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Stabilization of Networked Control Systems with Random Delays: A New Multirate Method ⋆

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Abstract: In this paper, stability issues for a class of Networked control systems (NCSs) with random delays is discussed where random delays are less than one sensor period or more than one sensor period but bounded. A new multirate method is proposed to formulate the union model for both short and long random delays. The sufficient conditions on the existence of stabilizing controllers are established when the transition probability matrix is known. V-K iteration approach is employed to calculate mode-independent and mode-dependent state-feedback gains.

1. INTRODUCTION

By Networked Control Systems (NCSs), we mean feedback control systems where networks, typically digital bandlimited serial communication channels, are used for the connections between spatially distributed system components like sensors and actuators to controllers see Lin et al. (2003). One defining feature of NCSs is that, instead of hardwiring the control devices with point to point connections, sensor, actuators, and controllers are all connected to the network as nodes. The primary advantages of NCSs are low cost, reduced system wiring, simple installation and maintenance, high reliability and ease of system diagnosis and maintenances see Bushnell (2001), Hu et al. (2003) and walsh et al. (2001). As a result, NCSs have been widely applied to many complicated control systems, such as aviation and aerospace fields, airplane manufacture see walsh et al. (2002).

Many researchers have paid attention on the study of the stability controller design for stabilization and performance achievement purposes for network control systems under the existence of network-induced delay. A stabilization problem of network control system is investigated by nilsson et al. (1998) when the network-induced delay is less than one sampling time. By using augmented state-space method, Xiao et al. (2000) converts a stabilization problem of NCSs with random delays into a stabilization problem of jump linear system governed by Markov chains such that the closed-loop system is a jump linear system. Under the frame of Markov characteristic of delay, Zhu et al. (2005) analyzes the stability of NCSs and gives the sufficient and necessary conditions of stochastic stability for NCSs. Zhang et al. (2005) considers the stabilization problem of NCSs on the condition of the sensor-to-controller and controller-to-actuator delays are modeled as two Markov chains. However, they are all under the assumption that NCSs work in the single rate mode. (Single rate mode mean states of plant sampled by sample period T_s and states of controllers by T_c are sampled by ideal samplers with the same sampling period, that is $T_s = T_c$, while the multirate mode means $T_s \neq T_c$).

NCSs worked in the single rate mode has some advantages such as simpler controller design and fewer possible jump states. However, it also has a disadvantage, that is, it can not map random delay to jump states exactly,which leads to weaker stable performances of system. For the purpose of exact corresponding, there are two choices, one is using faster sampling period under the single rate mode. However, as the sampling frequency increases so does the network congestion and hence the network induced delay and possibly packet loses; the other is multirate mode, in which the controller frequency is high enough to get a good response and the sensor frequency is low enough to avoid the loss of information. The study of multirate sampled-data systems has scored a great success in the past several years see Izadi et al. (2005), Hu et al. (2006),Wang et al. (2004) and Izadi et al. (2006). Using multirate method, Lin et al. (2003) consider stability and disturbance attenuation issues for a class of NCSs in the framework of switched systems when random delay less than one sample period. Georgiev et al. (2006) use multipoint packets to reduce network traffic and computation time of NCSs and the control problem for the multipoint-packet system is shown to equivalent to a multirate control problem, which is reduced to a synthesis problem with a constraint on the feedthrough matrix.

In this paper, we consider stabilization of NCSs with random induced-delay, wrong order of data packets and packet dropout. Using multirate method, a stabilization problem of NCSs with random delays that τ less than sensor sample period T_s or more than one sensor sample period T_s and less than nT_s (n is a finite positive integer) are treated as a stabilization problem of jump linear system. Then we develop robust multirate sampled-data control procedures for these jump systems.

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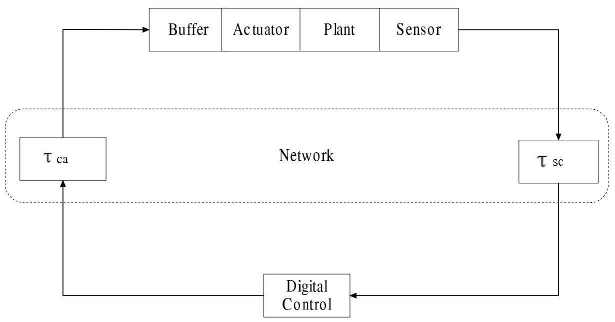

Fig. 1. Network control system with random delay

The organization of this paper is as follows. section 2 presents the mathematic model of NCSs with less than one sample period and more than one sample period but bounded delays separately. In section 3, a multirate controller is proposed to stabilize this control system. A simulation of network control of a cart and inverted pendulum with pendulum with short or long delays is shown in section 4 and conclusions are then followed in section 5.

2. NETWORKED CONTROL SYSTEM MODELING

Consider the NCSs in Fig.1,

The model of the NCSs discussed in this paper is shown in Fig. 1. For simplicity, only consider a random delays existing in feedback loop of sensor to controller, that is to say $\tau_{ca} = 0$.

We assume that the plant can be modeled as a continuous-time linear time-invariant system described by

$$\begin{cases} \dot{x}(t) = A^c x(t) + B^c u(t) \\ y(t) = C^c x(t) \end{cases} \quad (1)$$

where $x(t) \in \Re^n$ is the state variable, $u(t) \in \Re^m$ is control input, and $y(t) \in \Re^p$ is the controller output. $A^c \in \Re^{n\times n}, B^c \in \Re^{n\times m}$ and $C^c \in \Re^{p\times n}$ are the output matrix. For this NCSs, it is assumed that the plant output nodes (sensors) are clock-driven with sampling period T_s and that the actuator is also clock-driven with sampling period T_c. The controller reads the buffer periodically at a high frequency than the sampling frequency, say every $T_c = \frac{T_s}{N}$ for some integer N large enough. Whenever there is a new data in the buffer, then the controller will calculate the new control signal and transmit to the actuator. This proposed solution uses a multirate control loop, in which the actuation frequency is high enough to get a good response and the sensor frequency is low enough to avoid the loss of information.

When the network is inserted, the continuous system transforms to a partly discrete-time system. Because we do not assume the synchronization between the sampler and the digital controller, the control signal is no longer of constant value within finite sampling period. Therefore a sampling period has to be divided into subintervals corresponding to the controller's reading buffer period,$T = \frac{T_s}{N}$. Hence the continuous-time plant may be discretized into the following sampled-data systems

$$\begin{cases} x(k+1) = Ax(k) + \underbrace{[\,B\ B\ \cdots\ B\,]}_{N} \begin{bmatrix} u^1(k) \\ u^2(k) \\ \vdots \\ u^N(k) \end{bmatrix} \\ y(k) = Cx(k) \end{cases} \quad (2)$$

where $A = e^{A^c T_s}, B = \int_0^{\frac{T_s}{N}} e^{A^c t} B^c dt$ and $C = C^c$. N denote the number of subintervals which one sensor sampling period can be divided into. Note that for linear time-variant plant and constant-periodic sampling, the matrix A and B are constant.

2.1 Model of random delay which less than one sampling period

When random delay of NCSs is less than one sensor sampling period, the number d in (2) equals to 1. Let mode-independent feedback state controller of system is

$$u(k) = Kx(k - \tau_k), \tau_k \in \{0, 1\} \quad (3)$$

If we augment the state variable (Here, 0 and 1 express the multiple of sensor sampling period T_s),

$$\hat{x}(k) = \left[\, x^\top(k)\ x^\top(k-1) \,\right]^\top$$

during each sensor sampling period, the system is formulated as

$$\begin{cases} \hat{x}(k+1) = (\hat{A} + \hat{B}K)\hat{x}(k) \\ \hat{y}(k) = \hat{C}\hat{x}(k) \end{cases} \quad (4)$$

where

$$\hat{A} = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix}, \hat{B} = \begin{bmatrix} \gamma B & (N-\gamma)B \\ 0 & 0 \end{bmatrix}, \hat{C} = [\, C\ 0 \,]$$

and $C = C^c, \gamma \in \{0, 1, \cdots, d_s\}$, (here, $0, 1, \cdots, d_s$ denote the multiple of controller sampling period T_c). $0 \le d_s \le N$ expresses the maximum allowable delay. Note that $\gamma = 0$ implies $\tau_{sc} = T_s$, which corresponds to the previous "package dropout", while $\gamma = N$ implies $\tau_{sc} = 0$,which corresponds to the previous "no delay".

2.2 Model of long random delay

It is reasonable to assume that induced delay is random but bounded delay, which is integer multiple of the sensor sampling period. That is to say, $0 \le \tau_{sc} \le d_s \le \infty$ and d_s denotes the largest delay of τ_{sc}, measured by T_s. Because we have assumed that random delay of sensor to controller τ_{sc} has upper bounded, mode-dependent feedback state controller of system is

$$u(k) = K_{r_s(k)} x(k - r_s(k)) \quad (5)$$

where $r_s(k)$ is a bounded random integer sequence with $0 \le r_s(k) \le d_s < \infty$, and d_s is a finite delay. If we augment the state variable

$$\tilde{x}(k) = \left[\, x^\top(k)\ x^\top(k-1)\ \ldots\ x^\top(k-d_s) \,\right]^\top$$

where $\tilde{x}(k) \in \Re^{(d_s+1)\times n}$,then the closed-loop system is

$$\tilde{x}(k+1) = (\tilde{A} + \tilde{B}K_{r_s(k)})\tilde{x}(k) \quad (6)$$

where $\tilde{x}(k) \in \Re^{(d_s+1)n}$, then the closed-loop system is

$$\begin{cases} \tilde{x}(k+1) = (\tilde{A} + \tilde{B}K_{r_s(k)})\tilde{x}(k) \\ \tilde{y}(k) = \tilde{C}\tilde{x}(k) \end{cases} \tag{7}$$

$$\tilde{A} = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} \alpha_0 B & \alpha_1 B & \cdots & \alpha_{d_s} B \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\tilde{C} = [\, C \; 0 \; \cdots \; 0 \,]$$

and $C = C^c, \sum_{i=0}^{d_s} \alpha_i = d_s \times N$. Note that $\alpha_0 = d_s N$ implies $\tau_{sc} = 0$, which corresponds to the previous "no delay". It is clear that we can express different cases including package loss and bounded delay in NCSs through different combination of $\alpha_i, i \in \{0, 1, \cdots, d_s\}$. In the next section, we will formulate the above NCSs as a class of discrete-time jump linear systems.

3. STABILITY ANALYSIS

Motivated by the above analysis of NCSs whether short random delay or long random delay, we can describe them as a uniform form—a family of discrete-time linear systems described by the following difference equations

$$\tilde{x}(k+1) = \bar{A}_q \tilde{x}(k) \tag{8}$$

where $x(k) \in \Re^n$ is the state variable, and $\bar{A}_q \in \Re^{n \times n}$ is constant matrix indexed by $q \in U$, where the finite set $U = \{q_1, q_2, \cdots, q_n\}$ is called the set of modes.

For the NCSs which random delay is less than one sampling period, we can easily find all of its modes. From section 2.1 we can see, if $d_s < N$, this system has $d_s + 2$ different modes and if $d_s = N$, there are $N + 1$ modes. When random delay in NCSs is long delay (this is more common), the whole modes cannot be easily determined. So we develop Theorem 1 to calculate the whole number of modes.

Theorem 1. If closed system (7) has the maximum allowable delay $d_s T_s$ and controller sampling period $T_c = \frac{T_s}{N}$, then the whole modes of this system are

$$\sum_{i=1}^{min(d_s,N)} C_{d_s+1}^{i+1}[C_N^i - C_{N-1}^{i-1}] + C_{d_s+1}^1 \tag{9}$$

where i denotes the number of state signal existing in one sensor sampling period T_s.

For illuminate theorem 1 clearly, we introduce a definition as follows

Definition 1 If system (7) has sensor sampling period T_s and controller sampling period $T_c = \frac{T_s}{N}$, we call different combinations of finite state signals in N subintervals of T_s(every subinterval has only one state signal) as a class mode.

For example, if N equals to 3 and d_s equal to 2, then we have 7 class modes, which are shown in Fig. 2.

Proof. Here, we assume that controller is time-driven.

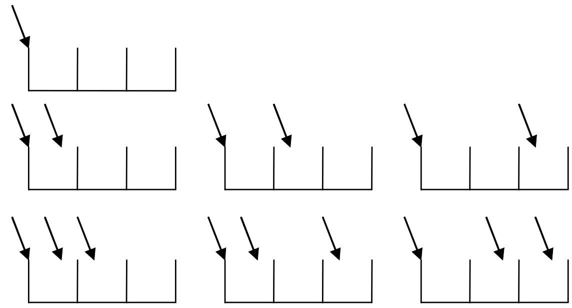

Fig. 2. Total numbers of class modes when $N = 3$ and $d_s = 2$

If $d_s < N$, the total number of class modes are

$$C_N^1 + C_N^2 + \cdots + C_N^{d_s}$$

Because the state signal which lies in subinterval $[(N-1)T_c, NT_c]$ of sensor sampling period does not action in interval $[kT_s, (k+1)T_s]$, but does action in subinterval $[0, T_c]$ of next sensor sampling period under time-driven mode, those class modes which have state signal in subinterval $[(N-1)T_c, NT_c]$ equals to those corresponding class modes which have not state signal in the last subinterval of sensor sampling period. We should take away those redundant class modes. They can be calculated by

$$C_{N-1}^1 + C_{N-1}^2 + \cdots + C_{N-1}^{d_s-1}$$

and plus one class mode which has not state signal in sensor sampling period , then different class modes are

$$\sum_{i=1}^{d_s}[C_N^i - C_{N-1}^{i-1}] + 1$$

If $d_s \geq N$, the total number of class modes are

$$C_N^1 + \cdots + C_N^N + C_N^{N+1} + \cdots + C_N^{d_s}$$

For $C_N^i, i \in \{N+1, \cdots, d_s\}$, numbers of state signals are larger than those of subintervals, which implies that at least existing a subinterval has more than one state signal. According to assumption that only the latest state signal is effective when more than one state signals in the same subinterval, these class modes equals to corresponding class modes which only preserve the latest state signal in those subinterval existing more than one signals. So the total class modes are

$$C_N^1 + C_N^2 + \cdots + C_N^N$$

and redundant class modes among them are

$$C_{N-1}^1 + C_{N-1}^2 + \cdots + C_{N-1}^{N-1}$$

and plus one class mode, the whole different class modes are

$$\sum_{i=1}^{N}[C_N^i - C_{N-1}^{i-1}] + 1$$

to sum up ,we get

$$\sum_{i=1}^{min(d_s,N)} [C_N^i - C_{N-1}^{i-1}] + 1$$

Among every class mode, combinations of different state signals make different modes, so every class mode has a set of modes, which can be given through formula $C^{i+1}_{d_s+1}$. So the total modes in one sensor sampling period equal to (9).

then this completes the proof of the theorem.

Remark 1: If controller is event-driven, the same results can also get.

Remark 2: From Fig.1 we can see, there is a buffer before actuator. Control signal in buffer is updated every controller sampling period T_c and we assume that only the latest control signal could be conserved. Using this method, package dropout and package transmission disorder phenomenon can also change to corresponding random delay. It is clearly that as the N increases, the effect using time-driven or event-driven controller under assumption mentioned above close to that using ideal event-driven controller. When $N \rightarrow \infty$, these two different approaches are equal.

Compared to (7) and (8), we can define the following linear time-varying system as s discrete-time jump linear system

$$\begin{cases} \tilde{x}(k+1) = \bar{A}_q \tilde{x}(k) \\ \tilde{y}(k) = \tilde{C}\tilde{x}(k) \end{cases} \tag{10}$$

where the signal q is called jumping signals. If random delay is less than one controller sampling period, then $\tilde{C} = [\, C \;\, 0 \,]$ and the set of U is given by $U = \{0, 1, \cdots .d_s, N\}$; If random delay is more than one controller sampling period, then $\tilde{C} = [\, C \; 0 \; 0 \cdots 0 \,]$, $q \in U$ can be calculated by theorem 1. Here, $\bar{A}_q = (\hat{A} + \hat{B}_q K)$ or $\bar{A}_q = (\tilde{A} + \tilde{B}_q K)$ and matrix variables $\hat{A}$, $\hat{B}_q$ and $\tilde{A}$, $\tilde{B}_q$ have the same meaning as (4) and (7) respectively. As for gains K, if we choose mode-independent controller, then $K = diag\{K, K, \cdots, K\}$; if we choose mode-dependent controllers, then $K = diag\{K_0, K_1, \cdots, K_{d_s}\}$.

As to the mean square stability of system (10), we develop the theorem 2 as follows

Theorem 2. Assume the jumped modes is decided by theorem 1 and the jump rules of jump linear system governed by transition probability matrix P of NCSs, then system (10) is mean square stability, if symmetric positive definite matrixes are found to satisfy

$$\sum_{i=0}^{q_{max}} p_{ji} \bar{A}_i^\top Q_j \bar{A}_i < Q_j, j = \{0, \cdots, q_{max}\} \tag{11}$$

where $p_{ji} \in P$,q_{max} denotes the maximum values of the set U of decided by theorem 1.

Proof. We use following Lyapunov function:

$$V(\tilde{x}(k), k) = \tilde{x}(k)^\top Q(\tau_k)\tilde{x}(k)$$

then we have

$$\begin{aligned} & E\left\{\Delta(\tilde{x}(k), k)\right\} \\ & = E\{\tilde{x}(k+1)^\top Q(\tau_{k+1})\tilde{x}(k+1) \mid \tilde{x}(k), \tau_k = i)\} \\ & - \tilde{x}(k)^\top Q(\tau_k)\tilde{x}(k) \\ & = E\{\tilde{x}(k)^\top \bar{A}_{\eta(k)}^\top Q(\tau_k)\bar{A}_{\eta(k)}\tilde{x}(k+1) \mid \tilde{x}(k), \tau_k = i)\} \end{aligned}$$

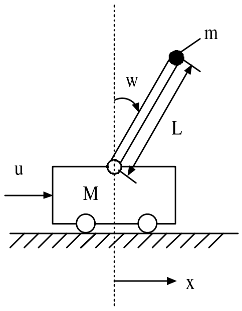

Fig. 3. Cart and inverted pendulum

$$\begin{aligned} & - \tilde{x}(k)^\top Q(i)\tilde{x}(k) \\ & = \tilde{x}(k)^\top \bar{A}_i^\top \sum_{j=0}^{q_{max}} p_{ij}\bar{A}_i \tilde{x}(k) \\ & - \tilde{x}(k)^\top Q_i \tilde{x}(k) \\ & = \tilde{x}(k)^\top \{\bar{A}_i^\top \sum_{j=0}^{q_{max}} p_{ij}\bar{A}_i - Q_i\}\tilde{x}(k) \end{aligned}$$

It is obvious that to ensure mean square stability we must satisfy

$$\bar{A}_i^\top \sum_{j=0}^{q_{max}} p_{ij}\bar{A}_i - Q_i < 0, i = \{0, \cdots, q_{max}\}$$

Compared theorem 3.1 of Xiao et al. (2000) we can get (11).

This completes the proof of the theorem.

Remark 3: The difficulty in this theorem is that the size of transition probability matrix P increase rapidly as the increase of maximum allowable delay and subinterval N in sampling period T_s and resolution of transition probability matrix P become harder and harder.

Using Theorem 2 and V-K iteration algorithm from Xiao et al. (2000), we can design mode-dependent or mode-independent controllers which make NCSs mean square stable.

4. NUMERICAL EXAMPLES

In this section, we use a numerical example to demonstrate the design procedure of multirate sampled-data control for the NCSs governed by Markovian jump system in single rate mode.

Example 1:Consider the cart and inverted pendulum problem in Fig.3, where M is the cart mass, L is the pendulum mass, m is the length from the point of rotation to the center of gravity of the pendulum, L is the cart position, x is the pendulum angular position, and w is the input force. The the state variables are $x_1 = w, x_2 = \dot{w}, x_3 = x$ and $x_4 = \dot{x}$ and assumption $M = 1.096kg, m = 0.109kg, L = 0.25m$. The sensor sampling time is $T_s = 0.03s$. Here, assume that random delay of NCSs is less than one sensor sampling period. Here, let controller sampling time is $T_c = \frac{1}{3}T_s, d_s = 2$, and $N = 3$.

The mode-independent controller is designed in discrete model,which is linearized when the pendulum is in the up-position ($w = 0$) .The state-space model is

$$x(k+1) = A_d x(k) + B_d u(k) \tag{12}$$

where

$$A_d = \begin{bmatrix} 1.0195 & 0.0302 & 0 & 0 \\ 1.3013 & 1.0195 & 0 & 0 \\ -0.0004 & -0.00001 & 1.0000 & 0.0300 \\ -0.0294 & -0.0004 & 0 & 1.0000 \end{bmatrix},$$

$$B_d = [\,-0.0002\ \ -0.0365\ \ 0.0000\ \ 0.0091\,]^\top$$

The cost function is given by

$$J = \sum_{k=0}^{\infty} (\frac{1}{2} x_k^\top T x(k) + u_k^\top u_k)$$

where $T = diag(1,1,1,1)$.
Using LMI tools, we design a LQR controller for the jump system. That is

$$K = [\,31.9183\ \ 4.9619\ \ 0.8835\ \ 2.1165\,]$$

It is clear that system has 4 modes, they are

$$A_q = \hat{A} + \hat{B}_q K, q = \{0,1,2,3\}$$

$$\hat{A} = \begin{bmatrix} A_d & 0 \\ I & 0 \end{bmatrix}, \hat{B}_0 = \begin{bmatrix} 3B_d & 0 \\ 0 & 0 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} 2B_d & B_d \\ 0 & 0 \end{bmatrix}$$

$$\hat{B}_2 = \begin{bmatrix} B_d & 2B_d \\ 0 & 0 \end{bmatrix}, \hat{B}_3 = \begin{bmatrix} 0 & 3B_d \\ 0 & 0 \end{bmatrix}$$

Assume initial delay distribution probability is π_0 and state transition probability matrix is P_E , delay distribution probability at sensor sampling time t_k is $\pi_k = \pi_0 P_E^k$.Let initial delay distribution probability is $\pi_0 = [\,0.3\ 0.3\ 0.3\ 0.1\,]$, initial state is $x_0 = [\,0.15\ 0\ 0\ 0\,]$, with the method that have developed from Zhu et al. (2005), state transition probability matrix is

$$P_E = \begin{bmatrix} 0.1 & 0.4 & 0.3 & 0.2 \\ 0.1 & 0.4 & 0.3 & 0.2 \\ 0.1 & 0.4 & 0.3 & 0.2 \\ 0.1 & 0.4 & 0.3 & 0.2 \end{bmatrix}$$

and initial controller is LQR controller K, we can get mode-independent controllers satisfied mean square stability of system. That is

$$K = [\,19.3228\ \ 3.8586\ \ 1.0109\ \ 3.4306\,]$$

Assume that random delay of NCSs is more than one sensor sampling period. Here, let controller sampling time is $T_c = \frac{1}{2}T_s, N = 2$, and the maximum allowable delay $d_s = 2$.

The mode-dependent controller is designed in discrete model,which is linearized when the pendulum is in the up-position $w = 0$. The state-space model is the same as (12), where

$$A_{d1} = \begin{bmatrix} 1.0195 & 0.0302 & 0 & 0 \\ 1.3013 & 1.0195 & 0 & 0 \\ -0.0004 & -0.00001 & 1.0000 & 0.0300 \\ -0.0294 & -0.0004 & 0 & 1.0000 \end{bmatrix},$$

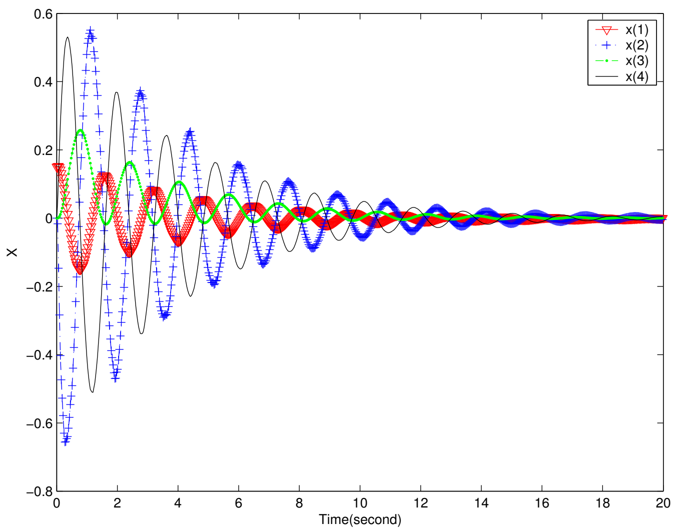

Fig. 4. States of closed-loop system when random delay less than one sensor period

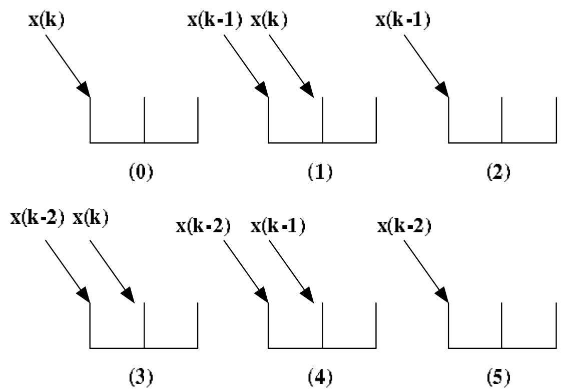

Fig. 5. Total modes under multirate mode

$$B_{d1} = [\,-0.0004\ \ -0.0548\ \ 0.0001\ \ 0.0137\,]^\top$$

According to theorem 1, we have 6 modes as

and

$$A_q = \tilde{A} + \tilde{B}_q K, q \in \{0,1,2,3,4,5\}.$$

$$\tilde{A} = \begin{bmatrix} A_{d1} & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}, \tilde{B}_0 = \begin{bmatrix} 2B_{d1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{B}_1 = \begin{bmatrix} B_{d1} & B_{d1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{B}_2 = \begin{bmatrix} 0 & 2B_{d1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{B}_3 = \begin{bmatrix} B_{d1} & 0 & B_{d1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{B}_4 = \begin{bmatrix} 0 & B_{d1} & B_{d1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{B}_5 = \begin{bmatrix} 0 & 0 & 2B_{d1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let initial delay distribution probability is $\pi_0 = [\,0.2\ 0.2\ 0.2\ 0.2\ 0.1\ 0.1\,]$, initial state is $x_0 = [\,0.15\ 0\ 0\ 0\,]$, expected state transition probability matrix is

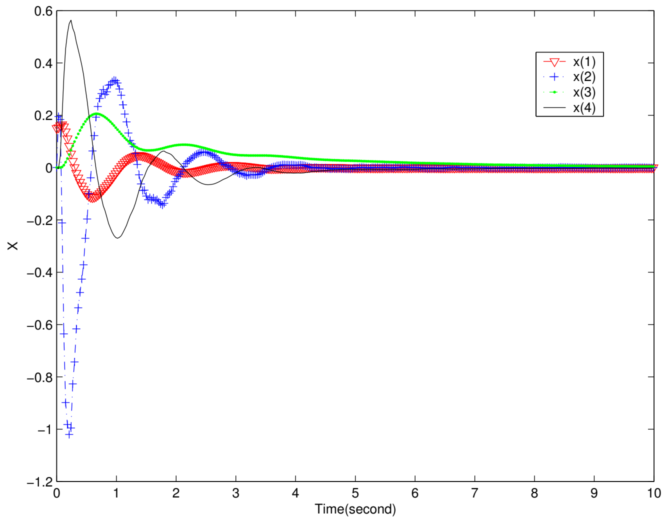

Fig. 6. States of closed-loop system when random delay more than one sensor period

$$P_E = \begin{bmatrix} 0.1 & 0.2 & 0.7 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.7 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0.7 & 0 \\ 0.1 & 0.2 & 0.7 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0.7 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0.7 & 0 \end{bmatrix}$$

And we design a LQR controller for the switched system without random delay (use $\tilde{B}_0$). That is

$$K = [\,31.8789 \;\; 4.9557 \;\; 0.9073 \;\; 2.1566\,]$$

and initial controller is LQR controller K, we can get mode-dependent controllers satisfied mean square stability of system as follows

$$K = \begin{bmatrix} K_0 & 0 & 0 \\ 0 & K_1 & 0 \\ 0 & 0 & K_2 \end{bmatrix}$$

and

$$\begin{aligned} K_0 &= [\,27.5193 \;\; 6.0256 \;\; 1.1762 \;\; 5.3342\,] \\ K_1 &= [\,26.0471 \;\; 7.0636 \;\; 2.1397 \;\; 7.0255\,] \\ K_2 &= [\,23.3650 \;\; 3.2273 \;\; 1.5858 \;\; 2.5774\,] \end{aligned}$$

The state trajectories of the closed-loop system caused by the discrete model and the obtained controllers are shown in Fig.6. It can be seen that the closed-loop system is mean square stable.

5. CONCLUSION

This note has presented a multirate method for the stabilization of a class of networked control system with random communication delays. By modeling analysis, the closed-loop systems can be expressed as jump systems. A new multirate method is proposed to formulate the union model for both short and long random delays. The sufficient conditions on the existence of stabilizing controllers are established when the transition probability matrix is known. A numerical example has been considered to illustrate the main results.Future work will focus on how to get expected transition probability matrix P_E under different multirate mode and comparison of performances between single rate mode and multirate mode.

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