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# A New Saturated Nonlinear PID Global Regulator for Robot Manipulators [1]

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**Abstract:** It is well known that industrial robots use the classical PID for positioning tasks. To the authors' knowledge, so far, there is not a proof of global regulation for such a controller. In the search of a practical PID regulator that be global, this paper proposes a new saturated nonlinear PID regulator for solving the problem of global regulation of robot manipulators with bounded torques. An approach based on Lyapunov theory is used for analyzing the global asymptotic stability. In this sense this proposal gives a step ahead in the search of a global asymptotic stability analysis for the practical PID. 

Keywords: Nonlinear PID, passivity, bounded control, asymptotic stability.

## 1. INTRODUCTION

Industrial robots are naturally equipped with classical PID controllers, which theoretically assure semiglobal asymptotic stability of the closed loop equilibrium for the regulation case (see e.g., Arimoto and Miyazaki, 1984; Arimoto *et al.*, 1990; Kelly, 1995; Ortega *et al.*, 1995; Alvarez–Ramirez *et al.*, 2000; Kelly *et al.*, 2005; Meza *et al.*, 2007). Uniformly and ultimately boundedness of the closed loop solutions can be concluded when the desired position is a function of time; some stability analyzes for this latter case can be found, for instance, in: Kawamura *et al.* (1988), Wen and Murphy (1990), Qu and Dorsey (1991), Rocco (1996), Cervantes and Alvarez–Ramirez (2001), Choi and Chung (2004) and Camarillo *et al.* (2008).

On the one hand, it is well known that the saturation phenomenon in robot control systems is intrinsically present when the actuators are driven by sufficiently large control signals. If this physical constraint is not considered in the controller design, it may lead to a lack of stability guarantee. Some works have been reported to solve this problem (Kelly, and Santibañez, 1996; Colbaugh, *et al.*, 1997a; Colbaugh, *et al.*, 1997b; Loria, *et al.*, 1997; Santibañez, and Kelly, 1997; Santibañez and Kelly, 1998b; Zergeroglu, *et al.*, 2000). More recently, new schemes dealing with this problem have been presented: Zavala-Rio and Santibañez (2006), Zavala-Rio and Santibanez (2007), Dixon (2007) and Alvarez–Ramirez, *et al.*, (2008), for the regulation case. Also for the tracking case some works have appeared in the control literature: Loria and Nijmeijer (1998), Dixon *et al.* (1999), Santibañez and Kelly (2001), Aguiñaga (2006), Moreno *et al.* (2008a) and Moreno *et al.*, (2008b). Some Ph.d. and M. Sc. thesis works dealing with the trajectory tracking with bounded inputs problem have been presented; see e.g., Loria (1996), Santibañez (1997), Licona (2002) and Aguiñaga (2006).

On the other hand, some global nonlinear PID regulators, which are based on Lyapunov and passivity theory, has been reported in (Arimoto, 1995; Kelly, 1998; Santibañez, and Kelly, 1998a; Meza and Santibañez, 1999) however, they do not take into account the effects of actuators saturation. To the best of the authors' knowledge, so far, a few saturated PID controllers have been reported; namely, two semiglobal saturated linear PID controller (Alvarez–Ramirez, *et al.*, 2003) and Alvarez–Ramirez, *et al.*, (2008), and two global saturated nonlinear PID controllers (Gorez, 1999; Meza *et al.*, 2005). The work introduced by Gorez (1999) was the first PID–like controller in assuring global regulation, the latter work, introduced in Meza *et al* (2005), also guarantees global regulation, but with the

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advantage of its structure is simpler than that presented in (Gorez, 1999).

In this paper, in the search of a practical PID regulator that be global, we propose a new saturated nonlinear PID regulator for robot manipulators with bounded torques. The structure of this new proposed controller is closer to the structure of the practical PID used in the industry. An approach based on Lyapunov theory is used for analyzing the global asymptotic stability. In this sense, this proposal gives a step ahead in the search of a global asymptotic stability analysis for the practical PID.

Throughout this paper, we use the notation $\lambda_{\mathrm{m}}\{A(\boldsymbol{x})\}$ and $\lambda_{\mathrm{M}}\{A(\boldsymbol{x})\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in \mathbb{R}^n$. By an abuse of notation, we define $\lambda_{\mathrm{m}}\{A\}$ as the greatest lower bound (infimum) of $\lambda_{\mathrm{m}}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$, that is, $\lambda_{\mathrm{m}}\{A\} := \inf_{\boldsymbol{x}\in\mathbb{R}^n} \lambda_{\mathrm{m}}\{A(\boldsymbol{x})\}$. Similarly, we define $\lambda_{\mathrm{M}}\{A\}$ as the least upper bound (supremum) of $\lambda_{\mathrm{M}}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$, that is, $\lambda_{\mathrm{M}}\{A\} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \lambda_{\mathrm{M}}\{A(\boldsymbol{x})\}$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ and that of matrix $A(\boldsymbol{x})$ is defined as the corresponding induced norm $\|A(\boldsymbol{x})\| = \sqrt{\lambda_{\mathrm{M}}\{A(\boldsymbol{x})^T A(\boldsymbol{x})\}}$.

## 2. DYNAMICS OF RIGID ROBOTS AND CONTROL PROBLEM FORMULATION

The dynamics of a serial $n$-link rigid robot, including the effect of viscous friction, can be written as (Spong and Vidyasagar, 1989):

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques, $F_v$ is the $n \times n$ diagonal matrix of viscous friction coefficients $f_{vi}$ for $i = 1, 2, \ldots, n$, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$, i.e.

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}}. \tag{2}$$

We assume that the links are joined together with revolute joints.

### 2.1 Properties of the Robot Dynamics

Three important properties of dynamics (1) are the following:

**Property 1.** (Koditschek, 1984; Ortega and Spong, 1989). The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy:

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right] \dot{\boldsymbol{q}} = \boldsymbol{0} \qquad \forall\ \boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n$$

and

$$\dot{M}(\boldsymbol{q}) = C(\boldsymbol{q},\dot{\boldsymbol{q}}) + C(\boldsymbol{q},\dot{\boldsymbol{q}})^T \qquad \forall\ \boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n.$$

Furthermore there exists a positive constant $k_{c_1}$ such that for all $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z} \in \mathbb{R}^n$ it has

$$\|\boldsymbol{c}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{z}\| \leq k_{c_1}\, \|\boldsymbol{y}\|\, \|\boldsymbol{z}\|.$$

$\diamond$

**Property 2.** (Craig, 1988). The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q} \in \mathbb{R}^n$. This means that there exist finite constants $\bar{g}_i \geq 0$ such that

$$\sup_{\boldsymbol{q}\in\mathbb{R}^n} |g_i(\boldsymbol{q})| \leq \bar{g}_i \qquad i = 1, \cdots, n. \tag{3}$$

where $g_i(\boldsymbol{q})$ stands for the elements of $\boldsymbol{g}(\boldsymbol{q})$. Equivalently, there exists a constant $k'$ such that

$$\|\boldsymbol{g}(\boldsymbol{q})\| \ \leq\ k' \quad \text{for all} \ \ \boldsymbol{q} \in \mathbb{R}^n.$$

Furthermore there exists a positive constant $k_g$ such that

$$\|\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{y})\| \leq k_g\|\boldsymbol{x} - \boldsymbol{y}\|.$$

for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$. $\diamond$

### 2.2 Problem Formulation

Consider the robot dynamic model (1). Assume that each joint actuator is able to supply a known maximum torque $\tau_i^{\max}$ so that:

$$|\tau_i| \leq \tau_i^{\max}, \qquad i = 1, \cdots, n \tag{4}$$

where $\tau_i$ stands for the $i$–entry of vector $\boldsymbol{\tau}$. We also assume that the maximum torque $\tau_i^{\max}$ of each actuator satisfies the following condition

$$\tau_i^{\max} > \bar{g}_i, \tag{5}$$

where $\bar{g}_i$ was defined in Property 2. This assumption means that the robot actuators are able to supply torques in order to hold the robot at rest for all desired joint position $\boldsymbol{q}_d \in \mathbb{R}^n$.

The control problem is to design a controller to compute the torque $\boldsymbol{\tau} \in \mathbb{R}^n$ applied to the joints, which satisfies the constraints (4), such that, the robot joint displacements $\boldsymbol{q}$ tend asymptotically toward the constant desired joint displacements $\boldsymbol{q}_d$.

## 3. SATURATION DEFINITIONS

Before presenting the main contribution of the paper, we recall some definitions of the continuous saturation functions that we use in our proposal.

**Definition 1.** (Kelly, 1998). $\mathcal{F}(m, \varepsilon, \boldsymbol{x})$ with $1 \geq m > 0$, $\varepsilon > 0$ and $\boldsymbol{x} \in \mathbb{R}^n$ denotes the set of all continuously differentiable increasing functions

$$\mathbf{sat}(\boldsymbol{x}) = [\mathrm{sat}(x_1)\ \mathrm{sat}(x_2)\ \cdots\ \mathrm{sat}(x_n)]^T$$

such that

- $|x| \geq |\mathrm{sat}(x)| \geq m|x|, \quad \forall x \in \mathbb{R} : |x| < \varepsilon$
- $\varepsilon \geq |\mathrm{sat}(x)| \geq m\varepsilon, \quad \forall x \in \mathbb{R} : |x| \geq \varepsilon$
- $1 \geq \frac{d\mathrm{sat}(x)}{dx} \geq 0, \quad \forall x \in \mathbb{R}$ $\diamond$

**Definition 2.** The hard saturation function $\mathbf{SAT}(\boldsymbol{x};\boldsymbol{k}) \in \mathbb{R}^{\mathbf{n}}$ is defined by

$$\mathbf{SAT}(\boldsymbol{x};\boldsymbol{k}) = \begin{bmatrix} \mathrm{SAT}(x_1;k_1) \\ \mathrm{SAT}(x_2;k_2) \\ \vdots \\ \mathrm{SAT}(x_n;k_n) \end{bmatrix},\ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix},$$

$\boldsymbol{k} = [\,k_1\; k_2 \ldots k_n\,]^T$, where $k_i$ is the $i$–th saturation limit, and

$$\mathrm{SAT}(x_i; k_i) = \begin{cases} x_i & \text{if } \; |x_i| \le k_i \\ k_i & \text{if } \; x_i > k_i \\ -k_i & \text{if } \; x_i < -k_i \end{cases}$$

for $\;i = 1, 2, \ldots n.$ ◇

Some useful properties of the above saturation functions are in order:

**Property 3.** (Kelly *et al.*, 2005). From Definition 1, it is easy to prove that:

$$\|\mathbf{sat}\,(\boldsymbol{x})\| \le \begin{cases} \|\boldsymbol{x}\| & \forall\, \boldsymbol{x} \in \mathbb{R}^n \\ \varepsilon\sqrt{n} & \forall\, \boldsymbol{x} \in \mathbb{R}^n \end{cases} \tag{6}$$

and

$$\|\mathbf{sat}\,(\Lambda\boldsymbol{x})\| \le \begin{cases} \|\Lambda\boldsymbol{x}\| & \forall\, \boldsymbol{x} \in \mathbb{R}^n \\ \varepsilon\sqrt{n} & \forall\, \boldsymbol{x} \in \mathbb{R}^n \end{cases} \tag{7}$$

where $\Lambda$ is a diagonal positive definite matrix. ◇

**Property 4.** (Kelly *et al.*, 2005). Similarly, from Definition 1, we have:

$$\left\|\frac{d}{dt}\,\mathbf{sat}\,(\boldsymbol{x})\right\| := \left\|\dot{\mathbf{sat}}\,(\boldsymbol{x})\right\| \le \|\dot{\boldsymbol{x}}\| \qquad \forall\, \boldsymbol{x} \in \mathbb{R}^n \tag{8}$$

and

$$\left\|\dot{\mathbf{sat}}\,(\Lambda\boldsymbol{x})\right\| \le \|\Lambda\dot{\boldsymbol{x}}\| \qquad \forall\, \boldsymbol{x} \in \mathbb{R}^n \tag{9}$$

where $\Lambda$ is a diagonal positive definite matrix. ◇

**Property 5.** From Definition 1 and 2 it is easy to see that, for all $\delta_i \ge \varepsilon$ with $i = 1, 2, \ldots, n$, we have:

$$\|\boldsymbol{x}\| \ge \|\mathbf{SAT}\,(\boldsymbol{x};\boldsymbol{\delta})\| \ge \|\mathbf{sat}\,(\boldsymbol{x})\| \qquad \forall\, \boldsymbol{x} \in \mathbb{R}^n \tag{10}$$

and

$$\|\Lambda\boldsymbol{x}\| \ge \|\mathbf{SAT}\,(\Lambda\boldsymbol{x};\boldsymbol{\delta})\| \ge \|\mathbf{sat}\,(\Lambda\boldsymbol{x})\| \qquad \forall\, \boldsymbol{x} \in \mathbb{R}^n \tag{11}$$

where $\Lambda$ is a diagonal positive definite matrix. ◇

**Property 6.** The integral of a hard saturation function $I(x_i) - \min_{x_i}\{I(x_i)\}$ is a positive definite function with a unique and global minimum at $x_i = b/k$, with $|b| < \delta_i$; where

$$I(x_i) := \int_b^{x_i} \left[\mathrm{SAT}\,(k\xi_i; \delta_i) - b\right] d\xi_i,$$

(see for instance Figure 1).

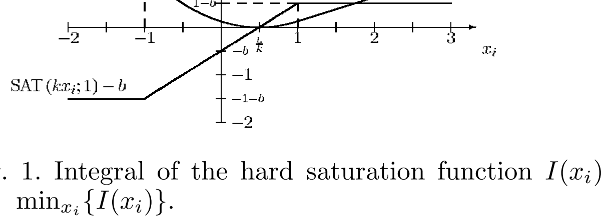


Fig. 1. Integral of the hard saturation function $I(x_i) - \min_{x_i}\{I(x_i)\}$.

## 4. SATURATED NONLINEAR PID GLOBAL REGULATOR

In this section we present a new saturated nonlinear PID controller to solve the set-point control problem of robot manipulators with actuator torque constraints.

### *4.1* Main Result

The proposed control law is given by

$$\boldsymbol{\tau} = K_p\mathbf{SAT}\left(B\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}}; \boldsymbol{\tau}'_{pd}\right) + \mathbf{SAT}\left(K_i\boldsymbol{w}; \boldsymbol{\tau}_w\right) \tag{12}$$

with

$$\boldsymbol{w} = \int_0^t \left[\alpha\mathbf{sat}\left(B\tilde{\boldsymbol{q}}\,(r)\right) - \dot{\boldsymbol{q}}\right] dr$$

where $\boldsymbol{\tau}'_{pd}$ and $\boldsymbol{\tau}_w$ are the respective vectors of saturation limits whose elements satisfy, from the real actuator limitations, $\tau_i^{\max} \ge k_{p_i}\tau'_{pd_i} + \tau_{w_i} \ge \overline{g}_i$, for $i = 1, 2, \ldots n$. $K_p$, $B$, $K_v$ and $K_i$ are $n \times n$ diagonal positive definite matrices whose elements are $k_{p_i}$, $b_i$, $k_{v_i}$, $k_{i_i}$ respectively with $i = 1, 2 \ldots n$, $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the position error vector, $\mathbf{sat}(\tilde{\boldsymbol{q}})$ was defined in Definition 1, $\alpha$ is a small positive constant suitably selected. $\mathbf{SAT}\left(B\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}}; \boldsymbol{\tau}'_{pd}\right) \in \mathbb{R}^n$ and $\mathbf{SAT}\left(K_i\boldsymbol{w}; \boldsymbol{\tau}_w\right) \in \mathbb{R}^n$ are the proportional–derivative and integral hard saturation functions, respectively, defined in Definition 2.

In the next paragraphs we analyze the stability of the equilibrium of the closed loop system formed by (12) and (1):

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{q}}} \\ \ddot{\boldsymbol{q}} \\ \\ \dot{\boldsymbol{w}} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M\,(\boldsymbol{q})^{-1}\left[K_p\mathbf{SAT}\left(B\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}}; \boldsymbol{\tau}'_{pd}\right)\right. \\ \left. +\mathbf{SAT}\left(K_i\boldsymbol{w}; \boldsymbol{\tau}_w\right) - C\left(\boldsymbol{q}, \dot{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} - F_v\dot{\boldsymbol{q}} - \boldsymbol{g}\left(\boldsymbol{q}\right)\right] \\ \alpha\mathbf{sat}\left(B\tilde{\boldsymbol{q}}\right) - \dot{\boldsymbol{q}} \end{bmatrix} \tag{13}$$

which is an autonomous differential equation whose unique equilibrium is: $\left[\tilde{\boldsymbol{q}}^T\;\; \dot{\boldsymbol{q}}^T\;\; \boldsymbol{w}^T\right]^T = \left[0\;\;\; 0\;\;\; K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)\right]^T$, provided that $\tau_{w_i} \ge \bar{g}_i$ for $i = 1, 2, \ldots, n$. Such an analysis is carried out using Lyapunov theory, and LaSalle's invariance principle.

Now, we are in position to introduce our main result:

**Proposition 1.** The Robot dynamics (1) in closed-loop with the control law (12), satisfying $\tau_{w_i} \ge \overline{g}_i$, $\lambda_{\mathrm{m}}\{K_p\} > k_{h2}$ and $\tau'_{pd_i} > \varepsilon$, where $k_{h2} = \frac{2k'}{\mathrm{sat}\left(\frac{2k'\lambda_{\mathrm{m}}\{B\}}{k_g}\right)}$, has a unique equilibrium given by $\left[\tilde{\boldsymbol{q}}^T\; \dot{\boldsymbol{q}}^T\; \boldsymbol{w}^T\right]^T = \left[0\;0\;K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)\right]^T \in \mathbb{R}^{3n}$, which is globally asymptotically stable, provided that $\alpha$ be suitably selected satisfying (14), shown at the top of the next page.

Furthermore the applied torques are bounded by $|\tau_i| \le \tau_i^{\max}$ for $i = 1, 2, 3 \ldots n$. ◇

**Proof.** Proposition 1 can be proven via Lyapunov theory. Toward this end we propose the following Lyapunov function candidate:

$$\alpha < \min\left\{\sqrt{\frac{2\beta\lambda_{\mathrm{m}}\{M\}}{\lambda_{\mathrm{M}}\{M\}^2}},\ \frac{4\lambda_{\mathrm{m}}\{F_v\}\left(\lambda_{\mathrm{m}}\{K_p\}-k_{h2}\right)}{\lambda_{\mathrm{M}}\{K_p\}^2\lambda_{\mathrm{M}}\{K_v\}^2+4\left(k_{c1}\sqrt{n}+\lambda_{\mathrm{M}}\{M\}\lambda_{\mathrm{M}}\{B\}\right)\left(\lambda_{\mathrm{m}}\{K_p\}-k_{h2}\right)}\right\} \tag{14}$$

$$\begin{aligned}V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w}) =& \underbrace{\frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} - \alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + h(\tilde{\boldsymbol{q}})}_{V_1(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})}\\ &+\underbrace{\sum_{i=1}^{n}\alpha\int_0^{\tilde{q}_i} f_{v_i}\mathrm{sat}(\beta_i r_i)\,dr_i}_{V_2(\tilde{\boldsymbol{q}})}\\ &+\underbrace{\sum_{i=1}^{n}\int_{g_i(q_d)}^{w_i}\left[\mathrm{SAT}(k_{i_i}r_i;\tau_{w_i})-g_i(q_d)\right]dr_i}_{V_3(\boldsymbol{w})}\\ &-I_{\min}\end{aligned} \tag{15}$$

where

$$\begin{aligned}h(\tilde{\boldsymbol{q}}) =& \sum_{i=1}^{n}\int_0^{\tilde{q}_i} k_{p_i}\mathrm{SAT}\left(\beta_i r_i;\tau'_{p_i}\right)dr_i\\ &+\mathcal{U}(\boldsymbol{q})-\mathcal{U}(\boldsymbol{q}_d)+\boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}}\end{aligned}$$

and $I_{\min}$ denotes the minimum value of $V_3(\boldsymbol{w})$, that is,

$$I_{\min} = -\sum_{i=1}^{n}\frac{g_i(q_{d_i})^2}{2}\left[\frac{(k_{i_i}-1)^2}{k_{i_i}}\right].$$

**Positive definiteness.** From Property 6, it is straightforward to see that $V_2(\tilde{\boldsymbol{q}})$ and $V_3(\boldsymbol{w})-I_{\min}$ are radially unbounded and positive definite functions in $\tilde{\boldsymbol{q}}$ and $\boldsymbol{w}-K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)$ respectively. Hence, in order to prove that $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w})$ is a radially unbounded and positive definite function, it remains to prove that $V_1(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is positive definite in $[\tilde{\boldsymbol{q}}^T\ \dot{\boldsymbol{q}}^T]$, to this end, notice that it is possible to below bound $V_1(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ by

$$\begin{aligned}V_1(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w}) \geq& \frac{1}{2}\lambda_{\mathrm{m}}\{M\}\|\dot{\boldsymbol{q}}\|^2\\ &-\alpha\lambda_{\mathrm{M}}\{M\}\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|\|\dot{\boldsymbol{q}}\|\\ &+\beta\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|^2\\ =& \left[\,\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|\quad\|\dot{\boldsymbol{q}}\|\,\right]P\begin{bmatrix}\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|\\ \|\dot{\boldsymbol{q}}\|\end{bmatrix}\end{aligned}$$

where $\beta$ is a positive constant satisfying $h(\tilde{\boldsymbol{q}}) \geq \beta\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|^2$ provided that $\lambda_{\mathrm{m}}\{K_p\} > k_{h2}$, and

$$P = \begin{bmatrix}\beta & -\frac{\alpha\lambda_{\mathrm{M}}\{M\}}{2}\\ -\frac{\alpha\lambda_{\mathrm{M}}\{M\}}{2} & \frac{\lambda_{\mathrm{m}}\{M\}}{2}\end{bmatrix}$$

which, due to $\alpha$ condition (14), results to be a positive definite matrix.

**Negative semidefiniteness.** The time derivative of the Lyapunov function candidate (15) along the trajectories of the closed loop system (13), after some algebraic manipulations and using Property 1, results:

$$\begin{aligned}&\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w}) = \left[\dot{\boldsymbol{q}}-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})\right]^T\times\\ &K_p\left[\mathbf{SAT}\left(B\tilde{\boldsymbol{q}}-K_v\dot{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right)-\mathbf{SAT}\left(B\tilde{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right)\right]\\ &-\dot{\boldsymbol{q}}^T F_v\dot{\boldsymbol{q}}-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T\left[\boldsymbol{g}(\boldsymbol{q}_d)-\boldsymbol{g}(\boldsymbol{q})\right]\\ &-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}}-\alpha\dot{\mathbf{sat}}(B\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}}\\ &-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T K_p\mathbf{SAT}\left(B\tilde{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right).\end{aligned} \tag{16}$$

Now we provide upper bounds on each of the terms of (16):

- $\dot{\boldsymbol{q}}^T K_p\left[\mathbf{SAT}\left(B\tilde{\boldsymbol{q}}-K_v\dot{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right)-\mathbf{SAT}\left(B\tilde{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right)\right] \leq 0$
- $-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T K_p\left[\mathbf{SAT}\left(B\tilde{\boldsymbol{q}}-K_v\dot{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right)-\mathbf{SAT}\left(B\tilde{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right)\right] \leq \alpha\lambda_{\mathrm{M}}\{K_p\}\lambda_{\mathrm{M}}\{K_v\}\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|\|\dot{\boldsymbol{q}}\|$
- $-\dot{\boldsymbol{q}}^T F_v\dot{\boldsymbol{q}} \leq -\lambda_{\mathrm{m}}\{F_v\}\|\dot{\boldsymbol{q}}\|^2$
- $-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T\left[\boldsymbol{g}(\boldsymbol{q}_d)-\boldsymbol{g}(\boldsymbol{q})\right] \leq \alpha k_{h2}\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|^2$
- $-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}} \leq \alpha k_{c_1}\sqrt{n}\|\dot{\boldsymbol{q}}\|^2$
- $-\alpha\dot{\mathbf{sat}}(B\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \leq \alpha\lambda_{\mathrm{M}}\{B\}\lambda_{\mathrm{M}}\{M\}\|\dot{\boldsymbol{q}}\|^2$
- $-\alpha\mathbf{sat}(B\tilde{\boldsymbol{q}})^T K_p\mathbf{SAT}\left(B\tilde{\boldsymbol{q}};\boldsymbol{\tau}'_{pd}\right) \leq -\alpha\lambda_{\mathrm{m}}\{K_p\}\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|^2$

where we have used the Property 2 – Property 5. Finally the time derivative $\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w})$ can be upper bounded by

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w}) \leq -\alpha\left[\,\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|\quad\|\dot{\boldsymbol{q}}\|\,\right]Q\begin{bmatrix}\|\mathbf{sat}(B\tilde{\boldsymbol{q}})\|\\ \|\dot{\boldsymbol{q}}\|\end{bmatrix}$$

where

$$Q = \begin{bmatrix}\lambda_{\mathrm{m}}\{K_p\}-k_{h2} & -\frac{\lambda_{\mathrm{M}}\{K_p\}\lambda_{\mathrm{M}}\{K_v\}}{2}\\ -\frac{\lambda_{\mathrm{M}}\{K_p\}\lambda_{\mathrm{M}}\{K_v\}}{2} & \frac{\lambda_{\mathrm{m}}\{F_v\}}{\alpha}-k_{c_1}\sqrt{n}-\lambda_{\mathrm{M}}\{M\}\lambda_{\mathrm{M}}\{B\}\end{bmatrix}$$

which is a positive definite matrix because we have assumed that $\lambda_{\mathrm{m}}\{K_p\} > k_{h2}$ and $\alpha$ satisfies (14), hence $\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w})$ is a globally negative semidefinite function. By using the fact that the Lyapunov function candidate (15) is a radially unbounded globally positive definite function and its time derivative is a globally negative semidefinite function we conclude that the equilibrium of the closed loop system (13) is stable. Finally, by invoking the LaSalle's invariance principle (see e.g., Khalil, 2002), the global asymptotic stability of the equilibrium is proven.

## 5. SIMULATION RESULTS

Using the SIMNON software, we tested our algorithm in the two revolute–joint robot manipulator used in (Reyes and Kelly, 1997). The desired joint positions were chosen as $q_{d1} = 90°$ and $q_{d2} = 60°$. The gains were tuned as $K_p = \mathrm{diag}\{2,2\}$ [Nm/deg], $K_i = \mathrm{diag}\{0.5, 0.5\}$ [Nm/deg], $K_v = \mathrm{diag}\{55, 8\}$ [sec], $\alpha = 1\times 10^{-5}$ [sec$^{-1}$], $B = \mathrm{diag}\{900, 100\}$, $\tau'_{pd_1} = 40$ [deg], $\tau'_{pd_2} = 3$ [deg], $\tau_{w_1} = 3.5$ [Nm] and $\tau_{w_2} = 2$ [Nm]. The maximum torques supplied

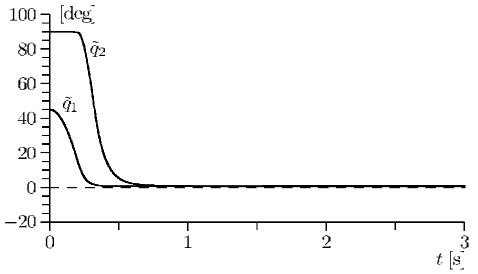


Fig. 2. Joint position errors for the saturated PID Control

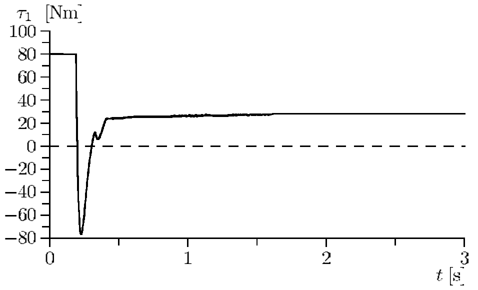


Fig. 3. Applied torque using the saturated PID Control

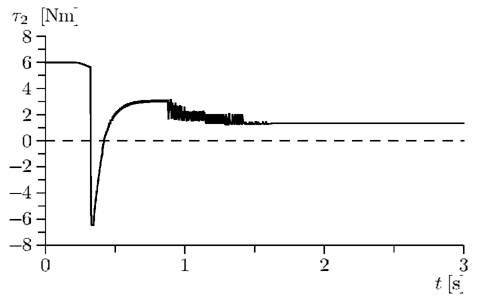


Fig. 4. Applied torque using the saturated PID Control

by the actuators are $\tau_1^{\max} = 150$ [Nm] and $\tau_2^{\max} = 15$ [Nm]. The parameters to be used are: $\lambda_{\mathrm{M}}\{M(\boldsymbol{q})\} = 5.03$ [kg m$^2$], $\lambda_{\mathrm{m}}\{M(\boldsymbol{q})\} = 0.102$ [kg m$^2$], $k_g = 1.406$ [Nm/deg], $k_{c_1} = 0.336$ [kg m$^2$], $k_{h_2} = 1.848$ [Nm/deg] and $\lambda_{\mathrm{m}}\{F_v\} = 0.175$ [Nm sec/deg]. The soft saturation function $\mathbf{sat}(\cdot)$ used in the simulation essays is the hyperbolic tangent, that is, $\mathbf{sat}(\cdot) = \mathbf{tanh}(\cdot)$.

The Figure 2 shows how the position errors converge to zero and the Figure 3 shows the torques for a period of three second. We can observe from Figure 3 and 4 that the proposed saturated nonlinear PID controller yields control inputs $|\tau_1| < \tau_1^{\max} = 150$ [Nm] and $|\tau_2| < \tau_2^{\max} = 15$ [Nm].

## 6. CONCLUSIONS

In this paper we have proposed a new saturated nonlinear PID regulator to solve the global regulation problem of robot manipulators with bounded torques.

By using particular properties of some saturation functions we have proposed a simple Lyapunov function candidate, which using the Lyapunov theory and the LaSalle's invariance principle, leads to conclude global asymptotic stability of the closed loop system.

The main relevance of this work lies in the step ahead that this paper gives in the search of a stability analysis that provides the conditions to prove global asymptotic stability for the practical PID in closed loop with robot dynamics.

It is also guaranteed that, regardless of initial conditions, the delivered torques remain inside prescribed limits.

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