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Recursive Subspace Model Identification Based On Vector Autoregressive Modelling

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Abstract: Recursive subspace model identification (RSMI) has been developed for a decade. Most of RSMIs are only applied for open loop data. In this paper, we propose a new recursive subspace model identification which can be applied under open loop and closed loop data. The key technique of this derivation of the proposed algorithm is to bring the Vector Auto Regressive with eXogenous input (VARX) models into RSMI. Numerical studies on a closed loop identification problem show the effectiveness of the proposed algorithm.

Keywords: Closed loop identification; subspace method; recursive algorithm.

1. INTRODUCTION

Nowadays, subspace model identification (SMI) is recognized to be very efficient to model multivariable systems. In off-line SMIs, the extended observability matrix or the estimation of state is derived from the singular value decomposition (SVD) of a certain matrix made of given input and output (I/O) data.

The recursive subspace identification problem has received much attention in the literature (Gustafsson et al. [1998], Oku & Kimura [2002], Mercère et al. [2007]). RSMI methods are mostly inspired by the off-line versions of subspace model identification techniques. The larger the number of I/O data is, the more computational complexity is necessary to perform SVD. Most RSMI algorithms apply certain updating techniques to avoid direct computation of the SVD. The most representative algorithm is to use the close relationship between SMI and sensor array signal processing (SAP) problems. The idea of applying subspace tracking algorithms to the RSMI problem was originally introduced in Gustafsson [1997]. Under the assumption that the order of the system to be identified is a priori known, Gustafsson et al. [1998] presented recursive algorithms which directly update an estimate of the extended observability matrix. More precisely, the Projection Approximation Subspace Tracking (PAST) algorithm and its instrumental variables version were applied and modified to derive an effective update of the signal subspace.

Also, the identification of closed loop by subspace methods has been an active research area in the past decade. Recent work is presented in (Jansson [2003], Qin & Ljung [2003], Chiuso & Picci [2005]). The work done by Jansson and Qin has been regarded as a significant advance in subspace identification of feedback systems (Chiuso & Picci [2005]). Due to the feedback control the future input is correlated with past output measurement or past noise, making the traditional SMIs biased. Therefore, most of the closed-loop SMIs try to decouple these two terms. The algorithms proposed by Jansson and Chiuso are based on identification of a predictor model. Chiuso [2007] had exploited the role which Vector Auto Regressive with eXogenous input (VARX) models plays in the mentioned algorithms.

As far as the author knows, there is no recursive (online) algorithm of closed loop subspace model identification. In this paper, we are concerned with on-line recursive subspace state-space system identification based on VARX model and PAST. The new algorithm is called 'VPC' by the meaning that the algorithm is based on VARX modelling and PAST under Closed loop data. The key point of this algorithm is to construct the same problem with SAP by exploiting VARX models which decouples the correlation between the future input and past output measurement or past noise. This algorithm can also be applied for open loop. Numerical studies on a closed loop identification problem show the effectiveness of the proposed algorithm.

This paper is organized as follows: in section 2, the problem formulation and notation are introduced, in section 3, the review of batch SMI for closed loop is described. Section 4 is dedicated to the main results that illuminate the new recursive algorithm based on VARX model and PAST. Finally, section 5 gives numerical examples to illustrate the effectiveness of the proposed algorithm. And in section 6, the conclusion is presented.

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2. PROBLEM FORMULATION AND NOTATION

Consider an n-th order linear time-invariant system in the innovation state space form which is equivalent with process form:

$$x_{t+1} = Ax_t + Bu_t + Ke_t \quad (1a)$$
$$y_t = Cx_t + Du_t + e_t \quad (1b)$$

Where $y_t \in R^l$,$u_t \in R^m$,$x_t \in R^n$ and $e_t \in R^l$ are the system output, input, state and innovation respectively. A, B, C and D are system matrices with appropriate dimensions. K is the Kalman filter gain.

The problem is to estimate recursively a state-space realization from the updates of the disturbed I/O data $u(t)$ and $y(t)$. We introduce the following assumption:
A1: (A,C) is observable.
A2: (A,[B K]) is controllabe.
A3: The eigenvalues of $A - KC$ are strictly inside the unit circle.
A4: The input u and innovation e are jointly stationary. The input signal is sufficient persistently exciting order Qin [2006]. For closed loop, there are

$$\overline{E}[e(k)e(l)^T] = R_e\delta_{kl}, \quad (2)$$
$$\overline{E}[e(k)u(l)^T] = 0, k > l, \quad (3)$$

where $\overline{E}$ is defined as in Ljung [1999] :

$$\overline{E}\{\cdot\} = \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} E\{\cdot\}$$

Which means that there is a feedback from $u(t)$ to $y(t)$.

For the purpose of identifying, we introduce some notations:

$$y_f(t) = \begin{bmatrix} y_t^T & y_{t+1}^T & \cdots & y_{t+f-1}^T \end{bmatrix}^T$$
$$y_p(t) = \begin{bmatrix} y_{t-p}^T & y_{t-p+1}^T & \cdots & y_{t-1}^T \end{bmatrix}^T$$
$$Y_f = [y_f(t) \; y_f(t+1) \; \ldots \; y_f(t+N-1)]$$
$$Y_p = [y_p(t) \; y_p(t+1) \; \ldots \; y_p(t+N-1)]$$

where $f > n$ and $p > n$ are a user defined integer which called future horizon and past horizon respectively. From the same way, we can formulate the $u_f(t)$, $e_f(t)$, $u_p(t)$ and $e_p(t)$. Also, we have the similar expression U_f, U_p, E_f and E_p.

By iterating the system equations, it is straightforward to get the extended state-space model (1), so an extended input-output equation can be formulated as following:

$$y_f(t) = \Gamma_f x(t) + H_f u_f(t) + G_f e_f(t) \quad (4)$$

For the Hankel matrix form of (4), we have

$$Y_f(t) = \Gamma_f X(t) + H_f U_f(t) + G_f E_f(t) \quad (5)$$
$$Y_p(t) = \Gamma_p X(t-p) + H_p U_f(t) + G_p E_p(t) \quad (6)$$

where

$$X(t) = [x(t) \; x(t+1) \; \cdots \; x(t+N-1)]$$
$$X(t-p) = [x(t-p) \; x(t-p+1) \cdots \; x(t-p+N-1)]$$

And Γ_f is the extended observability matrix of the system. H_f and G_f are two lower triangular Toeplitz matrices. Γ_f, H_f and G_f are formed:

$$\Gamma_i = \begin{bmatrix} C^T & CA^T & \cdots & (CA^{i-1})^T \end{bmatrix}^T$$

$$H_i = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix}$$

$$G_i = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ CAK & CK & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}K & CA^{i-3}K & \cdots & I \end{bmatrix}$$

By iterating (1), we also can obtain the following relation,

$$X(t) = A_k^p X(t-p) + \bar{L}_p Z_p(t) \quad (7)$$

where $Z_p = [Y_p^T \; U_p^T]^T$, $A_k = A - KC$, $B_k = B - KD$, and $\bar{L}_p = \begin{bmatrix} B_k & A_kB_k & \cdots & A_k^{p-1}B_k \end{bmatrix}$. For a large 'p', from the assumption A3($A_k^p \approx 0$, for a stable system), we can consider

$$X(t) \approx \bar{L}_p Z_p(t) \quad (8)$$

3. OVERVIEW OF BATCH CLOSED LOOP SMI

In the following, we briefly illustrate why most SMI methods for open loop identification can not be applied for closed-loop data. And how to solve this problem. The details refer to Qin [2006]. For most SMIs, the basic procedure is based on this projection:

$$Y_f(t)\Pi_{U_f}^{\perp} = \Gamma_f X(t)\Pi_{U_f}^{\perp} + H_f U_f(t)\Pi_{U_f}^{\perp} + G_f E_f(t)\Pi_{U_f}^{\perp} \quad (9)$$

where $\Pi_{U_f}^{\perp}$ represents the projection to the orthogonal complement of U_f and

$$\Pi_{U_f}^{\perp} = I - U_f^T(U_fU_f^T)^{-1}U_f \quad (10)$$

Furthermore, it's obviously that,

$$U_f\Pi_{U_f}^{\perp} = U_f(I - U_f^T(U_fU_f^T)^{-1}U_f) = 0 \quad (11)$$

Under the open loop condition, The input u and innovation e are uncorrelated. So the following comes into existence

$$E_f\Pi_{U_f}^{\perp} = E_f(I - U_f^T(U_fU_f^T)^{-1}U_f) = E_f \quad (12)$$

since

$$\frac{1}{N}E_fU_f^T \to 0 \; when \; N \to \infty$$

Substitute (10) and (12) in (9), we can simply (9),

$$Y_f(t)\Pi_{U_f}^{\perp} = \Gamma_f X(t)\Pi_{U_f}^{\perp} + G_f E_f(t) \quad (13)$$

Right multiply Z_p^T on both sides of (13),

$$Y_f(t)\Pi_{U_f}^{\perp}Z_p^T = \Gamma_f X(t)\Pi_{U_f}^{\perp}Z_p^T + G_f E_f(t)Z_p^T \quad (14)$$

Since $e(t)$ is uncorrelated with past input and output, where

$$\frac{1}{N}E_fZ_p^T \to 0 \; as \; N \to \infty$$

Formulation (14) become

$$Y_f(t)\Pi_{U_f}^{\perp}Z_p^T = \Gamma_f X(t)\Pi_{U_f}^{\perp}Z_p^T \quad (15)$$

From the above equation , we can estimate Γ_f from a SVD composition of $Y_f(t)\Pi_{U_f}^{\perp}Z_p^T$. Then we can estimate the system parameters.

But if the data are acquired under closed-loop condition. From assumption A4, we know that The input u and innovation e are correlated one way, this means $\frac{1}{N}E_fU_f^T \neq$

0 *as* $N \to \infty$. Then obviously we will derive a biased estimation of Γ_f from (15).

Most subspace identification for closed loop are trying to avoid the direct projection on U_f. The algorithms proposed by Jansson [2003], Chiuso & Picci [2005] are based on predictor model. Let us first rewrite (1) as follows:

$$x_{t+1} = A_k x_t + B_k u_t + K y_t \tag{16a}$$
$$y_t = C x_t + D u_t + e_t \tag{16b}$$

it is also clear that:

$$y_f(t) = \bar{\Gamma}_f x(t) + \bar{H}_f u_f(t) + \bar{G}_f y_f(t) + e_f(t) \tag{17}$$

where

$$\bar{\Gamma}_i = \begin{bmatrix} C^T & CA_k^T & \cdots & (CA_k^{i-1})^T \end{bmatrix}^T$$

$$\bar{H}_i = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CA_k B_k & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA_k^{i-2} B_k & CA_k^{i-3} B_k & \cdots & D \end{bmatrix}$$

$$\bar{G}_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CK & 0 & \cdots & 0 \\ CA_k K & CK & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA_k^{i-2} K & CA_k^{i-3} K & \cdots & 0 \end{bmatrix}$$

and from (8), $x(t) = \bar{L}_p z_p(t)$ then

$$y_f(t) - \bar{H}_f u_f(t) - \bar{G}_f y_f(t) \approx \bar{\Gamma}_f \bar{L}_p z_p(t) + e_f(t) \tag{18}$$

The main idea of these approaches is first to estimate $CA_k^i [B_k \;\; K] (i = 0, 1, \ldots, f-2)$ by exploiting VARX. Then form the $\bar{H}_f$ and $\bar{G}_f$. The detail is described in later section. After the estimation, system parameters can be derived similar as CCA (Jansson [2003]). As we can see that the most important trick for closed loop identification is that estimating the Toeplitz matrices $\bar{H}_f$ and $\bar{G}_f$ to decouple the correlation between $u_f(t)$ and $e_f(t)$. In recursive algorithm, the same philosophy is applied in a similar way.

4. RSMI BASED ON VARX AND PAST

The most important step in RSMI is the recursive update of the observability subspace (Gustafsson et al. [1998], Oku & Kimura [2002], Mercère et al. [2007]). The basic idea of solving this procedure is to use the close relationship between SMI and sensor array signal processing (SAP) problems. The idea of applying subspace tracking algorithms to the RSMI problem was originally introduced in Gustafsson [1997]. More precisely, the PAST algorithm (Yang [1995]) and its instrumental variables modification IVPAST (Gustafsson [1998]) were applied and modified to derive an effective update of the signal subspace.

In the array signal processing field, several adaptive algorithms were suggested as SVD alternatives to estimate the signal subspace. These techniques are based on the following data generation model,

$$r_t = S(t) m_t + n_t \tag{19}$$

In the above equation, the $n \times 1$ vector r_t denotes the observation, $S(t)$ is a deterministic $n \times p$ matrix, m_t is a random $p \times 1$ vector which denotes the source vector and n_t stands for noise. In the array signal processing field, several adaptive algorithms were suggested to estimate the signal subspace recursively.

Projection Approximation Subspace Tracking (PAST) algorithm Yang [1995] was proposed by Yang to deal with array signal processing problem.In this method, Yang introduced an unconstrained criterion to estimate the range of $A(t)$ as follows :

$$V(W) = E\|r - WW^T r\|^2 \tag{20}$$

where the matrix argument $W \in \mathscr{R}_{n \times p}$ and $n > p$. $\|\cdot\|$ is the Euclidean vector norm and $E[\cdot]$ is the expectation operator.

Yang had proved the global minimum of $V(W)$ is attained if and only if $W = QT$ where Q contains the n dominating eigenvectors of $R_r = E[rr^T]$. Here T is an arbitrary unitary matrix. Furthermore, all other stationary points are saddle points. From the minimization of (20), it provides an expression particular basis of $S(t)$. The expectation operator in (20) is replaced with exponentially weighted sum to obtain a recursive update.

$$V(W) = \sum_{k=1}^{t} \lambda^{t-k} \parallel r(k) - W(t) W^T(t) r(k) \parallel^2 \tag{21}$$

where λ is a forgetting factor$(0 < \lambda < 1)$. And replace R_r with $R_r(t) = \sum_{k=1}^{t} \lambda^{t-k} r(k) r^T(k)$. The key idea of PAST is to replace $W^T(k) r(k)$ with

$$h(k) = W^T(k-1) r(k) \tag{22}$$

This is so-called projection approximation. Substitute (22) for $W^T(t) r(k)$ in (21),

$$\bar{V}(W(t)) = \sum_{k=1}^{t} \lambda^{t-k} \parallel r(k) - W(t) h(k) \parallel^2 \tag{23}$$

then $V(W)$ can be minimized by

$$W(t) = R_{rh}(t) R_h^{-1}(t) \tag{24}$$

In Yang [1995] ,an efficient recursive RLS-like algorithm with $O(np)$ complexity has been given.

[Remark 1] We note that the PAST algorithm is derived by minimizing the modified cost function (23) instead of the original one (22).Hence, the columns of $W(t)$ are not exactly orthonormal. From the simulation results seen, this doesn't matter the estimation. If necessary , we can reorthonormalize $W(t)$ after each update.

Then the problem we are concerned with is how to transform the RSMI to the SAP problem. Denote $z_f(t) = y_f(t) - \bar{H}_f u_f(t) - \bar{G}_f y_f(t)$, then rewrite (18),

$$z_f(t) \approx \bar{\Gamma}_f x(t) + e_f(t) \tag{25}$$

The analogy between (25) and (19) is also obvious, then we can estimate $\bar{\Gamma}_f$ as following two steps:

Step1 : the update of the observation vector $z_f(t)$ from the input-output measurements as (25)

Step 2: the estimation of a basis of $\bar{\Gamma}_f$ from this observation vector as (19).

If the estimation of a basis of $\bar{\Gamma}_f$ had been obtained from the above steps, we can estimate the states and the system parameters A, B, C, D, K by some ordinary procedure which is described in van Overschee (1996).

Unfortunately, this matrix $\bar{H}_f$ and $\bar{G}_f$ is unknown. However, most MOESP algorithms are not suitable for closed loop identification. So these RSMIs are not suitable to extend to closed loop identification. In this paper, we will propose a new approach to update observation vector $z_f(t)$.

In this section, a direct recursive estimation method for closed-loop data is described. We call this algorithm as 'VPC'. Though we utilize the VARX model as Jansson' method, but totally in a different way. In this scenario for recursive estimation, the order n of a system to be identified is a priori known. For identifiability reasons we assume that $D = 0$, i.e. there is no direct feed through from u to y.

Iterating (1), the state space model can be recovered to the high order model as follows:

$$y_t \doteq \sum_{i=1}^{q} CA_k^i [B_k \ K] z_{k-i} + Du_t + e_t \tag{26}$$

Where $z_k = [u_k^T \ y_k^T]^T$. From above equation, we rewrite it as one long VARX model.

$$y_f(t) \doteq \sum_{i=1}^{q} CA_k^i [B_k \ K] z_p(t-i) + Du_f(t) + e_f(t) \tag{27}$$

From the long VARX model, we can estimate the (unstructured) estimates of $CA_k^i[B_k \ K](i = 0, 1, \ldots, q)$. note that q is large than f. Then form $\bar{H}_f$ and $\bar{G}_f$ from the large estimates.

It's obviously if we have estimated $\bar{H}_f$ and $\bar{G}_f$,

$$y_f(t) - \hat{\bar{H}}_f u_f(t) - \hat{\bar{G}}_f y_f(t) \approx \bar{\Gamma}_f x(t) + e_f(t) \tag{28}$$

and denote $z_f(t) = y_f(t) - \hat{\bar{H}}_f u_f(t) - \hat{\bar{G}}_f y_f(t)$,

$$z_f(t) \approx \bar{\Gamma}_f x(t) + e_f(t) \tag{29}$$

From the connection between the RSMI problem and PAST algorithm, we can obtain the estimation of $\bar{\Gamma}_f$ recursively. After the $\bar{\Gamma}_f$ have been estimated, it's easy to obtain the Kalman state $x(t)$. And according to Katayama [2005], we can obtain the system parameters matrices from least square algorithm.

Using the recursive update of the $z_f(t)$ shown in the previous subsection, the recursive algorithm proposed in this paper is derived as follows.

The state space model under closed loop data can be estimated recursively according to the procedure as follows.

1) Using the equation (22) to estimate $CA_k^i[B_k \ K](i = 0, 1, \ldots, q)$. Form $\bar{H}_f$ and $\bar{G}_f$ from the above estimates.

2) Update the observation vector $z_f(t) = y_f(t) - \hat{\bar{H}}_f u_f(t) - \hat{\bar{G}}_f y_f(t)$.

3) From the last estimates of $\bar{\Gamma}_f(t-1)$ to update the t time $\bar{\Gamma}_f(t)$.

4) Estimate $x(t)$ from (36) if we have obtained the estimate of $\bar{\Gamma}(t)$,then $x(t) = z_f(t)\hat{\bar{\Gamma}}_f(t)^\dagger$. where $(\cdot)^\dagger$ represent Moore-Penrose pseudo-inverse.

5) The corresponding system parameters A, B, C, D, K and R can be estimated from $X(t)$ and state space model (1), similar as Katayama [2005].

A detail of this algorithm is proposed in Appendix A.

[Remark 2] From the update procedure, the system parameters can be estimated recursively. But it has to point that for this algorithm only use of PAST algorithm not EIV-PAST as shown in Gustafsson et al. [1998]. Subspace tracking estimators typically require that the noise covariance matrix is proportional to the identity matrix. The noise $e(t)$ in the predictor model satisfies this condition only if it is spatially white. How to improve this strict condition is the main object of future research.

5. NUMERICAL EXAMPLES

In this section, the results of a numerical simulation are presented to illustrate the performances of the new recursive algorithm. The simulation example is a first order SISO system under closed-loop operation,

$$y(k) - ay(k-1) = u(k-1) + e(k) + 0.9e(k-1)$$

The feedback has the following structure:

$$u(k) = -\lambda y(k) + r(k)$$

where $\lambda = 0$ and $\lambda = 0.6$ for open and closed-loop operations respectively. And in this simulation we assume $a = 0.9$. The excitation signal r(k) is a moving average process:

$$r(k) = (1 + 0.8q^{-1} + 0.6q^{-2})r_0(k)$$

where $r_0(k)$ is zero mean white noise with unit variance. The process innovation e(k) is zero mean white noise with standard deviation $\sigma_e = 1.2$. Monte-Carlo experiments are conducted with 30 runs. Each run generates 1000 samples.

In this simulation, We choose the parameter $\beta = 0.99$. The future horizon and past horizon are chosen 10 and 25 for these three algorithms respectively. We will apply the following representative subspace algorithms in the literature: closed-loop algorithm by Chiuso & Picci [2005], a classical subspace algorithms CVA (MATLAB N4SID with CVA weighting).

As a result of the 30 simulation runs, for open loop case, Fig.1 illustrate the averages of estimated of a, respectively. Fig.2, illustrate the average of the 30 estimated frequency response of the transfer function of process. For closed loop case, Fig.3 illustrate the averages of estimated of a, respectively. Fig.4 illustrate the average of the 30 estimated frequency response of the transfer function of process.

As can be seen from these figures, VPC can yield consistent estimates for the process under closed-loop data. And from the bode plot, the VPC can give a satisfactory result and better performance than CVA algorithm, and has a similar result as the directly PBSID-opt algorithm. This illustrate the effectiveness of the proposed algorithm.

6. DISCUSSION AND CONCLUSION

In this paper, a new recursive subspace model identification algorithm for closed-loop data called VPC have been proposed. This algorithm is based on VARX model

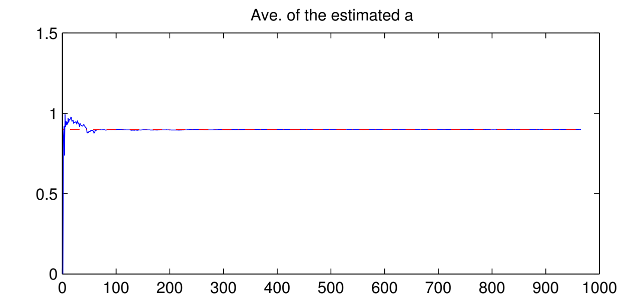

Fig. 1. Estimation results on a, show the average after 30 Monte Carlo trials. The dot lines represent the true value of a. Open loop case

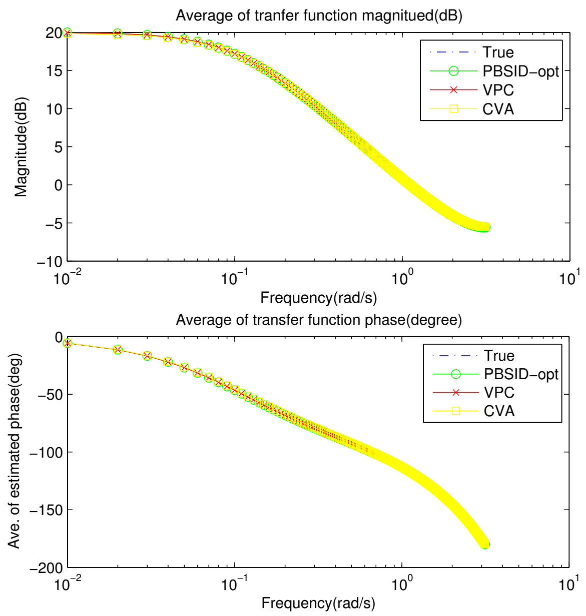

Fig. 2. Comparison of batch identification results w.r.t. the transfer function. Open loop case

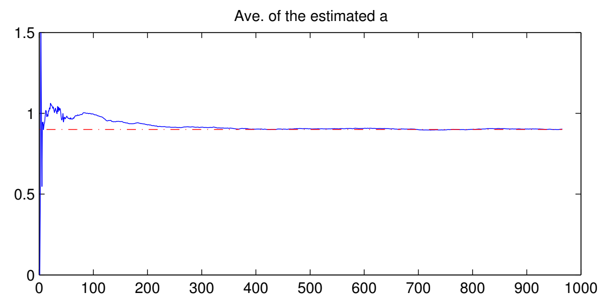

Fig. 3. Estimation results on a, show the average after 30 Monte Carlo trials. The dot lines represent the true value of a. Closed loop case

and PAST method. The VARX model plays a key role in deriving the recursive algorithm for closed loop data. The effectiveness of the proposed algorithm has been demonstrated by a numerical simulation. The future work will be improving the VPC for time-varying system and nonlinear system.

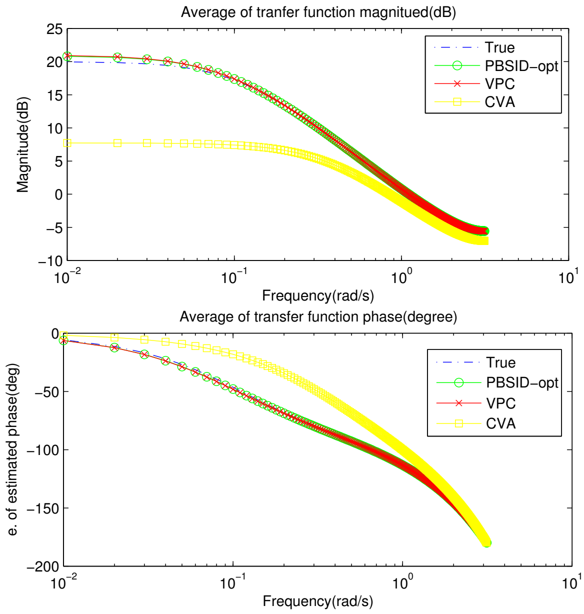

Fig. 4. Comparison of batch identification results w.r.t. the transfer function. Closed loop case

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Appendix A. VPC ALGORITHM

In this appendix the recursive updating formulas for VPC algorithm are given. Choose $P(0)$ and $\bar{\Gamma}_f(0)$ suitably, assume $\bar{\Gamma}_f(t-1)$ and $P(t-1)$ known. Assume the new data sample $(u_f(t),\ y_f(t))$ is acquired, and from $y_t \doteq \sum_{i=1}^{q} CA_k^i[B_k\ K]z_{k-i}+Du_t+e_t$, we obtain the estimation $\hat{\bar{H}}_f$ and $\hat{\bar{G}}_f$.

$$\begin{aligned}
\overline{z}_f(t) &= y_f(t) - \hat{\bar{H}}_f u_f(t) - \hat{\bar{G}}_f y_f(t) \\
y(t) &= \bar{\Gamma}_f(t-l)^T r(t) \\
h(t) &= P(t-1)y(t) \\
g(t) &= h(t)/[\beta + y^T(t)h(t)] \\
P(t) &= \frac{1}{\beta} Tri(P(t-1) - g(t)h^T(t)) \\
e(t) &= r(t) - \bar{\Gamma}_f(t-1)y(t) \\
\bar{\Gamma}_f(t) &= \bar{\Gamma}_f(t-1) + e(t)g^T(t) \\
x(t) &= z_f(t)\bar{\Gamma}_f(t)_f^{\dagger}
\end{aligned}$$

The operator $Tri(\cdot)$ indicates that only the upper (or lower) triangular part of $P(t) = R_y^{-1}(t)$ is calculated and its Hermitian transposed version is copied to the another lower (or upper) triangular part. Then from (1), use LS regression to estimate A,B,C,D,K.