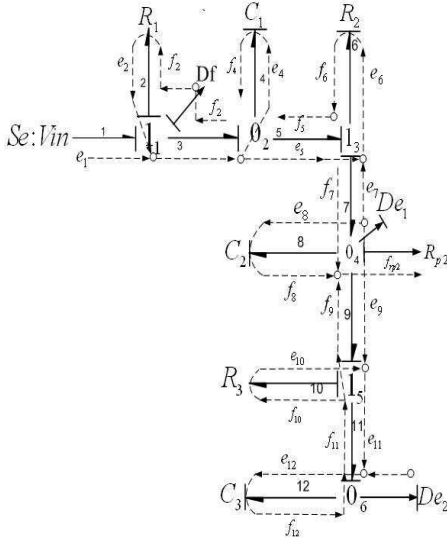


Fig. 2. System's DHBG

Figure 1 shows the electrical system illustrated in part I [1]. It has a switch (Sw), 7 components (parameters), and 3 sensors.  $D_f$  denotes the current (flow) sensor,  $\{De_1, De_2\}$  represent the voltage (effort) sensors that measure the respective nodes of the circuit shown in the figure. The corresponding HBG of the hybrid system is shown in Figure 2. The graph is enumerated for reference purposes. Controlled junction  $1_3$  models the switching behavior of the physical system (Sw). It is clear that this system has two modes, capture by the ON and OFF state of the switch.

To derive the GARR equations from the DHBG, we apply the GARR generation procedure. First, we define a boolean variable  $a$  for the controlled junction  $1_3$ . Then, we substitute the flow variables of bond  $\{5, 6, 7\}$  with  $\{af_5, af_6, af_7\}$ . With the help of causal paths shown in Figure 2, we apply covering path method for each junction to derive the GARR equations. The following GARR equations are derived from the constitutive relations of junction  $\{0_2, 0_4, 1_5\}$ .

For junction  $0_2$ , its constitutive equation is  $f_3 - f_4 - af_5 = 0$ .  $f_4$  and  $f_5$  are unknown. Note that  $f_5 = f_6$ . By covering casual paths  $4 - 3$  and  $5 - 6$ , one obtains  $f_3 - C_1 \frac{de_3}{dt} - \frac{e_6}{R_2} a = 0$ .  $\{f_2, f_3\}$  are measurable by sensor  $D_f$ , hence  $f_3 = D_f$ . Unknown  $e_3$  is given by the  $1_1$  junction's constitutive equation,  $e_3 = V_{in} - R_2 D_f$ . Based on the

constitutive equation of junction  $1_3$ ,  $e_6 = e_5 - e_7$ . Since  $e_7 = De_1$  and  $e_5 = e_3$ ,  $e_6$  can be written as  $e_6 = (V_{in} - R_2 f_2) - De_1$ . With equations  $(e_3)$  and  $(e_6)$ , the constitutive relation of  $0_2$  becomes

$$GARR_1 = D_f - C_1 \frac{d(V_{in} - R_1 D_f)}{dt} - \frac{1}{R_2} \{V_{in} - R_1 D_f - De_1\} a = 0 \quad (2)$$

Next, we consider the constitutive equation of Junction  $0_4$ . Since  $f_{rp2} \approx 0$ , the equation can be written  $af_7 - f_8 - f_9 = 0$ . In this case,  $\{f_7, f_8, f_9\}$  are unknown variables. For  $f_7$ , we cover the path  $7 - 6$  and we obtain  $f_7 = f_6 = \frac{e_6}{R_2}$ . And that leads to  $f_7 = (V_{in} - R_1 D_f - De_1) / R_2$ . By constitutive equation of  $C_1$  element,  $f_8 = C_2 \frac{dDe_1}{dt}$ . As for unknown  $f_9$ , we cover path  $9 - 11 - 12$  to yield  $f_9 = f_{11} = f_{12} = C_3 \frac{dDe_2}{dt}$ . Therefore,  $GARR_2$  can be written as

$$GARR_2 = \frac{1}{R_2} (V_{in} - R_1 D_f - De_1) a - C_2 \frac{d(De_1)}{dt} - C_3 \frac{d(De_2)}{dt} \quad (3)$$

The third GARR equation is derived from the constitutive equation of junction  $1_5$ ,  $e_9 - e_{10} - e_{11} = 0$ . Variables  $e_{11}$  and  $e_9$  are measurable by sensors  $De_2$  and  $De_1$ ; hence  $e_{11} = De_2$  and  $e_9 = De_1$ . In this case, it left unknown variable  $e_{10}$  to be solved. By covering path  $10 - 11 - 12$ , we obtain  $e_{10} = R_3 f_{12}$ . By constitutive equation of element  $C_3$ , we have  $f_{12} = C_3 \frac{d(De_2)}{dt}$ . And this leads to

$$GARR_3 = De_1 - R_3 C_3 \frac{d(De_2)}{dt} - De_2 \quad (4)$$

From this derivation, we yield three GARR equations that describes the behavior of the hybrid system. Now, to obtain the monitoring ability from the GARR equations, we present the FSM of the two modes.

$i = 1$	$GARR_1$	$GARR_2$	$GARR_3$	$D_b$	$I_b$
$R_1$	1	1	0	1	0
$C_1$	1	0	0	1	1
$R_2$	1	1	0	1	0
$C_2$	0	1	0	1	1
$R_3$	0	0	1	1	1
$C_3$	0	1	1	1	1
$V_{in}$	1	1	1	1	1

Table 3. FSM at mode 1

$i = 0$	$GARR_1$	$GARR_2$	$GARR_3$	$D_b$	$I_b$
$R_1$	1	0	0	1	0
$C_1$	1	0	0	1	0
$R_2$	0	0	0	0	0
$C_2$	0	1	0	1	1
$R_3$	0	0	1	1	1
$C_3$	0	1	1	1	1
$V_{in}$	1	0	0	1	0

Table 4. FSM at mode 0

Table 3 shows the FSM of the system at mode  $i = 1$  (i.e. when the Sw is ON) and Table 4 depicts the FSM of the system at mode  $i = 0$ . The monitoring ability gained from the  $\{D_b, I_b\}$  values of the two FSMs is summarized in Table 5.

$\theta$	Detectability	Isolability
$R_1$	all-mode	Nil
$C_1$	all-mode	Mode $i = 1$
$R_2$	Mode $i = 1$	Nil
$C_2$	all-mode	all-mode
$R_3$	all-mode	all-mode
$C_3$	all-mode	all-mode
$V_{in}$	all-mode	Mode $i = 1$

Table 5. Monitoring ability of the system's components

Table 5 shows that all components except  $R_2$  are all-mode detectable.  $R_2$  is not all-mode detectable but is weakly detectable at mode  $i = 1$ . The results show the consistency of property 2 with respect to controlled junction 1<sub>3</sub>. Components  $\{C_2, R_3, C_3\}$  are all-mode detectable and isolable. Finally,  $V_{in}$  is all-mode detectable and weakly isolable at mode  $i = 1$ .

### 5. QUANTITATIVE FDI FOR HYBRID SYSTEMS

This section presents a quantitative fault diagnosis framework called the Quantitative Bond Graph-based (QHBG) FDI system for real-time fault diagnosis. Figure 1 depicts

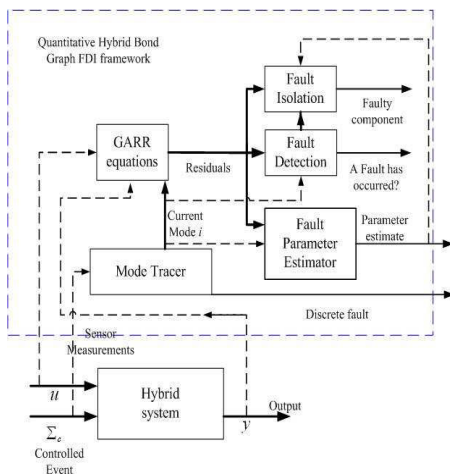


Fig. 3. Architecture of QHBG FDI system

the architecture of the QHBG diagnosis framework. The framework utilizes the global structural information of the HBG to design efficient and effective diagnosis algorithms for hybrid systems. This framework consists of a *GARR alarm generator*, a *fault detection module*, a *fault isolation*, a *fault estimator*, and a *mode tracer*. These modules are developed based on the HBG model of the monitored hybrid system. The mode tracer constantly determines the instantaneous operating mode using sensors and input information obtained from the hybrid system. This instantaneous mode information allows users to evaluate GARR equations for residuals effectively at all operating modes. With these residuals, we are able to detect, isolation, and estimate the size of a fault. The fault detection module decides whether or not a fault has occurred when the residual signals are non-zero. For those isolable parameters, the GARRs generate a set of unique residuals that

allow the faulty parameter to be identified. For a fault that is non-isolable parameter, the module will select a set of potential candidates. Finally, the parameter estimation module estimates the selected fault parameters to provide updated parameter's value. In this paper, we focus on the fault detection and isolation modules.

#### 5.1 Fault detection and isolation modules

The concept of using quantitative residuals for fault diagnosis is also applicable for hybrid systems. For simplicity, we define a binary coherence vector  $C = [c_1, \dots, c_m]$ . Each component  $c_l$  of  $C$  is a boolean variable whose value is obtained by the following simple decision rule:

$$c_l = \begin{cases} 1 & \text{if } |r_l| > \epsilon_l^i; \\ 0 & \text{otherwise} \end{cases} \quad \text{for } l = 1, \dots, m \quad (5)$$

where  $i$  denotes the instantaneous operating mode.

Similarly, when the hybrid system is fault-free, then the binary coherence vector  $C$  will be a zero vector. On the other hand, if the system is faulty, then the coherence vector will be a nonzero vector. Note that the threshold  $\epsilon_l^i$  is mode dependent, i.e., the threshold value changes with mode. This will reduce the tendency of false alarm and increase the fault detection time of the algorithm.

Consider a fault occurs at time  $k$ . To isolate the faulty parameter, we simply match the binary coherence vector  $C$  with the fault signatures of the FSM of the operating mode at time  $k$ . Under the assumption that the system can have only one fault, we can uniquely isolate the faulty parameter.

#### 5.2 A Case study: Simulation results

The electrical circuit presented in section 4.1 is used for simulations to evaluate the QHBG FDI framework developed in this paper. In this simulation, the QHBG is implemented to detect and isolate a fault simulated in the hybrid system. For illustration purposes, we let the nominal fault-free components' parameters  $\theta_n$  be  $\{R_1 = 1\Omega, C_1 = 1F, R_2 = 1\Omega, C_2 = 1F, R_3 = 1\Omega, C_3 = 1, V_{in} = 1\text{volts}\}$ . In this case study, the switching signal of the hybrid system is explicitly known; therefore, the mode tracer module of the framework is replaced by 1 to 1 mapping between the state of the switch to the state of controlled junction  $\mathbf{x} \in X = \{0, 1\}$ . For simplicity, the thresholds  $\epsilon_l^i$  are fixed at 0.01. A fault is simulated in  $R_3$  component where its parametric value changes abruptly from  $1\Omega$  to  $5\Omega$  in two runs. In both runs, the hybrid system has an initial mode  $i = 1$  (Sw=ON) and it switches to mode  $i = 0$  (Sw=OFF) at  $t = 5$  sec. In the first run, the fault is simulated at  $t = 2$  sec. In the second run, the fault started at  $t = 6$  sec. The residuals of the two runs are presented in following figures.

From the FSMs of the system, we have learned that component  $R_3$  is an isolable component where its fault signature is  $[r_1 \ r_2 \ r_3] = [0 \ 0 \ 1]$  for operating modes  $\forall i \in \{0, 1\}$ . That implies that we expect a change from zero value in  $r_3$ . Figure 4 depicts the simulated responses of the residuals in run 1. From the figure, we observe residual  $r_3$  changes significantly at  $t = 2$  sec. This is due to the

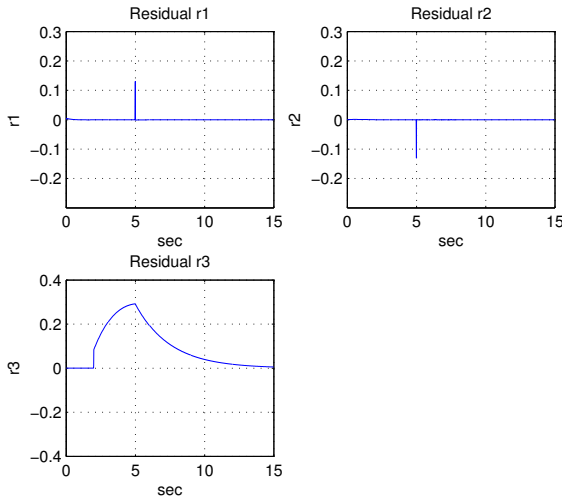


Fig. 4. Response of residuals in run 1

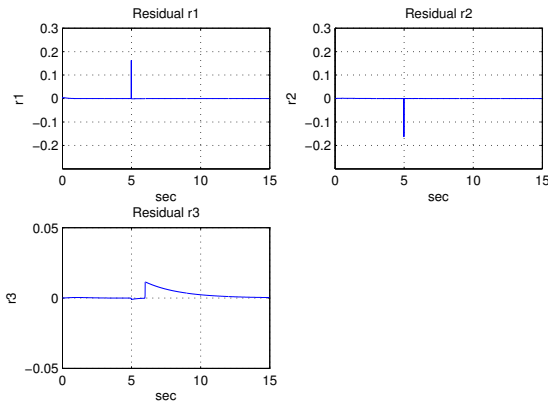


Fig. 5. Response of residuals in run 2

fault simulated at  $t = 2$  sec. The responses indicate the coherence binary vector at the time instant is  $C = [0 \ 0 \ 1]$  where the faulty component can be uniquely isolated from its FSM. The glitches shown in residuals  $\{r_1, r_2\}$  are due to the numerical differentiation error at  $t = 5$  sec which can be ignored during every mode change.

Figure 5 depicts the residuals of run 2 where the fault is simulated at  $t = 7$  sec when the system is at mode 0. Likewise in run 1, we expect the values of  $r_1$  and  $r_2$  to be negligible throughout the operation and residual  $r_3$  deviates from zero. The simulation results shown in Figure 5 confirms this behavior. One important point to note is that the magnitude of  $r_3$  due to the fault is significantly different at the two modes. This is due to the magnitude differences of the known variables in the GARR<sub>3</sub> between the two modes. It suggests that the threshold  $\epsilon_3^0$  can be chosen smaller than its counterpart  $\epsilon_3^1$  in order to have a less conservative and more reliable coherence binary vector  $C$  for fault detection and isolation.

In this study, we show that the component  $R_3$  is detectable and isolable at all modes based on the GARR equations. This results is consistent with the monitoring ability shown in Table 5. One physical implication of an all-mode detectable parameter is that its corresponding component requires energy interaction at all times to remain detectable.

This observation matches with our intuition since there is no way we are able to monitor the behavior of the component based on its constitutive equation and sensor if the component does not operate. An energy interaction within the component is essential for fault detectability and isolability.

## 6. CONCLUSIONS

In this paper, we develop an efficient monitoring ability analysis method and quantitative fault diagnosis system for hybrid systems based on hybrid bond graph (HBG). The developed methods are based on a set of unified constraint relations called Global Analytical Redundancy Relations (GARRs) which can be derived systematically from the Diagnostic Hybrid Bond Graph (DHBG) which we developed in part I of our work [1]. The efficient computation of the unified GARRs facilitates real-time implementation of the quantitative FDI system. Simulation based on an electrical hybrid system validates the quantitative fault diagnosis system.

## REFERENCES

- [1] C.B. Low, D. Wang, S. Arogeti, Z.J. Bing, Causality assignment and model approximation for quantitative hybrid bond graph-based fault diagnosis, In International Federation of Automatic Control (IFAC), submitted, Korea, 2008.
- [2] D.C. Karnopp. System dynamics: modeling and simulation of mechatronic systems. John Wiley, 2006.
- [3] M. Tagina, J. P. Cassar, G. D. Tanguy, M. Staroswiecki. Monitoring of Systems Modelled By Bond-Graphs. *International Conference on Bond Graph Modeling (ICBGM'95)*, pages 275–279, Las Vegas, 1995.
- [4] B.O. Bouamama, A.K. Samantaray, M. Staroswiecki, G. Dauphin-Tanguy. Derivation of Constraint Relations from Bond Graph Models for Fault Detection and Isolation. *International Conference on Bond Graph Modeling (ICBGM'03)*, pages 104–109, 2003.
- [5] A.K. Samantaray, K. Medjaher, B.O. Bouamama, M. Staroswiecki, G. Dauphin-Tanguy, Diagnostic bond graphs for online fault detection and isolation, *Simulation Modelling Practice and Theory*, volume 14, pages 237-262, 2006.
- [6] B.O. Bouamama, K. Medjaher, M. Bayart, A.K. Samantaray, B. Conrard. Fault detection and isolation of smart actuators using bond graphs and external model, *Control Engineering Practice*, volume 13, pages 159–175, 2005.
- [7] S. Narasimhan, G. Biswas. Model-based diagnosis of Hybrid Systems, *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, volume 37(3), pages 348–360, May, 2007.
- [8] P.J. Mosterman, G. Biswas, "A Theory of discontinuities in Physical System Models," *Journal of Franklin Institute*, 335(B), pp:401-439, 1998.