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Model-based Cold-start Speed Control Design for SI Engines

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Abstract: This paper presents a feedback deign approach to the cold-start speed control for spark ignition engines. First, in order to ensure successful combustion in the transient mode, a fuel injection controller is given based on the air charge estimation with inverse dynamics of fuel path, which is a dual sampling rate system, i.e. the estimation for the air charge is performed TDC-based, and the fuel injection command is delivered cycle-based, respectively. Then, a speed control scheme is proposed that provides a coordination between the spark advance and the throttle operation. A supervisor is exploited to management the multi-control laws. Finally, simulation results will be demonstrated which are carried out on a full scale 6-cylinder engine system simulator provided by the SICE benchmark problem.

1. INTRODUCTION

The speed control problem for internal combustion engines is not a new topic in the engine control community, since regulation of the idle speed is one of fundamental control specifications for engine management. However, precise speed control for spark ignition (SI) engine during transient operation mode is still a challenging issue. Particularly, as mentioned in Ohata, et al. (2007), a typical transient operation mode is the cold start in which speed control with high precision is difficult based on the measurable signals. As is shown in many references (e.g. Cho and Hedrick (1989), Stotsky, et al. (2000)), the idle speed controller is usually established based on the mean-value models of the engines, which describe the average characteristics in a given operating condition. The difficulty in speed control during the cold start lies on the variation of the models due to the dramatic changes of environmental temperature and the thermal conditions. For example, the wall wetting of injection fuel onto the inlet port and the intake valves causes unpredictable transient behavior of air-fuel mixing, and the air charging for individual cylinder can not be easily described by the mean-value model. However, in order to control the torque, which is provided by the multi-cylinders serially, with high precision under the air-fuel ratio (A/F) constraint for engine operation without misfiring, it is necessary to manage the behavior of individual cylinder air charging.

This paper deals with the speed control problem for internal combustion engines in the cold-start mode. The main purpose is to provide a solution to the SICE benchmark problem for engine cold start control, which is provided by the SICE Research Committee on Advanced Engine Control (Ohata, et al. (2007)). The characteristics of the proposed control approach are as follows. First, the control structure is a model-based feedback control with event-based switching, where for the air charging estimation and the fuel path, mean-valve models but with time-varying parameters are used in order to accommodate the transient behaviors. Second, the system is a multi-rate sampling control system. The control law for intake air dynamics via throttle operation is designed in continuous time domain, however, the fuel injection path is a cycle-based discrete time control for each individual cylinders with fixed phase delay in crankshaft angle, and the intake air estimation for individual cylinder is provided in TDC-based sampling rate, since each cylinder provides active torque during the expansion stroke only for one cycle, which start at the corresponding TDC and is determined by the intake air at the previous TDC sampling period. The proposed controller will be validated by the simulation results carried out on the simulator provided by the benchmark problem. Finally, it should be noted that the topic of emission control during cold start is not addressed in this paper. Indeed, the problem is more widely investigated in the automotive control community (Shaw (2002), Eng (2007)).

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A simplified physical structure of a four-stroke SI engine is shown in Fig. 1. In the engine with multi-cylinders, where we only sketched one of them for the sake of simplicity, the torque to drive the crankshaft rotational motion is provided by each cylinder serially along the crank angle, and the torque generated in each cylinder during its own expansion stroke is determined by the individual air charge and fuel injection mass during the corresponding induction stroke, which are influenced by the air inlet path and the fuel path, respectively. Thus, the engine speed dynamics should be divided into three parts: the intake air dynamics, the fuel dynamics and the crankshaft rotational dynamics. Typically, the actuating variables to the dynamics are chosen as the angle of throttle u_{th}, the fuel injection quantity u_f and the spark advance (SA) angle u_s, and the measurement signals for online control are engine speed

ω $[rad \cdot s^{-1}]$($\bar{\omega}$ $[rpm]$), and the air mass flow rate entering the intake manifold $\dot{m}_i$ $[g \cdot s^{-1}]$.

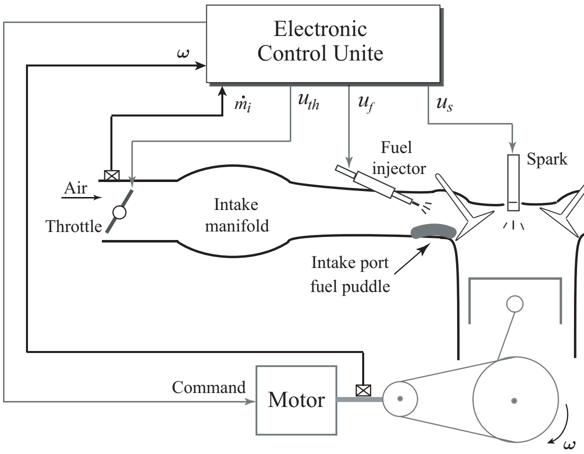

Fig. 1. Schematic representation of SI engine

Due to the complexity of the system, control is usually based on oriented mean-value models, which ignore the characteristics of individual cylinder and capture the average features of engine physics (Guzzella and Onder (2004), Heywood (1988)). In the following, we will briefly review and discuss these models from the references. In order to simplify the control scheme, we will exploit these models with some modifications to describe the dynamics more precisely during the cold-start mode.

First, the air inlet path dynamics is mainly determined by the pressure in the intake manifold p_m $[Pa]$. Following the ideal gas law, the dynamics can be characterized as (1) under isothermal condition (Guzzella and Onder (2004))

$$\dot{p}_m = \frac{RT_m}{V_m}(\dot{m}_i(u_{th}, p_m) - \dot{m}_o(p_m, p_c)) \tag{1}$$

where R is the gas constant, T_m $[K]$ denotes the temperature of manifold, V_m $[m^3]$ is the manifold volume, and p_c is the cylinder pressure.

In (1), $\dot{m}_o$ $[g{\cdot}s^{-1}]$ denotes the air mass flow rate leaving the manifold. Since the induction stroke changes sequentially according to the crankshaft angle, $\dot{m}_o$ is a nonlinear function obtained by applying the fluid flow model passing through an orifice, which is related to the ratio of the pressures in manifold and cylinders. A mean-value model for $\dot{m}_o$ can be found in Stotsky, et al. (2000) and Guzzella and Onder (2004) as follows:

$$\dot{m}_o = \frac{\rho_a V_c \eta}{4\pi p_a}\omega p_m \tag{2}$$

where ρ_a $[g \cdot m^{-3}]$ and p_a $[Pa]$ denote the atmosphere density and pressure, respectively, V_c $[m^3]$ is the volume of the cylinder, and η is the volumetric efficiency.

Second, the fuel injection dynamics is generally used to represent the fuel mass injected into each cylinder during the induction stroke, which is denoted by m_{fc} and a mean value model is (Aquino (1981))

$$\begin{cases} \dot{m}_f = -\tau m_f + \varepsilon u_f \\ m_{fc} = \tau m_f + (1-\varepsilon)u_f \end{cases} \tag{3}$$

where m_f is the fuel mass entering the intake port per induction stroke, u_f is the fuel injection command. ε represents the fraction of fuel deposited on the inlet port, and τ is the inlet port time constant.

When the air-fuel ratio λ in the cylinder satisfies the constraint condition for combustion, the crankshaft rotational dynamics from Newton's law is

$$J\dot{\omega} = \tau_e(\dot{m}_o, \lambda, SA) - \tau_f(\omega) \tag{4}$$

where τ_e is the engine torque and τ_f represents the friction torque. Due to the engine torque is generated serially with multi-cylinders, the description for the torque should be event-based discontinuous mathematical model, which will cause unfeasible complexity in the control design. Generally, the following mean-value computation of engine torque can be found in Stotsky, et al. (2000) as

$$\tau_e = \frac{a\rho_a V_c \eta}{4\pi p_a} p_m(t - t_d) f_\lambda(\cdot)[\cos(u_s')]^{2.875} \tag{5}$$

where a represents the maximum torque capacity which depends on the physical system parameters, t_d is the intake to torque production delay, which is a function of $\omega(t_d \simeq \pi/\omega)$, f_λ denotes the A/F influence for the mean-value engine torque, and $u_s' = u_s - MBT$ is the spark advance from MBT (the minimum SA for best torque).

The cold start operation is as follows. First, the crankshaft is driven by a starting motor to a constant rotational speed $\bar{\omega}_{mo} \pm \delta$ till a cylinder gets successful combustion, and the motoring mode will be continued up to maximum time T_{mo}. Once a cylinder gets successful combustion, the motor will turned off, and the driving torque will be switched from the motor to the engine. Therefore, the key to obtain a good starting speed performance is how to guarantee the successful combustion as quick as possible via managing the control inputs: u_{th}, u_f and u_s, and to regulate the engine speed at the desired idle speed ω_r with pre-specified error and without undesirable overshoot.

To perform feedback control with the models of the engine dynamics, the bottleneck is that the mean-value models mentioned above can not give enough information on the engine dynamics during the cold-start operation mode. Moreover, even if the A/F is under successful control to ensure combustion, open loop control, i.e. keeping the throttle opening constant determined with the model parameters in the static idle mode, can not provide satisfactory transient performance. Fig. 2 shows a simulation result where we just keep a constant spark advance $u_s = 20$ $[degree]$ and the throttle angle $u_{th} = 5.2$ $[degree]$ corresponding to the static mode of idle speed operation, and the fuel injection is controlled well to keep the combustion successfully. Obviously, to obtain satisfactory performance, advanced control is required.

The purpose of the SICE benchmark problem for cold-start speed control is to give an opportunity for the control community to challenge this problem. The benchmark problem is set with the specification: $\bar{\omega}_{mo} = 250$ $[rpm]$, the admissible motoring error $\delta = 50$ $[rpm]$, the maximum motoring time $T_{mo} = 1.5$ $[s]$, and the adjusting time for the speed regulation $T_g \le 1.5$ $[s]$, the idle speed set value $\bar{\omega}_r = 650$ $[rpm]$ with admissible error 50 $[rpm]$. In the next section, we will propose a model-based feedback control scheme with switching supervisor.

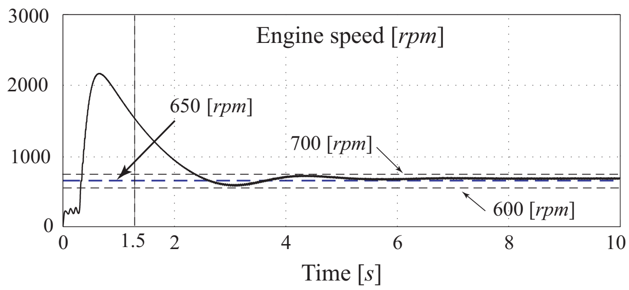

Fig. 2. Engine speed during cold start with constant control input

3. CONTROLLER DESIGN

In this section, we will describe the proposed design approach to the model-based feedback controller with switching supervisor design approach, which is expected to provide a solution for the SICE benchmark problem. The control system is indicated in Fig. 3. The whole system is divided into two main subsystems: one is the coordinated control between throttle and SA for the speed regulation, and the other one is for the management of A/F by the fuel injection control. The supervisor unit provides the necessary switching actions, the adjustment for the control parameters and the design reference model.

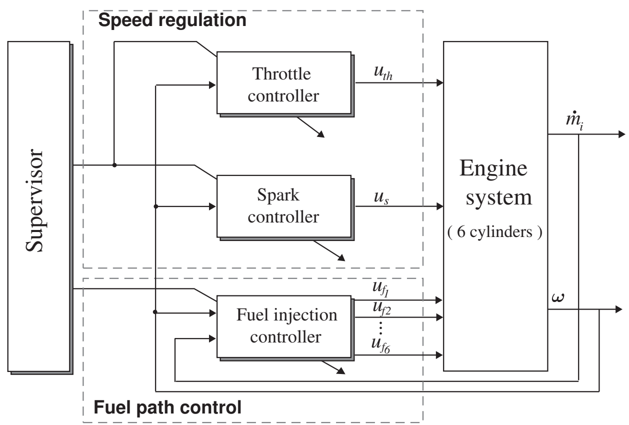

Fig. 3. Block diagram of the cold start control system

The speed control subsystem is designed on the models (1) and (4) in continuous time domain. The supervisor will coordinate the control action between u_s and u_{th} which provide a switching action from u_s to u_{th} based on speed error. On the other hand, the fuel control is formulated as an event-based discrete system under a multi-rate sampling algorithms: first, the air charge estimation is achieved at each TDC time, the sampling period $T_c = 2\pi/(3\omega)$ $[s]$ (i.e. 120 $[degree]$ in crankshaft angle) and l is used as the sampling sequence, then, the fuel injection control is designed with the sampling period $T_s = 4\pi/\omega$ $[s]$ (i.e. 720 $[degree]$ in crankshaft angle), since the control signals for each cylinder will be delivered once in a cycle, and the sampling sequence is denoted by k. Obviously, $k = \text{fix}((l-1)/6) + 1$.

The control algorithm for the fuel path is shown in Fig. 4, and the details for each block of the fuel control system and the speed regulation controller are shown in the following subsections.

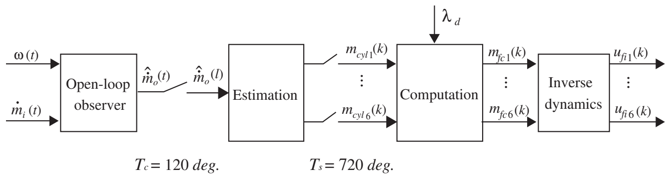

Fig. 4. Block diagram of the fuel path control system

3.1 Cylinder air charge estimation

The typical fueling injection for an SI engine is usually based on the estimation of the air charge into the cylinders, and then, on the basis of the estimation and the desired A/F, the necessary fuel amount to be injected is determined by the fueling control system. Since the air mass flow rate $\dot{m}_o$ is not measurable, we propose an open-loop observer (6) for $\dot{m}_o$ using the equation (2) and the air dynamic function (1) with the initial condition $\hat{p}_m(0) = p_a$.

$$\begin{cases} \dot{\hat{p}}_m = \dfrac{RT_m}{V_m}(\dot{m}_i - \hat{\dot{m}}_o) \\ \hat{\dot{m}}_o = \dfrac{\rho_a V_c \eta}{4\pi p_a}\omega \hat{p}_m \end{cases} \tag{6}$$

The effectiveness of the proposed observer has been tested in the simulation model provided by the SICE benchmark problem. The result is illustrated in Fig. 5, which shows that the estimation $\hat{\dot{m}}_o$ follows the actual value closely during the initial period.

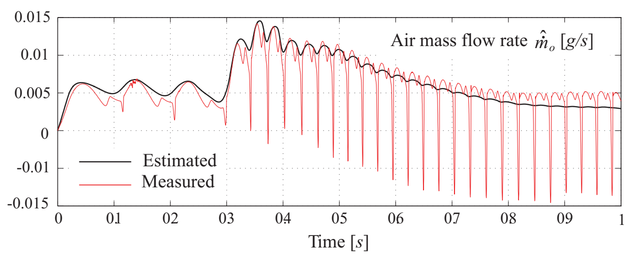

Fig. 5. Estimated air mass flow rate leaving manifold with open-loop observer

Generally, the fuel injection quantity for each cylinder should be depended on its air charge mass which is determined by the air flow passing through the intake valve of the cylinder during the whole induction stroke. Since the air and fuel are pumped into each cylinder synchronous, it is difficult to get the exact air mass and proper fuel injection command to satisfy the combustion condition for the A/F requirement. In the following, we propose an estimation method for the air charge mass $\hat{m}_{cyl}$ into each cylinder using $\hat{\dot{m}}_o$ and measured ω at each TDC as follows:

$$\hat{m}_{cyl}(l) = \hat{\dot{m}}_o(lT_c) \cdot \hat{t}_{TDC}(l) \tag{7}$$

where $\hat{m}_o(l)$ is obtained from the open-loop observer, and $\hat{t}_{TDC}(l) = 2\pi/(3\omega(lT_c))$ is a predicted time of induction time.

3.2 Fuel path control algorithm

From (3), the discrete time dynamic model for the fuel path is as follows,

$$\begin{cases} m_{fi}(k+1) = (1-\tau)m_{fi}(k) + \varepsilon u_{fi}(k) \\ m_{fc}(k) = \tau m_{fi}(k) + (1-\varepsilon)u_{fi}(k) \end{cases} \tag{8}$$

where $i = 1, 2, \cdots, 6$. The following fuel injection control law is derived based on the inverse dynamics of (8)

$$u_{fi}(k+1) = Au_{fi}(k) + Bm_{fci}(k) + Cm_{fci}(k+1) \tag{9}$$

where

$$A = -\frac{\tau\varepsilon - (1-\tau)(1-\varepsilon)}{1-\varepsilon}, \quad B = -\frac{1-\tau}{1-\varepsilon}, \quad C = \frac{1}{1-\varepsilon}$$

and $m_{fci}(k)$ is calculated with the desired air-fuel ratio λ_d by

$$m_{fci}(k) = \frac{\hat{m}_{cyl}(6(k-1)+i)}{\lambda_d} \tag{10}$$

It is well known that the amount of fuel required during the cold-start stage is significantly higher than the one needed for the warmed-up operation due to the little fuel vaporization during the first a few cycles (Eng (2007)). This means the fuel model parameters τ and ε, which are related to the temperature of individual cylinder and the engine speed are different from the steady state operation during cold start. Hence, we introduce the adjusting scheduling for the model parameters in (9) for temperature compensation during the first a few cycles.

3.3 Speed control with multi-input

If the A/F is under control, the cold-start engine speed control problem can be formulated as a multi-input single-output system with the throttle angle u_{th} and SA u_s as inputs and speed ω as output. The proposed speed control problem is closely related to idle speed regulation. Many papers (e.g. Hrovat and Sun (1997), Thornhill, et al. (2000)) have been published to solve this problem: the main control action includes PI controllers for the air loop with the integral portion as the core to achieve the desired speed, and proportional feedback control for the spark loop which is faster but with limited authority. We roughly divide the cold-start transient into two phases: first, the rapid engine acceleration, and then, the stage of idle speed regulation. In the following, we will describe the algorithms used.

We first introduce the following first-order system as the reference model, and which begins to work when a cylinder successfully ignited at $t = t_0$,

$$\dot{\omega}_d = -\tau_m(\omega_d - \omega_r), \quad t \geq t_0, \quad \omega_d(t_0) = \omega(t_0) \tag{11}$$

τ_m is a positive constant, $\omega_r = 650\pi/30$ $[rad \cdot s^{-1}]$, and ω_d is the reference trajectory for engine speed control. u_s is turned on at $t = t_0$ and turned off definitely at $t = t_1$ when the speed error $e_\omega = \omega - \omega_d$ satisfies $0 \leq \bar{e}_\omega(t_1) \leq 50$ $[rpm]$, and at the time $t = t_1$, the throttle control u_{th} is turned on. For the first stage, the control input is chosen as

$$\begin{cases} u_{th} = 0 \\ u_s = k_{p1}e_\omega \end{cases} \quad t_0 \leq t \leq t_1 \tag{12}$$

where k_{p1} is a given constant. The control during the first stage is expected to reduce engine torque output to reduce the overshoot of engine speed, which is due to the high pressure p_m. Hence, from physical consideration, the throttle opening is not necessary in order to drive the pressure p_m to the static value for idle speed as soon as possible, i.e. we set $u_{th} = 0$. The friction torque τ_f is simply modeled by the damping term $-D\omega$ only, where $D = D_0/J$ and D_0 is the damping coefficient. So that, from (4)and (5), the rotational dynamics can be written as

$$\dot{\omega} = -D\omega + cp_m(t - t_d)[\cos u_s']^{2.875} \tag{13}$$

where c is calculated by $c = \frac{a\rho_a V_c \eta}{J 4\pi p_a}$, and, we can compute u_s' from the inverse dynamics at $t = t_0$ as

$$\begin{aligned} \dot{\omega}_d(t_0) &= -D\omega_d(t_0) + c\hat{p}_m(t_0)[\cos(u_s'(t_0))]^{2.875} \\ &= -\tau_m(\omega_d(t_0) - \omega_r) \end{aligned} \tag{14}$$

where $\hat{p}_m(t_0)$ is obtained from the open-loop observer (6). In order to reduce the engine torque output, we can choose $a = 0.3a(t_0)$ in (14). Since from physical consideration, u_s' should be retarding from MBT and $e_\omega < 0$ during a few seconds after t_0, we choose $k_{p1} > 0$.

For the idle speed regulation, the feedback control law is given as follows:

$$\begin{cases} U_{th} = -k_{p2}\tilde{\omega} + u_{th}' \\ u_s' = 0, \qquad \text{i.e.,} \quad u_s = \text{MBT} \end{cases} \tag{15}$$

where k_{p2} is a constant, $\tilde{\omega} = \omega - \omega_r$, $U_{th} = (1 - \cos u_{th}) \cdot f_p(p_m, p_a)$ and $f_p(\cdot)$ is a nonlinear function used to represent the air flow passing through an orifice (Guzzella and Onder (2004)), and, the feedforward compensation term u_{th}' is a solution of the following equations:

$$\begin{cases} -D\omega_r + cp_{mr} = 0 \\ \alpha u_{th}' - \beta\omega_r p_{mr} = 0 \end{cases} \tag{16}$$

which gives the equilibrium of the engine dynamics represented by the mean-value model

$$\begin{cases} \dot{\omega} = -D\omega + cp_m(t - t_d) \\ \dot{p}_m = \alpha U_{th} - \beta\omega p_m \end{cases} \tag{17}$$

where the second equation is the air dynamics with $\dot{m}_i = s_0(1 - \cos u_{th})f_p(\cdot)$, $\alpha = \frac{RT_m s_0}{V_m}$, $\beta = \frac{RT_m}{V_m} \cdot \frac{\rho_a V_c \eta}{4\pi p_a}$ and s_0 is the area of the throttle. It should be noted that to perform this control law, the model parameters D and c are needed and a feasible way to determine the parameter values is to apply the identification technique to the model of rotational dynamics.

3.4 Stability and robustness analysis

Now, we discuss the convergence of the speed regulation control system. As shown in subsection 3.3, the controller is finally switched to (15) at $t = t_1$. Therefore, to show the convergence, we will prove the asymptotic stability of the error system consisting of (17) with the controller (15), which is presented by the following equations

$$\begin{cases} \dot{\tilde{\omega}} = -D\tilde{\omega} + c\tilde{p}_m(t - t_d) \\ \dot{\tilde{p}}_m = -(\alpha k_{p2} + \beta p_{mr})\tilde{\omega} - \beta\omega\tilde{p}_m \end{cases} \quad (18)$$

where $\tilde{p}_m = p_m - p_{mr}$. The conclusion is summarized as the following Proposition. Obviously, system (18) is with time-delay, and the delay-time t_d is time-varying depending on the engine speed ω. If we consider the region $\omega_0 < \omega$, the maximum value of t_d is determined: $r = \pi/\omega_0$ ($t_d \in [0, r]$). Hence, the proof for its stability is done using the Lyapunov-Razumikhin theorem (Hale and Lunel (1993)).

Proposition 1. *Let $q(> 1)$ be a sufficient small number and $\gamma > 0$ be a constant satisfying*

$$q^2\gamma^2 - 2D\gamma + c^2 < 0 \quad (19)$$

Consider the system (18) and the region: $\omega_{min} < \omega < +\infty$, where $\omega_{min} = q^2\gamma^2/(2\beta)$. If the feedback gain k_{p2} satisfies

$$\frac{-M(\omega_{min}) - \beta p_{mr}}{\alpha} < k_{p2} < \frac{M(\omega_{min}) - \beta p_{mr}}{\alpha} \quad (20)$$

then, system (18) is asymptotically stable at the origin, i.e. $\tilde{\omega} \to 0$ as $t \to \infty$ for any given initial condition, where $M(\omega) = \left[(2D\gamma - c^2 - q^2\gamma^2)(2\beta\omega - q^2\gamma^2)\right]^{1/2}$.

Proof: By Lyapunov-Razumihkin theorem, a time-delay system $\dot{x} = f(x(t - \tau))$, $\left(x(\tau) = x_0(\tau),\ \tau \in [0, r]\right)$ with $f(0) = 0$ is asymptotically stable if there exists a positive definite function $V(x)$ such that $\dot{V} \le -W(||x||)$, whenever $\max\limits_{0\le\tau\le r}\{||x(t - \tau)||\} < q \cdot ||x(t)||$ along any trajectory of the system, where $W(s) > 0$, $\forall s > 0$ is a continuous nondecreasing function and $q(> 1)$ is a given constant.

To prove the stability of (18) with Lyapunov-Razumikhin theorem, we need to show a positive definite function $V(\tilde{\omega}, \tilde{p}_m)$ that satisfies $\dot{V} \le -W(\tilde{\omega}, \tilde{p}_m)$, whenever

$$\max_{0\le t_d\le r}\{\tilde{\omega}^2(t - t_d) + \tilde{q}_m^2(t - t_d)\} < q \cdot (\tilde{\omega}^2(t) + \tilde{q}_m^2(t)) \quad (21)$$

where $q > 1$ is a given number. Construct a candidate Lyapunov-Razumihkin function as

$$V = \frac{\gamma}{2}\tilde{\omega}^2 + \frac{1}{2}\tilde{p}_m^2 \quad (22)$$

The time derivative of V along system (18) is

$$\begin{aligned}\dot{V} = &-D\gamma\tilde{\omega}^2 + \gamma c\tilde{p}_m(t - t_d)\tilde{\omega} - (\alpha k_{p2} + \beta p_{mr})\tilde{\omega}\tilde{p}_m \\ &-\beta\omega\tilde{p}_m^2\end{aligned} \quad (23)$$

Hence, along any trajectory of (18), when the Ruzumikhin condition (21) holds, $\dot{V}$ satisfies the following inequality

$$\begin{aligned}\dot{V} \le &-\left(D\gamma - \frac{c^2}{2} - \frac{q^2\gamma^2}{2}\right)\tilde{\omega}^2 - \left(\beta\omega - \frac{q^2\gamma^2}{2}\right)\tilde{p}_m^2 \\ &-(\alpha k_{p2} + \beta p_{mr})\,\tilde{\omega}\tilde{p}_m = -x^{\mathrm{T}}Qx\end{aligned} \quad (24)$$

where

$$Q = \begin{bmatrix} \dfrac{2D\gamma - c^2 - q^2\gamma^2}{2} & \dfrac{\alpha k_{p2} + \beta p_{mr}}{2} \\ \dfrac{\alpha k_{p2} + \beta p_{mr}}{2} & \dfrac{2\beta\omega - q^2\gamma^2}{2} \end{bmatrix}$$

and, taking the conditions (19) and (20) into account, the matrix Q is positive definite, i.e. the time derivative of the positive definite function (22) satisfies $\dot{V} \le -W(\tilde{\omega}, \tilde{p}_m)$, whenever the condition (21) holds. The convergence of $\tilde{\omega}$ follows from Lyapunov-Razumihkin theorem. □

Observing the proof of Proposition 1, it is obvious that if the system involves uncertainty, the stability can be guaranteed unless the derivative of V losses the negative definiteness whenever the Razumihkin condition holds. This motivates robustness consideration for uncertainties in the models.

Suppose the uncertainty is modeled as follows:

$$J\dot{\omega} = \tau_e - \tau_f + \Delta f(\omega, p_m) \quad (25)$$

where $\Delta f(\omega,\ \tilde{p}_m)$ represents the modeling error in torque generation. Then, the error system becomes

$$\begin{cases} \dot{\tilde{\omega}} = -D\tilde{\omega} + c\tilde{p}_m(t - t_d) + \Delta f(\omega, p_m) \\ \dot{\tilde{p}}_m = -(\alpha k_{p2} + \beta p_{mr})\tilde{\omega} - \beta\omega\tilde{p}_m \end{cases} \quad (26)$$

Assume that the uncertainty satisfies

$$||\Delta f(\omega, p_m)|| \le \rho||\tilde{p}_m(t - t_d)|| \quad (27)$$

where $\rho > 0$ is a known constant.

Proposition 2. *Let $q(> 1)$ is a sufficient small number and $\gamma > 0$ is a positive constant satisfying*

$$(\gamma^2 + \rho^2)q^2 - 2D\gamma + c^2 + \gamma^2 < 0 \quad (28)$$

Consider system (26) and the region: $\omega_{min} < \omega < +\infty$, where $\omega_{min} = q^2(\gamma^2 + \rho^2)/(2\beta)$, if the feedback gain k_{p2} satisfies

$$\frac{-N(\omega_{min}) - \beta p_{mr}}{\alpha} < k_{p2} < \frac{N(\omega_{min}) - \beta p_{mr}}{\alpha} \quad (29)$$

system (26) is asymptotically stable at the origin for any Δf satisfying condition (27), where $N(\omega) = \left[(2D\gamma - c^2 - \gamma^2 - (\gamma^2 + \rho^2)q^2)(2\beta\omega - q^2(\gamma^2 + \rho^2))\right]^{1/2}$.

Proof: The procedure is similar to the proof for the proposition 1, and we omit the details in this paper. □

4. SIMULATION RESULTS

The values of the simulation parameters for the SICE benchmark problem are shown in Table 1. The fuel injection control is under (6), (7), (9) and (10), and in order to compensate the fuel requirement of the cold engine, during the first 3 cycles, the parameters for the individual cylinders are scheduled and shown in Table 2.

Table 1. Simulation parameters

R	287	V_c	$3 \times 10^{-3}[L]$
p_a	$1.01 \times 10^{-5}[Pa]$	η	1
ρ_a	$1.1837[g \cdot m^{-3}]$	τ	0.01
T_m	$298[K]$	ε	0.1
V_m	$6 \times 10^{-3}[L]$	λ_d	14.5

Table 2. Parameter compensation for fuel path

	cycle 1	cycle 2	cycle 3	cycle 4 ...
τ	0.08	0.6	0.2	0.01 ...
ε	0.5	0.1	0.1	0.1 ...

During the acceleration stage, the time constant of reference model (14) is set as $\tau_m = 16$, the control gain is $k_{p1} = 0.05$ and the initial value for SA is set as $u_s(0) = 10$ for simplicity. For the regulation of idle speed, SA is set as MBT= 20 $[degree]$. The reference values of the model (17) are: $D = 0.3$, $c = 0.0015$, $\alpha = 2.312 \times 10^7$, $\beta = 0.098$ and the control gain $k_{p2} = 0.2$. The results in Fig. 6 show that the design specification is satisfied. To test the robustness of the proposed control system, the value of the model parameter c is changed to $c = 0.001$, and k_{p2} is set as 0.8. The results in Fig. 7 indicate that the control system has robustness.

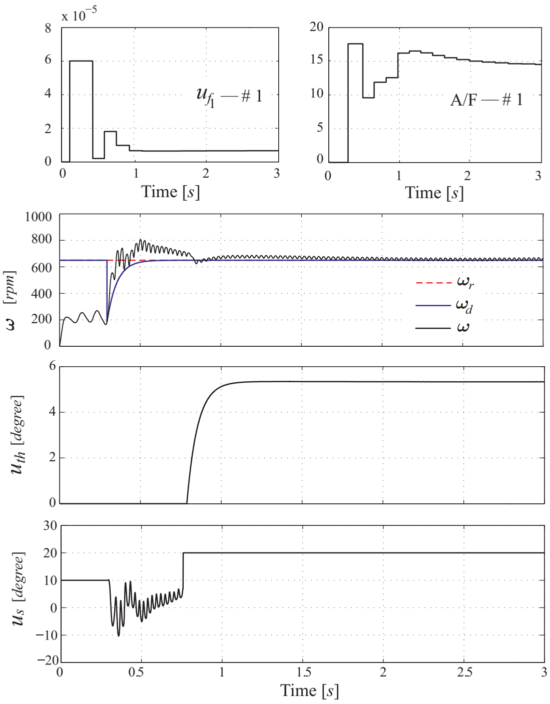

Fig. 6. Simulation results of cold-start speed control

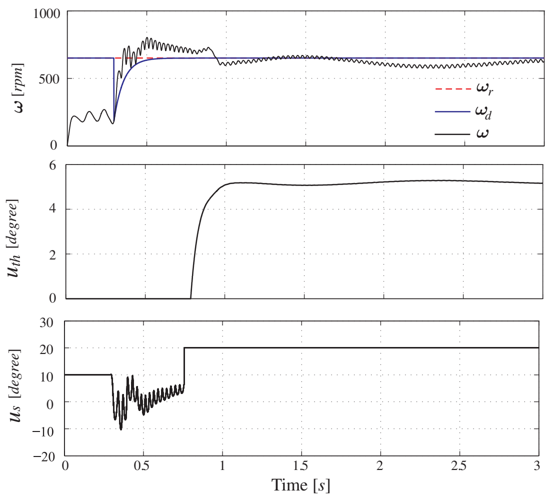

Fig. 7. Robustness test results

5. CONCLUSION

The cold-start control problem is investigated for SI engines. Here, we only focused on the speed control performance. From the view of control, the speed control system for SI engines is a multi-input single-output system, and there are two challenging issues: to keep combustion successfully and to regulate the engine speed without undesired overshoot. Also, as is well-known, the torque generation process is with speed-depended time delay.

In this paper, a model-based feedback control scheme is presented. For the combustion event management, a discrete time control with a dual sampling rate is proposed, i.e. the fuel injection command is delivered cycle-based with a TDC-based provided individual cylinder air charging estimation. For the speed regulation, a coordinated feedback controller is presented between the spark advance control in the transient mode and the throttle operation in the idle speed regulation mode. Furthermore, the stability and the robustness of the error system in the speed regulation mode are discussed with the Lyapunov-Razimikhin theorem. Finally, the proposed control scheme is tested in the simulator provided by the SICE benchmark for engine cold-start problem. It is shown that the control scheme satisfies the speed control performance specified by the SICE benchmark problem.

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