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A comparative analysis of nonlinear control approaches for non-minimum phase processes

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Abstract: The control of nonlinear non-minimum phase processes by using traditional geometric approaches has lead to the development of specific design methods, such as the statically equivalent output approach. On the other hand, alternative design methods based on physical models, energy and mass balances, and the concept of passivity have been applied to a wide range of electrical and electromechanical systems. However, their application to process control has been limited. In this work, a comparative analysis of both design control approaches is presented. A standard continuous stirred tank reactor example is used to illustrate the differences among the approaches and the performances attained in both cases.

Keywords: Passivity-based control, Nonlinear systems, Non-minimum phase systems.

1. INTRODUCTION

Non-minimum phase processes pose one of the most challenging problems for many controller design techniques. For instance, linearization techniques rely on generating the inverse of the process, and therefore its application to non-minimum phase processes requires the use of special design methods. A popular approach, presented in Niemiec and Kravaris (2003), is the use of the notion of statically equivalent outputs, this outputs redefinition makes the non-minimum phase system minimum phase as far as the controller is concerned. These results guarantee internal stability of the closed-loop system locally around the equilibrium point. However, this technique relies on a linearized model of the system, and therefore the range of operation is limited to a region around the linearizing point. Interconnection and Damping Assignment - Passivity Based Control (IDA-PBC) introduced in Ortega et al. (2002), represents an attractive alternative to design nonlinear controllers compared to more traditional methods like the one based on differential geometric concepts. In the IDA-PBC framework, no inversion of the system dynamics is made, therefore, these methods can be applied to both minimum and non-minimum phase systems. The port-controlled Hamiltonian (PCH) models (van der Schaft, 2004) describe the systems by defining matrices that precisely captures the interconnection and damping structure of the system. Thus, a control design can achieve decoupled outputs by assigning a proper interconnection matrix between the outputs in closed-loop. Even though the wide application of these techniques to mechanical and electromechanical systems there have been just a couple of examples illustrating their use in process control (Sira-Ramírez and Angulo-Nunez, 1997; Sbárbaro and Ortega, 2005; Johnsen and Allgöwer, 2006). In this work an IDA-PB controller is designed to decouple the outputs of a non-minimum phase continuous stirred tank reactor (CSTR) over a wide range of operational points. The performance of the controller is compared, by means of numerical simulations, with a controller based on synthetic-outputs (Niemiec and Kravaris, 2003). This paper is organized as follows: Section 2 and 3 give the preliminaries for equivalent output linearization and IDA-PBC respectively. Sections 4 presents a comparative example. Some simulation results are presented in Section 5 and finally, in Section 6, some closing remarks are given.

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2. EQUIVALENT OUTPUTS APPROACH

Consider a non-minimum phase process with a mathematical model of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x), \end{aligned} \tag{1}$$

where x denotes the vector of state variables, u denotes the manipulated input vector, and y denotes the controlled output vector. It is assumed that $x \in X \subset \Re^n$ is a connected open set that includes x^*, $u \in U \subset \Re^m$ that includes u^*, $f, g : X \to \Re^n$ and $h : X \to \Re^m$ are real analytic functions, and (x^*, u^*) denotes the nominal steady-state (equilibrium) pair of the process, that is, $f(x^*) + g(x^*)u^* = 0$. The relative order of a controlled output y_i, is denoted by r_i, where r_i is the smallest integer for which $[L_{g_i} L_f^{r_i-1} h_i(x) \dots L_{g_m} L_f^{r_i-1} h_i(x)] \not\equiv [0 \dots 0]$, where L_{g_i} and L_f are Lie derivative operators. The characteristic matrix for a system of the form of (1) with finite relative orders r_i is defined as

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$$C(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}.$$

It is assumed that $\mathrm{rank}(g) = m$ and $\det(C) \neq 0$. If the nonlinear system of (1) is non-minimum phase, an input/output linearizing model-state feedback controller may induce closed-loop instability. An approach based on the notion of statically equivalent outputs, that makes the non-minimum phase system minimum phase as far as the controller is concerned, is proposed in Niemiec and Kravaris (2003). Let $h_{a_1} \dots h_{a_m}$ be auxiliary output maps with the following properties:

(1) The system
$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y_a &= h_a(x), \end{aligned} \tag{2}$$
is locally hyperbolically minimum-phase.
(2) The outputs $y_i = h_i(x)$ and $y_{a_i} = h_{a_i}(x)$ are statically equivalent in the sense that $h_i(x^*) = h_{a_i}(x^*)$ for $i = 1, \dots, m$.

If such output maps can be found, controlling the output y_i to a constant set point can be accomplished by controlling the output y_{a_i} to the identical set point. In this case, input/output linearizing state feedback may be based on the minimum phase auxiliary output maps, and the following model-state-feedback controller

$$\begin{aligned} u_a = &\{[\gamma_{1r_{a_1}} \dots \gamma_{mr_{a_m}}] C_a\}^{-1} \\ &\left\{ (y_{sp} - y) + (h - h_a) - \sum_{i=1}^{m} \sum_{k=1}^{r_{a_i}} \gamma_{ik} L_f^k h_{a_i} \right\}, \end{aligned}$$

achieves local asymptotic stability and zero steady-state error for an open-loop locally hyperbolically stable system (1), where y_{sp} is the desired set-point, r_{a_i} are the relative orders of the auxiliary outputs y_{a_i}, C_a is the corresponding characteristic matrix and $\gamma_{ik} \in \Re^m$ are constant parameters that satisfy $\det[\gamma_{1r_{a_1}} \dots \gamma_{mr_{a_m}}] \neq 0$.

2.1 Construction of statically equivalent outputs

The first step in the derivation of the auxiliary outputs is the construction of $(n-m)$ independent functions that vanish on the equilibrium manifold. Using these $(n-m)$ vanishing functions, a large class of outputs $h_{a_i}(x), i = 1, \dots, m$, are constructed, which are statically equivalent to $h_i(x), i = 1, \dots, m$, and depend on a number of arbitrary weighting parameters that are selected to place the transmission zeros of the linearization of (2) such that the controller exhibits a local minimum phase behavior. Assuming that the $m \times m$ matrix

$$G = \begin{bmatrix} g_{1,n-m+1}(x) & \cdots & g_{m,n-m+1}(x) \\ \vdots & & \vdots \\ g_{1,n}(x) & \cdots & g_{m,n}(x) \end{bmatrix}$$

is nonsingular, the following scalar functions $\eta_1(x), \dots, \eta_{n-m}(x)$, that vanishes at the equilibrium can be constructed as

$$\begin{aligned} \eta_j(x) &= f_j(x) - \\ &[g_{1,j}(x) \dots g_{m,j}(x)] G^{-1}(x) \begin{bmatrix} f_{n-m+1}(x) \\ \vdots \\ f_n(x) \end{bmatrix} \\ j &= 1, \dots, n-m, \end{aligned} \tag{3}$$

and the following class of output map can be considered
$$h_a(x) = h(x) + \Lambda \eta(x), \tag{4}$$
where
$$\Lambda = \begin{bmatrix} \lambda_{1,1} & \cdots & \lambda_{1,n-m} \\ \vdots & & \vdots \\ \lambda_{m,1} & \cdots & \lambda_{m,n-m} \end{bmatrix} \text{ and } \eta(x) = \begin{bmatrix} \eta_1(x) \\ \vdots \\ \eta_{n-m}(x) \end{bmatrix},$$
with $\lambda_{i,j}$ being arbitrary real numbers, or, more generally, arbitrary state-dependent weight functions $\lambda_{i,j}(x)$. The output map (4) are statically equivalent to $h(x)$, in the sense that $h_a(x^*) = h(x^*)$. Values of $\lambda_{i,j}$ that assign the zeros of the linear approximation of (2) to desired locations $z_1^d, \dots, z_{n-m}^d$ are determined from
$$\Lambda = -(CS)(NS)^{-1},$$
where S is the $n \times (n-m)$ matrix which solves
$$SP - AS = BQ,$$
with
$$A = \frac{\partial f}{\partial x}(x^*) + \sum_{i=m}^{m} u_i^* \frac{\partial g_i}{\partial x}(x^*), \; B = [g_1(x^*) \dots g_m(x^*)],$$
$$C = \frac{\partial h}{\partial x}(x^*), \text{ and } N = \frac{\partial \eta}{\partial x}(x^*).$$
P and Q being $(n-m) \times (n-m)$ and $m \times (n-m)$ matrices, respectively, and P is such that P and A do not have common eigenvalues, (Q, P) is an observable pair, and the eigenvalues of P are the desired transmission zeros of system (2).

3. THE IDA-PBC METHODOLOGY

IDA-PBC was introduced in Ortega et al. (2002) as a procedure to control physical systems described by PCH models of the form
$$\begin{aligned} \dot{x} &= (J - R)\nabla H + gu \\ y &= g^T \nabla H. \end{aligned} \tag{5}$$

Since its introduction, the IDA-PBC methodology has been applied to a wide range of non-linear system. The key step in this design methodology is the solution of a partial differential equation (PDE) that guarantees closed-loop stability of the controller. However, it is also possible, as described in section 3.2, to use simplified procedures which do not require solving any PDE to define the stability of the controlled system.

3.1 Exact Matching IDA-PBC

Consider the system (1), and assume that it can be modeled as a PCH of the form (5), and there exist matrices
$$J_d = J + J_a$$

$$R_d = R + R_a$$

and $g^{\perp}(x)_{(n-m)\times m}$, a function

$$H_d = H + H_a$$

where $J_d(x)_{n\times n} = -J_d(x)^{\top}_{n\times n}$ and $R_d(x)_{n\times n} = R_d(x)^T_{n\times n} \geq 0$, are the desired closed loop interconnection and damping matrices respectively, $H_d(x) : \Re^n \to \Re$, is the desired total stored energy, that verify the PDE

$$g^{\perp}(J - R)\nabla H = g^{\perp}(J_d - R_d)\nabla H_d \tag{6}$$

and the relation

$$(J - R)\nabla H + gu = (J_d - R_d)\nabla H_d \tag{7}$$

where $g^{\perp}(x)$ is a full-rank left annihilator of $g(x)$, i.e., $g^{\perp}(x)g(x) = 0$, and H_d is such that the following conditions holds

$$\textit{Equilibrium assignment, } \nabla H_d(x^*) = 0$$

$$\textit{Lyapunov stability, } \nabla^2 H_d(x^*) \geq 0$$

with $x^* \in \Re$ the equilibrium to be stabilized. Then, the closed-loop system (5) with $u = \beta(x)$, where

$$\beta(x) = (g^{\top}g)^{-1}g^{\top}\left[(J_d - R_d)\nabla H_d - (J - R)\nabla H\right] \tag{8}$$

takes the PCH form

$$\dot{x} = (J_d - R_d)\nabla H_d \tag{9}$$

with x^* a (locally) stable equilibrium. It will be asymptotically stable if, in addition, x^* is an isolated minimum of H_d and the largest invariant set under the closed-loop dynamics (9) contained in

$$\left\{x \in \Re^n \mid \nabla H_d^{\top} R_d \nabla H_d = 0\right\}$$

equals $\{x^*\}$. An estimate of its domain of attraction is given by the largest bounded level set $\{x \in \Re^n | \; H_d(x) \leq c\}$. The conditions (6) y (7), can be summarized in one equation whose solution guarantee the closed-loop stability and gives the control action:

$$Q\dot{x} = Q(J_d - R_d)\nabla H_d \tag{10}$$

where $Q(x) = [\, g^{\perp}(x) \;\; g^{\top}(x)\,]^{\top}$. The closed-loop dynamic is given by

$$\dot{x} = Q^{-1}Q(J_d - R_d)\nabla H_d = (J_d - R_d)\nabla H_d.$$

The stability of the controller is determined by the derivative of $H_d(x)$ being less or equal to zero along the trajectories of x (Ortega and García-Canseco, 2004). If (6) holds, the derivative becomes

$$\dot{H}_d = -\nabla H_d^{\top} R_d \nabla H_d.$$

This is a quadratic expression and therefore always smaller or equal to zero. The controller design methodology resumed above achieves stabilization by rendering the system passive with respect to a desired storage function and injecting damping. It should be noticed that in this design procedure the key step is the solution of (6).

3.2 Non-Exact Matching IDA-PBC

If (6) does not hold, (10) becomes

$$Q\dot{x} = Q(J_d - R_d)\nabla H_d + [\,\delta^{\top} \;\, 0 \, \cdots \, 0\,]^{\top}$$

where $[\,\delta^{\top} \;\, 0 \cdots 0\,]^{\top}$ is a $n \times 1$ matrix, and $\delta(x)$ corresponds to the error of rank $n - m$ in (6) and is given by

$$\delta = g^{\perp}\left[(J_d - R_d)\nabla H_d - (J - R)\nabla H\right]. \tag{11}$$

In this case, the closed-loop dynamic with control action (8) is

$$\dot{x} = (J_d - R_d)\nabla H_d + Q^{-1}[\,\delta^{\top} \;\, 0 \, \cdots \, 0\,]^{\top}. \tag{12}$$

The derivative of H_d along the trajectories of x is in the non-matching case

$$\dot{H}_d = -\nabla H_d^{\top} R_d \nabla H_d + \nabla H_d^{\top} Q^{-1}[\,\delta^{\top} \;\, 0 \, \cdots \, 0\,]^{\top},$$

this means that the stability of the closed-loop system (12) depends on the error (11), and this one in turn depends on the non-matching solution of (6). The following propositions provide the conditions guaranteeing local and global stability of a non-matching IDA-PB controller (Ramírez, 2008).

Proposition 1. Consider a PCH system as in (5), with a non-matching IDA-PB controller with control action given by (8), and a neighborhood $D = \{x \in \Re^n \mid \|x\|_2 < r\}$ of x^* where $r > 0$ and such that the following conditions holds

(1) $x^* \in D$ is an assignable equilibrium point of (5) and an isolated minimum of $H_d(x)$,
(2) $\forall x \in D, \quad \nabla H_d^{\top} R_d \nabla H_d \geq \nabla H_d^{\top} Q^{-1}[\,\delta^{\top} \;\, 0 \, \cdots \, 0\,]^{\top}$,
(3) and the largest invariant set under the closed-loop dynamics (12) contained in $S = \{x \in D | \nabla H_d^{\top} R_d \nabla H_d + \nabla H_d^{\top} Q^{-1}[\,\delta^{\top} \;\, 0 \, \cdots \, 0\,]^{\top} = 0\}$ equals $\{x^*\}$,

then the closed-loop system (12) is asymptotically stable.

Proposition 2. Consider a PCH system as in (5), with a non-matching IDA-PB controller that holds proposition 1, if $D = \{x \in \Re^n \mid \|x\|_2 < r\}$ where $r \to \infty$, then the closed-loop system (12) is globally asymptotically stable.

4. A COMPARATIVE EXAMPLE: CONTROL OF A NONMINIMUM-PHASE CHEMICAL REACTOR

From the descriptions of both approaches, it is possible to notice that the design using IDA-PBC offers the following advantages over the synthetic output approach: it is much simpler, does not rely on linearization and provides a much clear insight into the structure of the problem. In order to illustrate further the differences between these two methods a classical example is presented. Let's consider a non-isothermal CSTR with series/parallel reactions taking place (Niemiec and Kravaris, 2003; Guay et al., 2005; Antonelli and Astolfi, 2003). The reactor model consists of mole balances for species A and B, and an energy balance for the reactor:

$$\begin{aligned}
\dot{x}_1 &= -k_1(x_3)x_1 - k_3(x_3)x_1^2 + (x_{10} - x_1)u_1 \\
\dot{x}_2 &= k_1(x_3)x_1 - k_2(x_3)x_2 - x_2u_1 \\
\dot{x}_3 &= \vartheta(x) + \frac{u_2}{\rho C_p} + (x_{30} - x_3)u_1,
\end{aligned}$$

C_A ,C_B	Molar concentrations of A and B
T	Reactor temperature
F/V	Dilution rate
Q_H	Rate of heat added or removed per unit volume
C_p	Heat capacity of the reacting mixture
ρ	Density of the reacting mixture
ΔH	Heat of reaction
E	Activation energy
R	Joule constant

Table 1. CSTR Parameters

with

$$\vartheta(x) = -\frac{\Delta H_1 k_1(x_3)x_1 + \Delta H_2 k_2(x_3)x_2 + \Delta H_3 k_3(x_3)x_1^2}{\rho C_p}.$$

The rate coefficients k_i are dependent on the reactor temperature via the Arrhenius equation

$$k_i(x_3) = k_{i0} \exp \frac{E_i}{Rx_3} \;\; i = 1, 2, 3.$$

The parameters are summarized in table 1. The control objective (Niemiec and Kravaris, 2003), is to maintain the outputs $y_1 = x_3 = T$ and $y_2 = x_2 = C_B$ at set points by manipulating the dilution rate $u_1 = F/V$ and the rate of heat addition or removal per unit volume $u_2 = Q_H$. Initially, the reactor is operating at a steady-state of $x_1^* = 1.25 mol/l$, $x_2^* = 0.90 mol/l$, and $T^* = 407.15K$, which corresponds to $u_1^* = 19.52/h$ and $u_2^* = -451.51 kJ/(lh)$. Around this steady-state, the process is locally asymptotically stable with eigenvalues of -96.465 and $-33.154 \pm 9.815i$. The transmission zero of the linearized system is found to be +122.71. This indicates that the process is locally non-minimum phase around the given steady-state due to the right-half plane transmission zero.

Control Using Synthetic Outputs Since $n - m = 1$, there is only one independent function that vanishes on the equilibrium manifold. One such function can be generated according to (3),

$$\eta(x) = -x_2(k_1x_1 + k_3x_1^2) + (x_{10} - x_1)(k_1x_1 - k_2x_2).$$

Statically equivalent outputs for the given system can be constructed as $y_{a_1} = x_3 + \lambda_1\eta$ and $y_{a_2} = x_2 + \lambda_2\eta$. Following the procedure in Niemiec and Kravaris (2003), defining $[\,\gamma_{11}\ \gamma_{21}\,]$ as $diag\{\gamma, \gamma\}$, the following characteristic matrix

$$C_a(x) = \begin{bmatrix} L_{g_1}L_f^0 h_{a_1} & L_{g_2}L_f^0 h_{a_1} \\ L_{g_1}L_f^0 h_{a_2} & L_{g_2}L_f^0 h_{a_2} \end{bmatrix} = \begin{bmatrix} C_{a_{11}} & C_{a_{12}} \\ C_{a_{21}} & C_{a_{22}} \end{bmatrix},$$

with,

$$\begin{aligned}
C_{a_{11}} &= \lambda_1(x_{30} - x_3)\nu_1 - \lambda_1 x_2\nu_2 + (x_{30} - x_3)(1 + \lambda_1\nu_3), \\
C_{a_{12}} &= (1 + \lambda_1\nu_3)/(\rho C_p), \\
C_{a_{21}} &= \lambda_2(x_{30} - x_3)\nu_1 - x_2(1 + \lambda_2\nu_2) + \lambda_2(x_{30} - x_3)\nu_3, \\
C_{a_{22}} &= \lambda_2\nu_3/(\rho C_p), \\
\nu_1 &= -x_2(k_1 + k_3 2x_1) - (k_1x_1 - k_2x_2) + k_1(x_{10} - x_1), \\
\nu_2 &= -(k_1x_1 + k_3x_1^2) - k_2(x_{10} - x_1), \\
\nu_3 &= -k_1E_1(-x_1x_2 + x_1(x_{10} - x_1)) - \\
&\quad\; k_2E_2(-x_2(x_{10} - x_1)) - k_3E_3(x_1^2x_2),
\end{aligned}$$

and the Lie derivatives

$$\begin{aligned}
L_f^1 h_{a_1} &= -\lambda_1(k_1x_1 + k_3x_1^2)\nu_1 + \\
&\quad \lambda_1(k_1x_1 - k_2x_2)\nu_2 + \vartheta(1 + \lambda_1\nu_3), \\
L_f^1 h_{a_2} &= -\lambda_2(k_1x_1 + k_3x_1^2)\nu_1 + \\
&\quad (k_1x_1 - k_2x_2)(1 + \lambda_2\nu_2) + \lambda_2\vartheta\nu_3,
\end{aligned}$$

defines the control law

$$[u_{a_1}(x)\;\; u_{a_2}(x)]^\top, \tag{13}$$

where

$$\begin{aligned}
u_{a_1} &= \\
&\Big[C_{a_{22}}(\gamma(x_3 - h_{a_1}) - L_f^1 h_{a_2}) - \\
&\quad C_{a12}(\gamma(x_2 - h_{a_2}) - L_f^1 h_{a_1})\Big]\frac{1}{C_{a_{11}}C_{a_{22}} - C_{a_{12}}C_{a_{21}}}, \\
u_{a_2} &= \\
&\Big[-C_{a_{21}}(\gamma(x_3 - h_{a_1}) - L_f^1 h_{a_2}) + \\
&\quad C_{a11}(\gamma(x_2 - h_{a_2}) - L_f^1 h_{a_1})\Big]\frac{1}{C_{a_{11}}C_{a_{22}} - C_{a_{12}}C_{a_{21}}}.
\end{aligned}$$

In order to place the transmission zero at -122.71 (the reflection of the process zero with respect to the imaginary axis), the following values for the weights $\lambda_1 = -5.45375 \times 10^{-6}$ and $\lambda_2 = 2.77156 \times 10^{-3}$, and $\gamma = 0.01$, are used in Niemiec and Kravaris (2003).

PCH Representation The first step in the application of the design methodology detailed in section 3, is to express the CSTR process as a PCH system. Writing down the systems equations as a PCH model is not direct. The fact that the enthalpy ΔH_1, has opposite sign compared to the others enthalpies, make it impossible to include the term ϑ in the damping matrix R directly. Observing that ΔH_3 is the most negative of the system enthalpies, and that the divisions $\frac{\Delta H_1}{\Delta H_3}$ y $\frac{\Delta H_2}{\Delta H_3}$ are less than one, a new state variable can be obtained dividing $\dot{x}_3$ by $-\frac{\rho C_p}{\Delta H_3}$. Thus $\dot{\bar{x}}_3$ is

$$\begin{aligned}
\dot{\bar{x}}_3 = \; & \frac{1}{\Delta H_3}(\Delta H_1 k_1 x_1 + \Delta H_2 k_2 x_2 - u_2) + k_3 x_1^2 - \\
& \frac{\rho C_p}{\Delta H_3}(x_{30} - x_3)u_1,
\end{aligned}$$

with the new rate coefficient

$$k_i(\bar{x}_3) = k_{i0} \exp\left(-\frac{E_i \Delta H_3}{R\bar{x}_3\rho C_p}\right) \quad i = 1, 2, 3.$$

Introducing this change of variables the following equivalent model is obtained

$$\begin{aligned}
\dot{x}_1 &= -k_1x_1 - k_3x_1^2 + (x_{10} - x_1)u_1 \\
\dot{x}_2 &= k_1x_1 - \left[\frac{\Delta H_2}{\Delta H_3}k_2x_2 + \left(1 - \frac{\Delta H_2}{\Delta H_3}\right)k_2x_2\right] - x_2u_1 \\
\dot{\bar{x}}_3 &= \frac{\Delta H_1}{\Delta H_3}k_1x_1 + \frac{\Delta H_2}{\Delta H_3}k_2x_2 + k_3x_1^2 - \frac{1}{\Delta H_3}u_2 - \\
&\quad \frac{\rho C_p}{\Delta H_3}(x_{30} - x_3)u_1
\end{aligned}$$

From the transformed model the following PCH system is considered

$$H = x_1 + x_2 + \bar{x}_3. \tag{14}$$

$$J = \begin{bmatrix} 0 & -k_1x_1 & -k_3x_1^2 \\ k_1x_1 & 0 & -\dfrac{\Delta H_2}{\Delta H_3}k_2x_2 \\ k_3x_1^2 & \dfrac{\Delta H_2}{\Delta H_3}k_2x_2 & 0 \end{bmatrix}, \quad (15)$$

$$R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \left(1-\dfrac{\Delta H_2}{\Delta H_3}\right)k_2x_2 & 0 \\ 0 & 0 & -\dfrac{\Delta H_1}{\Delta H_3}k_1x_1 \end{bmatrix}, \quad (16)$$

$$g = \begin{bmatrix} x_{10}-x_1 & 0 \\ -x_2 & 0 \\ -\dfrac{\rho C_p}{\Delta H_3}(x_{30}-x_3) & -\dfrac{1}{\Delta H_3} \end{bmatrix}, \quad (17)$$

Since ΔH_3 is negative, the term $-\frac{\Delta H_1}{\Delta H_3}k_1x_1$ becomes positive and the matrix R is positive defined.

IDA-PBC Synthesis The desired performance can be achieved by shaping the interconnection and damping matrices of the closed-loop PCH system. Its desirable that the dynamics of the controlled system are just given by dissipative elements; i.e.

$$\dot{x} = -R_d\nabla H_d.$$

Such performance can be achieved selecting a null desired interconnection matrix. Notice that in the open-loop PCH system part of the dissipation for x_1 and x_2 occurs through the natural interconnection matrix J. As a null closed-loop interconnection matrix is selected, the elements of the original interconnection structure must be introduced in the closed-loop damping matrix. Taking this under consideration, a decoupled closed-loop system can be obtained using the following matrices

$$J_a = -J,$$

$$R_a = \begin{bmatrix} k_1x_1+k_3x_1^2 & 0 & 0 \\ 0 & \dfrac{\Delta H_2}{\Delta H_3}k_2x_2 & 0 \\ 0 & 0 & \left(1+\dfrac{\Delta H_1}{\Delta H_3}\right)k_1x_1 \end{bmatrix}.$$

The closed-loop system is characterized by the following desired interconnection and damping matrices

$$J_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; R_d = \begin{bmatrix} k_1x_1+k_3x_1^2 & 0 & 0 \\ 0 & k_2x_2 & 0 \\ 0 & 0 & k_1x_1 \end{bmatrix}. \quad (18)$$

In Ramírez (2008), several general "non-exact matching" storage functions that achieves (locally) stable IDA-PB controllers for a wide range of PCH systems were proposed. One of them is the *logarithmic* storage function which achieves the closed-loop desired storage function

$$H_d(x) = \sum_{i=1}^{n}\kappa_i(x_i - x_i^*\ln x_i), \quad (19)$$

with closed-loop gradient

$$\nabla H_d(x) = \left[\,\kappa_1\left(1-\frac{x_1^*}{x_1}\right)\;\kappa_2\left(1-\frac{x_2^*}{x_2}\right)\;\cdots\;\kappa_n\left(1-\frac{x_n^*}{x_n}\right)\,\right]^\top. \quad (20)$$

The control law can be directly obtained replacing (14),(15),(16),(17),(18) and (20) in (8)

$$\beta(x) = [\,\beta_1(x)\;\beta_2(x)\,]^\top, \quad (21)$$

where β_1 and β_2 are respectively given by

$$\beta_1(x) = -\frac{(x_{10}-x_1)(k_1x_1-k_3x_1^2)(\tilde{x}_1-1)}{(x_{10}-x_1)^2+x_2^2} - \frac{x_2\,(k_1x_1-k_2x_2(\tilde{x}_2-1))}{(x_{10}-x_1)^2+x_2^2},$$

$$\beta_2(x) = \rho C_p(x_{30}-x_3)\beta_1(x) - \frac{1}{\Delta H_3}\left(k_1x_1\left(\tilde{x}_3-\frac{\Delta H_1}{\Delta H_3}\right)+\frac{\Delta H_2}{\Delta H_3}k_2x_2+k_3x_1^2\right),$$

and $\tilde{x}_i$ is $\kappa_i\left(1-\frac{x_i^*}{x_i}\right)$, with $i=1,2,3$. In order to obtain a closed-loop profile similar to the synthetic outputs controller (13), the values used for the controllers parameters where $\kappa_1 = 2.5$, $\kappa_2 = 2.5$ and $\kappa_3 = 4000$.

Closed-loop Stability The stability analysis of the closed loop system, requires the use of a full-rank left annihilator $g^\perp$ that solves the PDE (6). A possible choice is

$$g^\perp = \begin{bmatrix} x_2 \\ x_{10}-x_1 \\ 0 \end{bmatrix}^\top.$$

For an asymptotically stable closed-loop system proposition 1 must be fulfilled for $x \in \Re^3$. The first condition, on the storage function, holds by construction of (19). The remaining conditions, depends on the shaped interconnection and damping matrices. For this example, the stability conditions becomes

$$\Big\{\forall x \in \Re^2 | \dot{H}_d = -(k_1x_1+k_3x_1^2)\tilde{x}_1^2 - k_2x_2\tilde{x}_2^2 - k_1x_1\tilde{x}_3^2 - \phi x_2(k_1x_1+k_3x_1^2)(\tilde{x}_1-1) - \phi(x_{10}-x_1)(k_1x_1+k_2x_2(\tilde{x}_2-1)) \le 0\Big\}$$

where $\phi = \frac{x_2\tilde{x}_1+(x_{10}-x_1)\tilde{x}_2}{x_2^2+(x_{10}-x_1)^2}$, and the largest invariant set under the closed-loop dynamics of

$$\{x\in\Re^3 \mid \dot{H}=0\} = \{x^*\}.$$

A local asymptotic stability study can be carried out by linearizing the closed-loop system with (21), and analyzing all the eigenvalues of the Jacobian matrix

$$\frac{\partial}{\partial x}\left((J_d-R_d)\nabla H_d + Q^{-1}[\,\delta\;0\;\cdots\;0\,]^\top\right)$$

which must have real part strictly negative.

5. NUMERICAL SIMULATIONS

Figure 1 and figure 2 shows the performance of the synthetic-output controller and the IDA-PB controller for step changes in x_2 set-point while maintaining the set-point of x_3 at $407.15K$. The closed-loop responses for both controllers are similar around the equilibrium point used in the design of (13), but the synthetic-output controller loses the design properties as the set-point of x_2 moves

C_{A0}	5.0 mol/l	k_{10}	$1.287 \times 10^{1}2$ h
T_0	403.15 K	k_{20}	$1.287 \times 10^{1}2$ h
C_p	3.01 kJ/(kg K)	k_{30}	9.403×10^{9} l/(mol h)
ρ	0.94342 kg/l	E_1/R	-9758.3 K
ΔH_1	4.20 kJ/mol	E_2/R	-9758.3 K
ΔH_2	-11.00 kJ/mol	E_3/R	-8560.0 K
ΔH_3	-41.85 kJ/mol		

Table 2. CSTR Numerical Parameters

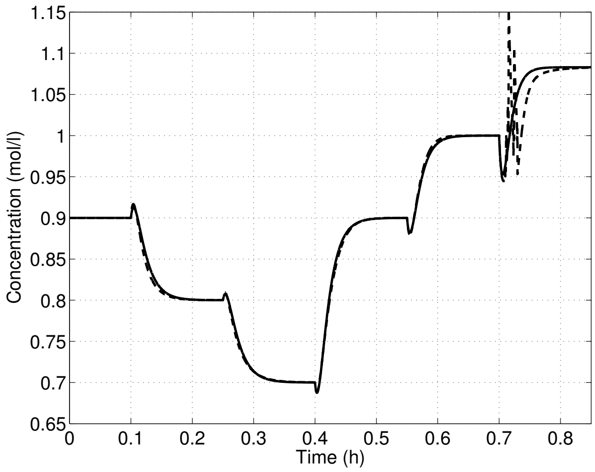

Fig. 1. Performance of controllers for x_2, IDA-PBC (solid), synthetic-output (dashed)

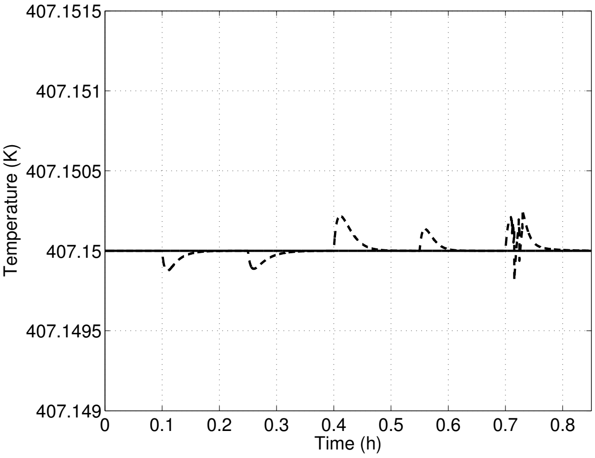

Fig. 2. Performance of controllers for x_3, IDA-PBC (solid), synthetic-output (dashed)

away from that equilibrium. Even though the change in x_3 can be considered negligible with the synthetic-output controller, the decoupling is not perfect. However, the IDA-PB controller (21) achieves exact decoupling of the outputs. The numerical values used in the simulation are summarized in table 2.

6. CONCLUSION

The comparative analysis carried out shows that the IDA-PBC approach poses considerable advantages over synthetic-output linearization. The design based on PCH models enables to easily identify the interconnection structure of the process and achieve decoupled outputs, while the more complex linearization methodology have no direct physical interpretation. The IDA-PBC design does not rely on local approximations (linearization of the dynamics), and therefore the stability region can cover a wide range of operational conditions. Additionally, by using the non exact-matching IDA-PBC approach it is not necessary to find a solution for a PDE, this simplifies the design of the controller since the desired closed-loop PCH structure can be selected regarding to physical properties of the process. The stability of the IDA-PB controller is verified by a linear analysis and the performance have been compared using a classical example with the input-(synthetic)output linearization controller proposed in Niemiec and Kravaris (2003). The numerical simulations have shown that the controllers exhibits similar response around the nominal equilibrium point. As the operational conditions change the IDA-PB controller maintains the desired closed-loop behavior unlike the synthetic output controller and also provides exact decoupling of the process outputs. Further research could compare the IDA-PBC approach with methods not based on linearization as the one presented in Ball et al. (2004).

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