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Bias-compensation based method for errors-in-variables model identification

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Abstract: It is well known that least-squares (LS) method gives biased parameter estimates when the input and output measurements are corrupted by noise. One possible approach for solving this bias problem is the bias-compensation based method such as the bias-compensated least-squares (BCLS) method. In this paper, a new bias-compnesation based method is proposed for identification of noisy input-output system. The proposed method is based on compensation of asymptotic bias on the instrumental variables type (IV-type) estimates by making use of noise covariances estimates. In order to obtain the noise covariances estimates, an overdetermined system of equations is introduced, and the noise covariances estimation algorithm is derived by solving this overdetermined system of equations. From the combination of the parameter estimation algorithm and the noise covariances estimation algorithm, the proposed bias-compensated instrumental variables type (BCIV-type) method can be established. The results of a simulated example indicate that the proposed algorithm provides good estimates.

Keywords: Estimation; Identification; Errors-In-Variables Model

1. INTRODUCTION

Recently, consistent estimation methods for identification of linear discrete-time system in the presence of input and output noises, which is usually called "errors-in-variables" (EIV) model, have received much attention because of its important applications in signal processing, communications and control systems.

Several methods have been proposed to estimate unknown parameters of EIV model. Joint Output (JO) method (Söderström [1981]) and Koopmans-Levin (KL) method (Fernando and Nicholson [1985]) require *a priori* knowledge about the values of variances or the ratio to measurements noises.

Bias-compensated least-squares (BCLS) method is proposed by Sagara et al. (Sagara and Wada [1977]) and it has been extended by Wada et al. (Wada et al. [1990]) to the input-output noise case without any *a priori* knowledge of noise variances. BCLS method based on compensation of asymptotic bias on the least-squares (LS) estimates by making use of noise variances estimates is very efficient method for estimation of noisy input-output system parameters. In recent years, BCLS method has been developed to improve the estimation accuracy and several recursive algorithms have been proposed (Jia et al. [2001], Ikenoue et al. [2005]).

On the other hand, another method named bias-eliminated least-squares (BELS) method has been proposed by Zheng et al. (Zheng and Feng [1989]) in which the different estimation method of asymptotic bias is used and further developed to be the efficient method (Zheng [1999, 2002]) to treat bias problem in noisy input-output system identification.

In this paper, a new bias-compnesation based method is proposed for identification of EIV model in the case where the input and output measurements are corrupted by colored noise. The proposed method is based on compensation of asymptotic bias on the instrumental variables type (IV-type) estimates by making use of noise covariances estimates. In order to obtain the noise covariances estimates, an overdetermined system of equations is introduced, and the noise covariances estimation algorithm is derived by solving this overdetermined system of equations. From the combination of the parameter estimation algorithm and the noise covariances estimation algorithm, the proposed bias-compensated instrumental variables type (BCIV-type) method can be established. The results of a simulated example indicate that the proposed algorithm provides good estimates.

This paper is organized as follows. In section 2, the problem statement is presented and the BCLS method is described. In section 3, the IV-type estimator is introduced and the BCIV-type estimator is derived for estimating unknown parameters of EIV model and it can be learned that the unknown noise covariances must be estimated in order to obtain consistent estimates of parameters. In section 4, the noise covariances estimation algorithm is derived by solving the overdetermined system of equations, and the BCIV-type method is established. Moreover, the recursive BCIV-type algorithm is described. The simulation results are presented in section 5 and finally section 6 gives the conclusion.

2. PROBLEM STATEMENT

Consider the parameter estimation problem of single-input single-output linear discrete-time system described as follows:

$$A(q^{-1})y_t = B(q^{-1})u_t \tag{1}$$

where u_t and y_t are the true input and output, q^{-1} is shift operator, $q^{-1}u_t = u_{t-1}$, and the polynomials $A(q^{-1})$ and $B(q^{-1})$ are defined by

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n} \tag{2}$$

$$B(q^{-1}) = b_1q^{-1} + \cdots + b_nq^{-n} \ . \tag{3}$$

It is assumed that $A(z)$ has all zeros outside the unit circle and $A(z)$ and $B(z)$ have no common factors. Let z_t and w_t be the noise-corrupted measurements of y_t and u_t, respectively, i.e.

$$z_t = y_t + e_t, \ w_t = u_t + d_t \tag{4}$$

where e_t is the output measurement noise and d_t is the input measurement noise. The measurement noises e_t and d_t are assumed to be zero-mean colored noise with unknown covariances

$$r_{ee}(k) = E[e_te_{t-k}], \ (k = 0, \ \pm 1, \ \pm 2, \cdots) \tag{5}$$

$$r_{dd}(k) = E[d_td_{t-k}], \ (k = 0, \ \pm 1, \ \pm 2, \cdots) \tag{6}$$

and described as

$$e_t = H_e(q^{-1})\omega_{e,t} \tag{7}$$

$$d_t = H_d(q^{-1})\omega_{d,t} \tag{8}$$

where $E[\cdot]$ stands for mathematical expectation, $\omega_{e,t}$ and $\omega_{d,t}$ are zero-mean white noises with unknown variances $\sigma^2_{\omega_e}$ and $\sigma^2_{\omega_d}$, and $H_e(q^{-1})$, $H_d(q^{-1})$ take up any form of rational function. The true input u_t is a zero-mean stationary random process with finite variance, and u_t, d_t and e_t are assumed to be statistically independent of each other.

Substituting (4) into (1) yields

$$A(q^{-1})z_t = B(q^{-1})w_t + v_t \tag{9}$$

where v_t is a composite noise defined by

$$v_t = A(q^{-1})e_t - B(q^{-1})d_t \ . \tag{10}$$

Define some vectors as

$$\boldsymbol{\theta}^T = [\boldsymbol{a}^T, \ \boldsymbol{b}^T] = [a_1 \cdots a_n, \ b_1 \cdots b_n] \tag{11}$$

$$\begin{aligned}\boldsymbol{p}_t^T &= [-\boldsymbol{z}_t^T, \ \boldsymbol{w}_t^T] \\ &= [-z_{t-1} \cdots - z_{t-n}, \ w_{t-1} \cdots w_{t-n}]\end{aligned} \tag{12}$$

$$\begin{aligned}\boldsymbol{q}_t^T &= [-\boldsymbol{y}_t^T, \ \boldsymbol{u}_t^T] \\ &= [-y_{t-1} \cdots - y_{t-n}, \ u_{t-1} \cdots u_{t-n}]\end{aligned} \tag{13}$$

$$\begin{aligned}\boldsymbol{r}_t^T &= [-\boldsymbol{e}_t^T, \ \boldsymbol{d}_t^T] \\ &= [-e_{t-1} \cdots - e_{t-n}, \ d_{t-1} \cdots d_{t-n}]\end{aligned} \tag{14}$$

then (4), (9) and (10) can be written as

$$\boldsymbol{p}_t = \boldsymbol{q}_t + \boldsymbol{r}_t \tag{15}$$

$$z_t = \boldsymbol{p}_t^T\boldsymbol{\theta} + v_t \tag{16}$$

$$v_t = e_t - \boldsymbol{r}_t^T\boldsymbol{\theta} \ . \tag{17}$$

Let the equation error ξ_t for an estimate $\widehat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$ be defined as

$$\xi_t = \widehat{A}(q^{-1})z_t - \widehat{B}(q^{-1})w_t = z_t - \boldsymbol{p}_t^T\widehat{\boldsymbol{\theta}} \tag{18}$$

where the polynomials $\widehat{A}(q^{-1})$ and $\widehat{B}(q^{-1})$ are defined by

$$\widehat{A}(q^{-1}) = 1 + \widehat{a}_1q^{-1} + \cdots + \widehat{a}_nq^{-n} \tag{19}$$

$$\widehat{B}(q^{-1}) = \widehat{b}_1q^{-1} + \cdots + \widehat{b}_nq^{-n} \tag{20}$$

and

$$\widehat{\boldsymbol{\theta}}^T = [\widehat{\boldsymbol{a}}^T, \ \widehat{\boldsymbol{b}}^T] = [\widehat{a}_1 \cdots \widehat{a}_n, \ \widehat{b}_1 \cdots \widehat{b}_n] \ . \tag{21}$$

Minimizing the sum of squared equation error ξ_t yields the least-squares (LS) estimate of $\boldsymbol{\theta}$

$$\widehat{\boldsymbol{\theta}}_{LS,N} = \widehat{\boldsymbol{R}}_{pp,N}^{-1}\widehat{\boldsymbol{r}}_{pz,N} \tag{22}$$

where

$$\widehat{\boldsymbol{R}}_{pp,N} = \frac{1}{N}\sum_{t=1}^{N}\boldsymbol{p}_t\boldsymbol{p}_t^T \tag{23}$$

$$\widehat{\boldsymbol{r}}_{pz,N} = \frac{1}{N}\sum_{t=1}^{N}\boldsymbol{p}_tz_t \ . \tag{24}$$

From the assumption of e_t and d_t, the composite noise v_t defined by (10) is not white. Hence the LS estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$ has a bias asymptotically. The asymptotic result of the LS estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$ is obtained as

$$\boldsymbol{\theta}_{LS} = \boldsymbol{\theta} - \boldsymbol{R}_{pp}^{-1}\boldsymbol{Q}\boldsymbol{\gamma} \tag{25}$$

where

$$\boldsymbol{\theta}_{LS} = \lim_{N\to\infty}\widehat{\boldsymbol{\theta}}_{LS,N} = \boldsymbol{R}_{pp}^{-1}\boldsymbol{r}_{pz} \tag{26}$$

$$\boldsymbol{R}_{pp} = \lim_{N\to\infty}\widehat{\boldsymbol{R}}_{pp,N} = E[\boldsymbol{p}_t\boldsymbol{p}_t^T] \tag{27}$$

$$\boldsymbol{r}_{pz} = \lim_{N\to\infty}\widehat{\boldsymbol{r}}_{pz,N} = E[\boldsymbol{p}_tz_t] \tag{28}$$

$$\boldsymbol{Q} = \begin{bmatrix}\boldsymbol{Q}_1 + \boldsymbol{Q}_2 \\ \boldsymbol{Q}_3\end{bmatrix} \tag{29}$$

$$\boldsymbol{Q}_1 = \boldsymbol{a}\boldsymbol{i}_1^T + \sum_{j=1}^{n-1}\left[(\boldsymbol{S}_n)^j + (\boldsymbol{S}_n^T)^j\right]\boldsymbol{a}\boldsymbol{i}_{j+1}^T \tag{30}$$

$$\boldsymbol{Q}_2 = [\boldsymbol{0}_n \ \boldsymbol{I}_n \ \boldsymbol{O}_n] \tag{31}$$

$$\boldsymbol{Q}_3 = \boldsymbol{b}\boldsymbol{i}_{n+2}^T + \sum_{j=1}^{n-1}\left[(\boldsymbol{S}_n)^j + (\boldsymbol{S}_n^T)^j\right]\boldsymbol{b}\boldsymbol{i}_{j+n+2}^T \tag{32}$$

$$\boldsymbol{i}_j = \boldsymbol{I}_{2n+1}(:, j) \ (j = 1, \ \cdots, \ 2n+1) \tag{33}$$

$$\boldsymbol{S}_n = \begin{bmatrix}\boldsymbol{0}_{n-1}^T & 0 \\ \boldsymbol{I}_{n-1} & \boldsymbol{0}_{n-1}\end{bmatrix} \tag{34}$$

$$\boldsymbol{\gamma} = [r_{ee}(0), \cdots, r_{ee}(n), r_{dd}(0), \cdots, r_{dd}(n-1)]^T \tag{35}$$

$\boldsymbol{I}_n$ is an $n \times n$ identity matrix, $\boldsymbol{O}_n$ is an $n \times n$ zero matrix and $\boldsymbol{0}_n$ is an $n \times 1$ zero vector.

One possible approach for solving this bias problem is the bias-compensation principle based method such as the bias-compensated least-squares (BCLS) method (Wada et al. [1990], Jia et al. [2001], Ikenoue et al. [2005]) and the bias-eliminated least-squares (BELS) method (Zheng and Feng [1989], Zheng [1999, 2002]). From (25), it can be expected that a consistent estimate of $\boldsymbol{\theta}$ can be obtained by compensating for the asymptotic bias in the LS estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$. Hence the BCLS estimate $\widehat{\boldsymbol{\theta}}_{BCLS,N}$ is given by following equation.

$$\widehat{\boldsymbol{\theta}}_{BCLS,N} = \widehat{\boldsymbol{\theta}}_{LS,N} + \widehat{\boldsymbol{R}}_{pp,N}^{-1}\widehat{\boldsymbol{Q}}_{BCLS,N-1}\widehat{\gamma}_N \tag{36}$$

where $\widehat{\gamma}_N$ denotes the estimate of γ at time instant N, and $\widehat{Q}_{BCLS,N}$ denotes the estimate of Q whose elements are composed of the BCLS estimate $\widehat{\theta}_{BCLS,N}$.

If the noise covariance vector γ is known, the consistent estimate for noisy input-output system can be obtained via (36) simply. But in more general case, the noise covariances are unknown, it is necessary to estimate them firstly. The noise covariances estimation algorithm has been proposed for the case where the input noise is white noise and the output noise is colored noise (Zheng [2002]), and several algorithms have been proposed for the case where the input and output measurements are corrupted by white noise (Wada et al. [1990], Jia et al. [2001], Zheng and Feng [1989], Zheng [1999], Ikenoue et al. [2005]).

3. INSTRUMENTAL VARIABLES TYPE ESTIMATOR

In this section, a new bias-compensation principle based method is considered by using the instrumental variables type estimator. Introduce a vector η_t of dimention $m \geq 2n$. Now, let us consider the instrumental variables type (IV-type) estimate $\widehat{\theta}_{IVt,N}$ defined by the following equation

$$\frac{1}{N}\sum_{t=1}^{N}\eta_t\left(z_t - p_t^T\widehat{\theta}_{IVt,N}\right) = \mathbf{0}_m \ . \tag{37}$$

In general, the IV-type vector η_t has higher dimention than $2n$, (37) gives an overdetermined system and has no exact solution. Solving (37) in a least-squares sense yields

$$\widehat{\theta}_{IVt,N} = \left(\widehat{R}_{\eta p,N}^T W \widehat{R}_{\eta p,N}\right)^{-1}\widehat{R}_{\eta p,N}^T W \widehat{r}_{\eta z,N} \tag{38}$$

where

$$\widehat{R}_{\eta p,N} = \frac{1}{N}\sum_{t=1}^{N}\eta_t p_t^T \tag{39}$$

$$\widehat{r}_{\eta z,N} = \frac{1}{N}\sum_{t=1}^{N}\eta_t z_t \tag{40}$$

and W is a positive definite weighting matrix (no weighting, that is $W = I_m$, is one possible choice). $\widehat{\theta}_{IVt,N}$ exists if $\widehat{R}_{\eta p,N}$ is full rank. The elements of the IV-type vector η_t can be chosen in various ways. Choosing all elements of the IV-type vector η_t as signals uncorrelated with a composite noise v_t, then $\widehat{\theta}_{IVt,N}$ becomes the well-known extended instrumental variable estimate of θ (Söderström and Mahata [2002]). The matrix $\widehat{\Phi}_N$

$$\widehat{\Phi}_N = \widehat{R}_{\eta p,N}^T W \widehat{R}_{\eta p,N} \tag{41}$$

may often become ill-conditioned in the case where all elements of the IV-type vector η_t are chosen as signals uncorrelated with a composite noise v_t, so it is necessary to choose an IV-type vector η_t so that at least one element of the vector η_t is correlated with v_t and hence the matrix $\widehat{\Phi}_N$ becomes well-conditioned. However, choosing at least one element of the vector η_t correlated with v_t, then the IV-type estimate $\widehat{\theta}_{IVt,N}$ has a bias asymptotically.

Substituting (16) into (38) yields

$$\widehat{\theta}_{IVt,N} = \theta + \left(\widehat{R}_{\eta p,N}^T W \widehat{R}_{\eta p,N}\right)^{-1}\widehat{R}_{\eta p,N}^T W \widehat{r}_{\eta v,N} \tag{42}$$

where

$$\widehat{r}_{\eta v,N} = \frac{1}{N}\sum_{t=1}^{N}\eta_t v_t \ . \tag{43}$$

Taking limit of (42) yields

$$\theta_{IVt} = \theta + h_{IVt} \tag{44}$$

where

$$\theta_{IVt} = \lim_{N\to\infty}\widehat{\theta}_{IVt,N} = \left(R_{\eta p}^T W R_{\eta p}\right)^{-1} R_{\eta p}^T W r_{\eta z} \tag{45}$$

$$R_{\eta p} = \lim_{N\to\infty}\widehat{R}_{\eta p,N} = E[\eta_t p_t^T] \tag{46}$$

$$r_{\eta z} = \lim_{N\to\infty}\widehat{r}_{\eta z,N} = E[\eta_t z_t] \tag{47}$$

and h_{IVt} is the asymptotic bias of the IV-type estimate $\widehat{\theta}_{IVt,N}$ defined as

$$h_{IVt} = \left(R_{\eta p}^T W R_{\eta p}\right)^{-1} R_{\eta p}^T W r_{\eta v} \tag{48}$$

where

$$r_{\eta v} = \lim_{N\to\infty}\widehat{r}_{\eta v,N} = E[\eta_t v_t] \ . \tag{49}$$

Using (17), $r_{\eta v}$ can be expressed as follows:

$$\begin{aligned} r_{\eta v} &= E[\eta_t v_t] \\ &= E[\eta_t(e_t - r_t^T\theta)] \\ &= \widetilde{d} - \widetilde{D}\theta \\ &= -\widetilde{Q}\widetilde{\gamma} \end{aligned} \tag{50}$$

where $\widetilde{d} = E[\eta_t e_t]$, $\widetilde{D} = E[\eta_t r_t^T]$, $\widetilde{Q}$ is an $m \times p$ matrix whose elements are composed of the parameter θ, and $\widetilde{\gamma}$ is a $p \times 1$ noise covariace vector. From (44), (48) and (50), the asymptotic bias h_{IVt} can be expressed as follows:

$$h_{IVt} = \theta_{IVt} - \theta = -\left(R_{\eta p}^T W R_{\eta p}\right)^{-1} R_{\eta p}^T W \widetilde{Q}\widetilde{\gamma} \ . \tag{51}$$

From (44), it can be expected that a consistent estimate of θ can be obtained by compensating for the asymptotic bias h_{IVt} in the IV-type estimate $\widehat{\theta}_{IVt,N}$. From (51), estimate of the asymptotic bias h_{IVt} at time instant N becomes

$$\widehat{h}_{IVt,N} = -\left(\widehat{R}_{\eta p,N}^T W \widehat{R}_{\eta p,N}\right)^{-1}\widehat{R}_{\eta p,N}^T W \widehat{\widetilde{Q}}_{N-1}\widehat{\widetilde{\gamma}}_N \tag{52}$$

where $\widehat{\widetilde{Q}}_N$ and $\widehat{\widetilde{\gamma}}_N$ denote the estimates of $\widetilde{Q}$ and $\widetilde{\gamma}$ at time instant N, respectively. Hence the bias compensated instrumental variables type (BCIV-type) estimate $\widehat{\theta}_{BCIVt,N}$ is given by

$$\begin{aligned} \widehat{\theta}_{BCIVt,N} &= \widehat{\theta}_{IVt,N} - \widehat{h}_{IVt,N} \\ &= \widehat{\theta}_{IVt,N} + \\ &\left(\widehat{R}_{\eta p,N}^T W \widehat{R}_{\eta p,N}\right)^{-1}\widehat{R}_{\eta p,N}^T W \widehat{\widetilde{Q}}_{BCIVt,N-1}\widehat{\widetilde{\gamma}}_N \end{aligned} \tag{53}$$

where $\widehat{\widetilde{Q}}_{BCLS,N}$ denotes the estimate of $\widetilde{Q}$ whose elements are composed of the BCIV-type estimate $\widehat{\theta}_{BCIVt,N}$. Practically the noise covariance vector $\widetilde{\gamma}$ is unknown, it is necessary to estimate $\widetilde{\gamma}$.

It is possible to write the structures of $\widetilde{d}$, $\widetilde{D}$ and $\widetilde{Q}$. For example, assumed that the IV-type vector η_t is given by

$$\eta_t = [w_{l,t+n+l}^T,\ w_t^T,\ w_{l,t}^T]^T \tag{54}$$

where

$$w_{l,t} = [w_{t-n-1}\ \cdots\ w_{t-n-l}]^T \tag{55}$$

and $m = n + 2l$. Then, using the assumption of e_t, d_t, it is easily shown that $\tilde{\boldsymbol{d}}$ and $\tilde{\boldsymbol{D}}$ become as follows:

$$\tilde{\boldsymbol{d}} = E[\boldsymbol{\eta}_t e_t] = \begin{bmatrix} E[\boldsymbol{w}_{l,t+n+l}e_t] \\ E[\boldsymbol{w}_t e_t] \\ E[\boldsymbol{w}_{l,t}e_t] \end{bmatrix}$$

$$= \begin{bmatrix} E[\boldsymbol{d}_{l,t+n+l}e_t] \\ E[\boldsymbol{d}_t e_t] \\ E[\boldsymbol{d}_{l,t}e_t] \end{bmatrix} = \begin{bmatrix} \mathbf{0}_l \\ \mathbf{0}_n \\ \mathbf{0}_l \end{bmatrix} = \mathbf{0}_{n+2l} \tag{56}$$

$$\tilde{\boldsymbol{D}} = E[\boldsymbol{\eta}_t \boldsymbol{r}_t^T]$$

$$= \begin{bmatrix} -E[\boldsymbol{w}_{l,t+n+l}\boldsymbol{e}_t^T] & E[\boldsymbol{w}_{l,t+n+l}\boldsymbol{d}_t^T] \\ -E[\boldsymbol{w}_t \boldsymbol{e}_t^T] & E[\boldsymbol{w}_t \boldsymbol{d}_t^T] \\ -E[\boldsymbol{w}_{l,t}\boldsymbol{e}_t^T] & E[\boldsymbol{w}_{l,t}\boldsymbol{d}_t^T] \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{O}_{l\times n} & E[\boldsymbol{d}_{l,t+n+l}\boldsymbol{d}_t^T] \\ \boldsymbol{O}_{n\times n} & E[\boldsymbol{d}_t \boldsymbol{d}_t^T] \\ \boldsymbol{O}_{l\times n} & E[\boldsymbol{d}_{l,t}\boldsymbol{d}_t^T] \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{O}_{(n+2l)\times n} & \boldsymbol{R}_{\tilde{d}d} \end{bmatrix} \tag{57}$$

where

$$\boldsymbol{d}_{l,t} = [d_{t-n-1} \ \cdots \ d_{t-n-l}]^T \tag{58}$$

$$\boldsymbol{R}_{\tilde{d}d} = \begin{bmatrix} E[\boldsymbol{d}_{l,t+n+l}\boldsymbol{d}_t^T] \\ E[\boldsymbol{d}_t \boldsymbol{d}_t^T] \\ E[\boldsymbol{d}_{l,t}\boldsymbol{d}_t^T] \end{bmatrix}$$

$$= \left[\begin{array}{cccc} r_{dd}(l) & r_{dd}(l+1) & \cdots & r_{dd}(n+l-1) \\ \vdots & \vdots & \ddots & \vdots \\ r_{dd}(1) & r_{dd}(2) & \cdots & r_{dd}(n) \\ \hline r_{dd}(0) & r_{dd}(1) & \cdots & r_{dd}(n-1) \\ r_{dd}(1) & r_{dd}(0) & \cdots & r_{dd}(n-2) \\ \vdots & \vdots & \ddots & \vdots \\ r_{dd}(n-1) & r_{dd}(n-2) & \cdots & r_{dd}(0) \\ \hline r_{dd}(n) & r_{dd}(n-1) & \cdots & r_{dd}(1) \\ \vdots & \vdots & \ddots & \vdots \\ r_{dd}(n+l-1) & r_{dd}(n+l-2) & \cdots & r_{dd}(l) \end{array}\right] . \tag{59}$$

Additionally, $\tilde{\boldsymbol{D}}\boldsymbol{\theta} - \tilde{\boldsymbol{d}}$ can be expressed as follows:

$$\tilde{\boldsymbol{D}}\boldsymbol{\theta} - \tilde{\boldsymbol{d}} = \begin{bmatrix} \boldsymbol{O}_{(n+2l)\times n} & \boldsymbol{R}_{\tilde{d}d} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix} - \mathbf{0}_{n+2l}$$

$$= \boldsymbol{R}_{\tilde{d}d}\boldsymbol{b}$$

$$= \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\gamma}} \tag{60}$$

thus $\tilde{\boldsymbol{Q}}$ and $\tilde{\boldsymbol{\gamma}}$ becomes as follows:

$$\tilde{\boldsymbol{Q}} = \tilde{\boldsymbol{b}}\tilde{\boldsymbol{i}}_1^T + \sum_{j=1}^{n+l-1} \left[(\boldsymbol{S}_{n+2l})^j + (\boldsymbol{S}_{n+2l}^T)^j\right] \tilde{\boldsymbol{b}}\tilde{\boldsymbol{i}}_{j+1}^T \tag{61}$$

$$\tilde{\boldsymbol{b}} = [\mathbf{0}_l^T, \ \boldsymbol{b}^T, \ \mathbf{0}_l^T]^T \tag{62}$$

$$\tilde{\boldsymbol{i}}_j = \boldsymbol{I}_{n+l}(:, j) \ (j = 1, \ \cdots, \ n+l) \tag{63}$$

$$\tilde{\boldsymbol{\gamma}} = [r_{dd}(0), \ \cdots, \ r_{dd}(n+l-1)]^T \tag{64}$$

Since the dimention of the noise covariance vector $\tilde{\boldsymbol{\gamma}}$ is $p = n + l$ in the case where the input noise d_t is colored noise, $n + l$ noise covariances must be estimated for the BCIV-type estimate. However, using the IV-type vector $\boldsymbol{\eta}_t$ given by (54), there can be avoided the necessity to estimate the output noise covariances. Thus only the input noise covariances are to be determined. Moreover, if the input noise d_t is white noise with unknown variance σ_d^2, the structures of (59), (61) and (64) can be written more simply.

$$\boldsymbol{R}_{\tilde{d}d} = \begin{bmatrix} \boldsymbol{O}_{l\times n} \\ \sigma_d \boldsymbol{I}_n \\ \boldsymbol{O}_{l\times n} \end{bmatrix} \tag{65}$$

$$\tilde{\boldsymbol{Q}} = \tilde{\boldsymbol{b}} = [\mathbf{0}_l^T, \ \boldsymbol{b}^T, \ \mathbf{0}_l^T]^T \tag{66}$$

$$\tilde{\boldsymbol{\gamma}} = \sigma_d^2 \ . \tag{67}$$

If the input and output measurements are corrupted by white noise, it is necessary to estimate the input and output noise variances in the BCLS method (Wada et al. [1990], Jia et al. [2001], Zheng and Feng [1989], Zheng [1999], Ikenoue et al. [2005]). Moreover, if the input noise is white noise and the output noise is colored noise, it is necessary to estimate the input noise variance and $n + 1$ output noise covariances in Zheng's method (Zheng [2002]). On the contrary, it can be learned from (67) that if the input noise is white noise, the BCIV-type estimate requires only the input noise variance estimate.

Since the IV-type vector $\boldsymbol{\eta}_t$ defined by (54) has no causality, it is necessary to set $t = t - l + 1$ and use the following equations instead of $\boldsymbol{\eta}_t$, $\boldsymbol{p}_t$ and z_t in the real applications.

$$\boldsymbol{\eta}_t \ \rightarrow \ \boldsymbol{\eta}_{t-l+1} \tag{68}$$

$$\boldsymbol{p}_t \ \rightarrow \ \boldsymbol{p}_{t-l+1} \tag{69}$$

$$z_t \ \rightarrow \ z_{t-l+1} \ . \tag{70}$$

4. BIAS-COMPENSATION METHOD FOR IV-TYPE ESTIMATOR

In this section, the estimation algorithm of the noise covariances vector $\tilde{\boldsymbol{\gamma}}$ is proposed for the BCIV-type estimate, and the BCIV-type method is derived. Multiplying (16) by the IV-type vector $\boldsymbol{\eta}_t$, and taking expectation yield

$$E[\boldsymbol{\eta}_t z_t] = E[\boldsymbol{\eta}_t(\boldsymbol{p}_t^T\boldsymbol{\theta} + v_t)] = E[\boldsymbol{\eta}_t \boldsymbol{p}_t^T]\boldsymbol{\theta} + E[\boldsymbol{\eta}_t v_t]$$

$$= E[\boldsymbol{\eta}_t \boldsymbol{p}_t^T]\boldsymbol{\theta} - \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\gamma}} \ . \tag{71}$$

Thus, we can obtain the following overdetermined system of equations

$$\boldsymbol{r}_{\eta z} = \boldsymbol{R}_{\eta p}\boldsymbol{\theta} - \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\gamma}} \ . \tag{72}$$

Solving (72) for $\boldsymbol{\theta}$ in a least-squares sense yields

$$\boldsymbol{\theta} = \left(\boldsymbol{R}_{\eta p}^T \boldsymbol{W} \boldsymbol{R}_{\eta p}\right)^{-1} \boldsymbol{R}_{\eta p}^T \boldsymbol{W} \left(\boldsymbol{r}_{\eta z} + \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\gamma}}\right)$$

$$= \boldsymbol{\theta}_{IVt} + \left(\boldsymbol{R}_{\eta p}^T \boldsymbol{W} \boldsymbol{R}_{\eta p}\right)^{-1} \boldsymbol{R}_{\eta p}^T \boldsymbol{W} \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\gamma}} \ . \tag{73}$$

Equation (73) implies that, for a given $\tilde{\boldsymbol{\gamma}}$, the parameter vector $\boldsymbol{\theta}$ can be obtained by solution of overdetermined system of equations (72), and replacing (73) by the estimates at time instant N yields the BCIV-type estimate $\hat{\boldsymbol{\theta}}_{BCIVt,N}$ in (53).

Moreover, solving (72) for $\tilde{\boldsymbol{\gamma}}$ in a least-squares sense yields

$$\tilde{\boldsymbol{\gamma}} = \left(\tilde{\boldsymbol{Q}}^T \boldsymbol{X} \tilde{\boldsymbol{Q}}\right)^{-1} \tilde{\boldsymbol{Q}}^T \boldsymbol{X} \left(\boldsymbol{R}_{\eta p}\boldsymbol{\theta} - \boldsymbol{r}_{\eta z}\right) \tag{74}$$

where $\boldsymbol{X}$ is an $m \times m$ positive definite weighting matrix. Equation (74) implies that, for a given $\boldsymbol{\theta}$, the noise covariances vector $\tilde{\boldsymbol{\gamma}}$ can be obtained by solution of overdetermined system of equations (72). Thus, by replacing (74) by the estimate at time instant N, the estimate of noise covariances vector $\tilde{\boldsymbol{\gamma}}$ at time instant N can be obtained by

$$\widehat{\widehat{\gamma}}_N = \left(\widehat{\widehat{\boldsymbol{Q}}}^T_{BCIVt,N\text{-}1} \boldsymbol{X} \widehat{\widehat{\boldsymbol{Q}}}_{BCIVt,N\text{-}1} \right)^{-1} \times$$
$$\widehat{\widehat{\boldsymbol{Q}}}^T_{BCIVt,N\text{-}1} \boldsymbol{X} \left(\widehat{\boldsymbol{R}}_{\eta p,N} \widehat{\boldsymbol{\theta}}_{BCIVt,N\text{-}1} - \widehat{\boldsymbol{r}}_{\eta z,N} \right) . \quad (75)$$

Based on the above discussion, the bias-compensation method for the IV-type estimator can be established from the combination of the BCIV-type parameter estimation algorithm (53) and the noise covariances estimation algorithm (75). To perform adaptive identification of noisy input-output system, the following recursive BCIV-type algorithm may be applied.

The recursive BCIV-type algorithm

Step 0: Set the initial values of the algorithm. Choose the weighting matrices $\boldsymbol{W}$ and $\boldsymbol{X}$, the IV-type vector $\boldsymbol{\eta}_t$.

Step 1: Calculate the IV-type estimate.

$$\widehat{\boldsymbol{\theta}}_{IVt,N} = \widehat{\boldsymbol{\theta}}_{IVt,N\text{-}1} + \boldsymbol{K}_N \left(\boldsymbol{g}_N^T - \boldsymbol{\Psi}_N^T \widehat{\boldsymbol{\theta}}_{IVt,N\text{-}1} \right)$$

where

$$\boldsymbol{g}_N = \left[\, (N\text{-}1) \widehat{\boldsymbol{r}}^T_{\eta z,N\text{-}1} \boldsymbol{W} \boldsymbol{\eta}_N \;\; z_N \, \right]$$
$$\boldsymbol{\Psi}_N = \left[\, (N\text{-}1) \widehat{\boldsymbol{R}}^T_{\eta p,N\text{-}1} \boldsymbol{W} \boldsymbol{\eta}_N \;\; \boldsymbol{p}_N \, \right]$$
$$\boldsymbol{K}_N = \boldsymbol{M}_{N\text{-}1} \boldsymbol{\Psi}_N \left(\boldsymbol{\Lambda}_N + \boldsymbol{\Psi}_N^T \boldsymbol{M}_{N\text{-}1} \boldsymbol{\Psi}_N \right)^{-1}$$
$$\boldsymbol{\Lambda}_N = \begin{bmatrix} 0 & 1 \\ 1 & \boldsymbol{\eta}_N^T \boldsymbol{W} \boldsymbol{\eta}_N \end{bmatrix}^{-1} = \begin{bmatrix} -\boldsymbol{\eta}_N^T \boldsymbol{W} \boldsymbol{\eta}_N & 1 \\ 1 & 0 \end{bmatrix}$$
$$\boldsymbol{M}_N = \boldsymbol{M}_{N\text{-}1} - \boldsymbol{K}_N \boldsymbol{\Psi}_N^T \boldsymbol{M}_{N\text{-}1} .$$

Step 2: Calculate the covariance matrix and the covariance vector.

$$\widehat{\boldsymbol{R}}_{\eta p,N} = \widehat{\boldsymbol{R}}_{\eta p,N\text{-}1} + \frac{1}{N} \left(\boldsymbol{\eta}_N \boldsymbol{p}_N^T - \widehat{\boldsymbol{R}}_{\eta p,N\text{-}1} \right)$$
$$\widehat{\boldsymbol{r}}_{\eta z,N} = \widehat{\boldsymbol{r}}_{\eta z,N\text{-}1} + \frac{1}{N} \left(\boldsymbol{\eta}_N z_N - \widehat{\boldsymbol{r}}_{\eta z,N\text{-}1} \right) .$$

Step 3: Calculate the noise covariances estimates.

$$\widehat{\widehat{\gamma}}_N = \left(\widehat{\widehat{\boldsymbol{Q}}}^T_{BCIVt,N\text{-}1} \boldsymbol{X} \widehat{\widehat{\boldsymbol{Q}}}_{BCIVt,N\text{-}1} \right)^{-1} \times$$
$$\widehat{\widehat{\boldsymbol{Q}}}^T_{BCIVt,N\text{-}1} \boldsymbol{X} \left(\widehat{\boldsymbol{R}}_{\eta p,N} \widehat{\boldsymbol{\theta}}_{BCIVt,N\text{-}1} - \widehat{\boldsymbol{r}}_{\eta z,N} \right) .$$

Step 4: Calculate the BCIV-type estimate.

$$\widehat{\boldsymbol{\theta}}_{BCIVt,N} = \widehat{\boldsymbol{\theta}}_{IVt,N} + N^2 \boldsymbol{M}_N \widehat{\boldsymbol{R}}^T_{\eta p,N} \boldsymbol{W} \widehat{\widehat{\boldsymbol{Q}}}_{BCIVt,N\text{-}1} \widehat{\widehat{\gamma}}_N .$$

Step 5: Set $N = N + 1$ and repeat from step 1 until the chosen stop criterion is satisfied.

Initial value selection: Initial values at $N = 0$ are given as $\widehat{\boldsymbol{\theta}}_{IVt,0} = \boldsymbol{0}_{2n}$, $\boldsymbol{M}_0 = \rho \boldsymbol{I}_{2n}$ (ρ is a large number), $\widehat{\boldsymbol{R}}_{\eta p,0} = \boldsymbol{O}_{m \times 2n}$, $\widehat{\boldsymbol{r}}_{\eta z,0} = \boldsymbol{0}_m$. Initial value of the BCIV-type estimate is given as $\widehat{\boldsymbol{\theta}}_{BCIVt,T} = \widehat{\boldsymbol{\theta}}_{IVt,T}$, ($0 < T < L$: L is a number for idling).

5. SIMULATION RESULTS

5.1 The case where the input noise and the output noise are colored noise

Computer simulation which compares the proposed BCIV-type algorithm with the LS algorithm has been carried out. Consider the following second-order system:

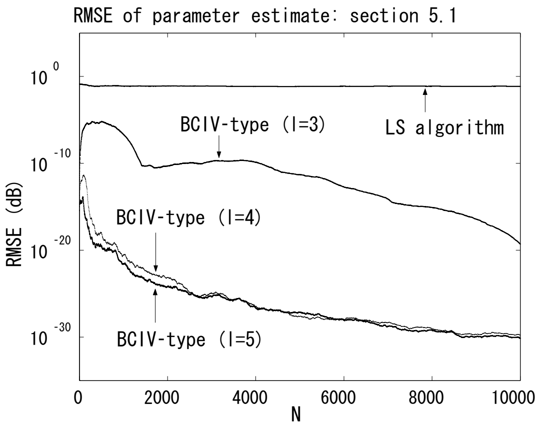

Fig. 1. RMSE of parameter estimates (section 5.1).

Table 1. Mean values, standard deviations of estimates for $N = 10000$ (section 5.1).

True value	LS algorithm	BCIV-type ($l = 3$)	BCIV-type ($l = 4$)	BCIV-type ($l = 5$)
$a_1 = -1.5$	−0.7046 ±0.0162	−1.5130 ±0.0934	−1.5004 ±0.0159	−1.5000 ±0.0122
$a_2 = 0.7$	0.0567 ±0.0133	0.7121 ±0.0860	0.7004 ±0.0120	0.7001 ±0.0085
$b_1 = 1.0$	0.9997 ±0.0326	0.9936 ±0.0734	0.9950 ±0.0285	0.9971 ±0.0313
$b_2 = 0.5$	1.9223 ±0.0421	0.4726 ±0.1575	0.4952 ±0.0548	0.4969 ±0.0518

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{1.0q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} . \quad (76)$$

The noise free input u_t is assumed to be generated as $u_t = 0.9u_{t-1} + \epsilon_t$ where ϵ_t is a zero-mean white noise with unit variance. The colored output noise e_t is assumed to be generated as $e_t = -0.8e_{t-1} + \omega_{e,t} + 0.3\omega_{e,t-1}$ where $\omega_{e,t}$ is a zero-mean white noise with variance $\sigma^2_{\omega,e} = 1.8898$, which yields SNR $= 10\log_{10}(E[y_t^2]/E[e_t^2]) = 20$ [dB]. The colored input noise d_t is assumed to be generated as $d_t = 0.6d_{t-1} + \omega_{d,t} + 0.2\omega_{d,t-1}$ where $\omega_{d,t}$ is a zero-mean white noise with variance $\sigma^2_{\omega,d} = 0.0263$, which yields SNR $= 10\log_{10}(E[u_t^2]/E[d_t^2]) = 20$ [dB]. The IV-type vector $\boldsymbol{\eta}_t$ is chosen as in (54). In particular, the cases of $l = 3,\ 4,\ 5$, which yield $m = 8,\ 10,\ 12$ respectively, are examined. Weighting matrices are chosen as $\boldsymbol{W} = \boldsymbol{I}_m$ and $\boldsymbol{X} = \boldsymbol{I}_m$. Computer simulation for comparison is carried out through $M = 100$ independent runs with a data length of 10000. Fig. 1 gives a plot of the root mean squared error (RMSE) which is defined by

$$\text{RMSE} = 20\log_{10} \sqrt{\frac{1}{M} \sum_{k=1}^{M} \frac{\|\widehat{\boldsymbol{\theta}}_{k,t} - \boldsymbol{\theta}\|^2}{\|\boldsymbol{\theta}\|^2}} \text{ [dB]} \quad (77)$$

where $\widehat{\boldsymbol{\theta}}_{k,t}$ denotes the estimate of $\boldsymbol{\theta}$ at time step t in the kth independent run. Table 1 provides the mean values, the standard deviations of estimates for $N = 10000$.

Simulation results indicate that the LS method gives biased results. On the contrary, the proposed BCIV-type method can give consistent estimate. Especially, the resulting BCIV-type estimates in the cases of $l = 4,\ 5$ are more accurate than those obtained with $l = 3$.

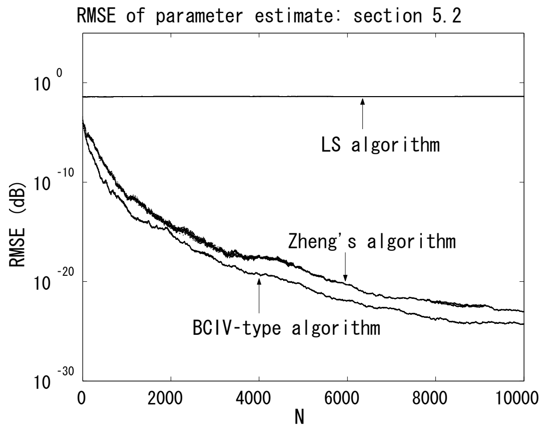

Fig. 2. RMSE of parameter estimates (section 5.2).

Table 2. Mean values, standard deviations of estimates for $N = 10000$ (section 5.2).

True value	LS algorithm	Zheng's algorithm	BCIV-type algorithm
$a_1 = -1.5752$	-0.5332 ± 0.0080	-1.5369 ± 0.0622	-1.5518 ± 0.0572
$a_2 = 0.6065$	-0.4009 ± 0.0080	0.5658 ± 0.0651	0.5791 ± 0.0674
$b_1 = 0.1699$	0.1806 ± 0.0059	0.1601 ± 0.0401	0.1645 ± 0.0315
$b_2 = 0.1438$	0.2163 ± 0.0059	0.1391 ± 0.0395	0.1411 ± 0.0286

5.2 The case where the input noise is white noise and the output noise is colored noise

By computer simulation, the proposed BCIV-type algorithm is compared with the LS algorithm and Zheng's algorithm (Zheng [2002]). Consider the folowing second-order system:

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.169901q^{-1} + 0.143831q^{-2}}{1 - 1.575157q^{-1} + 0.606531q^{-2}} . \quad (78)$$

The noise free input u_t is assumed to be generated as $u_t = \epsilon_t - 0.3\epsilon_{t-1} + 0.5\epsilon_{t-2} - 0.7\epsilon_{t-3} + 0.9\epsilon_{t-4}$ where ϵ_t is a zero-mean white noise with variance $1/(1 + 0.3^2 + 0.5^2 + 0.7^2 + 0.9^2) = 0.3788$. The colored output noise e_t is assumed to be generated as $e_t = e_{t-1} - 0.4e_{t-1} + \omega_{e,t} - 0.87\omega_{e,t-1} + 0.57\omega_{e,t-2}$ where $\omega_{e,t}$ is a zero-mean white noise with variance $\sigma^2_{\omega,e} = 0.2264$, which yields SNR $= 10\log_{10}(E[y_t^2]/E[e_t^2]) = 10$ [dB]. The noise variance of white input noise d_t is set as $\sigma_d^2 = 0.1$ which yields SNR $= 10\log_{10}(E[u_t^2]/E[d_t^2]) = 10$ [dB]. The IV-type vector $\boldsymbol{\eta}_t$ is chosen as in (54), where $l = 4$ which yields $m = 10$. Weighting matrices are chosen as $\boldsymbol{W} = \boldsymbol{I}_m$ and $\boldsymbol{X} = \boldsymbol{I}_m$. Computer simulation for comparison is carried out through $M = 100$ independent runs with a data length of 10000. Fig. 2 gives a plot of the RMSE which is defined by (77). Table 2 provides the mean values, the standard deviations of estimates for $N = 10000$.

Simulation results indicate that the LS method gives biased results. On the contrary, the Zheng's method and the proposed BCIV-type method can give consistent estimate. Though the BCIV-type method requires only the input noise variance estimate, the resulting parameter estimates obtained by the BCIV-type method is more accurate than those obtained by the Zheng's method.

6. CONCLUSIONS

In this paper, the method of consistent estimation of noisy input-output system has been studied. A new bias-compensation based method has been proposed for EIV model identification. The proposed BCIV-type method consists of the parameter estimation algorithm which is based on compensation of asymptotic bias on the IV-type estimates, and the noise covariances estimation algorithm which is based on an overdetermined system of equations. The proposed method can treat not only the white input-output noise case but also the colored input-output noise case. It is demonstrated that the proposed method can give consistent parameter estimate via simulation results.

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