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A two level hierarchical control structure for optimizing a rougher flotation circuit

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Abstract: The control of rougher flotation circuits represents a challenging control problem due to the non linearities, multiple inputs-multiple outputs and the wide variety of disturbances acting on the system. Many concentrators rely on regulatory control loops to maintain a stable operation and on the plant operators to find the best operational results. As a mean of using the operator's knowledge in a consistent manner Expert Systems have been proposed to emulate the current practices in flotation plants. In this work, an algorithmic supervisory system, as opposed to rule-based systems, is proposed and analyzed using simulations of an industrial rougher flotation circuit.

Keywords: Flotation circuits, Hierarchical control, Level control, Optimization.

1. INTRODUCTION

The control of flotation circuits plays an important role in the final operational results of any concentrator plant. In order to deal with the complex nature of this process several authors have proposed the use of systematic advanced algorithms such as adaptive self-tuning control algorithms, Jamsa-Jounela (1992), and predictive control strategies for controlling the metallurgical performance of the flotation circuits, Hodouin et al. (2000). Notwithstanding the benefits of these advanced strategies, the current practices in many concentrators consider a two level hierarchical control strategy. The lower level of regulatory control loops for stabilizing the system and the upper level dealing with the metallurgical optimization. The later is usually carried out by the plant operators. Since the regulatory control loops have a strong effect on the final metallurgical results, there has been a renovated interest in improving their performance, Jamsa-Jounela et al. (2003), Stenlund and Medvedev (2002), Hulbert (1996).

The optimization of the flotation circuits operation by means of Expert Systems has been proposed as a supervisory level by Osorio et al. (1999), Jamsa-Jounela (1988). Under this scheme a set of rules are designed together with plant operators and engineers to keep a final copper concentrate grade over a minimum value and the final copper tailing grade below a maximum value. This approach has provided good results, but in practice it requires additional efforts, during its lifetime, in order to keep the rule-based system updated. In addition, its tuning can be time consuming and the overall design process can not be coupled with the regulatory control loop design.

The aim of this work is to present a more systematic and integrated form for designing supervisory systems, where the dynamics can be taken into account during the design stage.

Many contributions have been made during the last two decades in the field of model-based on-line optimization methods based on a two hierarchical level structure. The classical Real Time Optimization (RTO) approach has been one of the most widely studied. This approach considers the use of steady state models in the optimization level to determine the optimal set-points to the regulatory level. This means that the control system must wait for steady-state conditions to perform the optimization, limiting its performance in the face of disturbances. Many authors have suggested alternative methods, based for instance on evolutive strategies and disturbance estimations, Sequeira et al. (2002). As pointed out in Saez, Cipriano and Ordys (2002), predictive control approach has the flexibility to be used as a building block in this application. Furthermore, predictive control tools are readily available in many modern distributed control systems, Qin et al. (2003). The use of model predictive controller as supervisory control system has been analyzed in Saez, Cipriano and Ordys (2002) and Lee et al. (2000), where the supervisory algorithm has the same sampling time as that of the lower level control strategy. However, in practice the data used by supervisory systems has a slower sampling time. In addition, some kind of data processing may also be required by the supervisor level in order to avoid gross errors and to ensure consistency of the data. This additional data processing must be taken into account in the design of the supervisory strategy.

In the context of flotation circuits, a hierarchical optimal control strategy was proposed by Zaragoza and Herbst (1988) based on an economic static objective function. The optimal control variables were found by using the Nelder-

Mead Algorithm. As this approach is based on steady state data, it inherits the drawbacks of the classical RTO approaches. This works proposes an on-line optimization method based on the use of dynamic data and taking into account the required data processing.

This paper is organized as follows: Section 2 describes the proposed hierarchical control structure. Section 3 describes the model used to test the level of performance that can be attained by this approach. In section 4, some simulations results demonstrate the flexibility of the proposed approach. Finally, in section 5 some conclusions are given.

2. HIERARCHICAL CONTROL STRUCTURE

The hierarchical structure considers a process which has been decomposed into two sub-systems. One describing the relationship between the control variables and some internal measured variables and the second part describing the part of the process concerning with the optimization. Usually, the supervisory level has a different sampling time (larger) than that of the regulatory control level and considers some data processing stages such as filtering, data reconciliation, among others.

The local controllers are such that try to enforce the equality $y_m(t) = r$, in spite of the disturbances z.

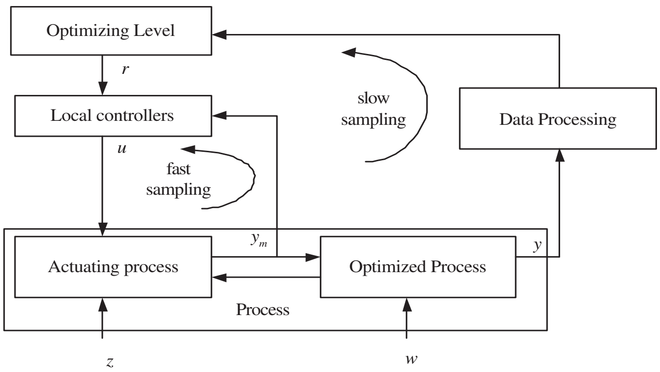

Fig. 1. Hierarchical control structure

The load disturbance w represents changes in the incoming mineralogical composition, particle liberation, degree of oxidation, etc. The the feed flowrate, z, is a disturbance coming from the grinding stage, u are the valve openings, y_m are the tank levels and r are their corresponding set-points.

The optimization objective is the minimization of the copper tailing grade in each flotation bank given a desired final copper concentrate grade L_C^*. The cost function P at a given sampling time,k, is expressed as follows:

$$P(kT) = |(L_C^*(kT) - L_C(kT))| + \lambda \sum_{j=1}^{N} |L_{Tcu,j}(kT)| \quad (1)$$

where λ is a weighting constant and L_C represents the cumulative copper concentrate grade resulting by adding the concentrate mass flowrate of each bank, which is usually measured on-line in flotation plants. The copper tailing grade of each bank is denoted by $L_{Tcu,j}$. Further details concerning the equations for calculating L_C and $L_{Tcu,j}$ are provided in section 3.

3. DYNAMIC MODEL OF A ROUGHER FLOTATION CIRCUIT

The state equations of the dynamic models are based on the mass and volume balance equations described in Casali et al. (2002), Perez-Correa et al. (1998).

The mass balance corresponding to the mineralogical class i in the bank j is given by:

$$\frac{dM_{SP_{ij}}}{dt} = G_{ST_{j-1}} L_{T_{i(j-1)}} - \widetilde{k}_{ij} M_{SP_{ij}} - \quad (2)$$

$$(\frac{Q_{Tj}}{A_T h_{P_j}}) M_{SP_{ij}} \quad (3)$$

where $M_{SP_{ij}}$ is the Mass of solid in the pulp of the mineralogical class i in the bank j in [t], h_{P_j} the level of bank j in [mm], $\widetilde{k}_{ij}$ is the flotation constant of the mineralogical class i in bank j, [h^{-1}], Q_{T_j} is the tailing volumetric flow rate of bank j, [m^3/h], $Q_{C_j}$is the concentrate volumetric flow rate of bank j in [m^3/h], $Q_{F_j}$is the feeding volumetric flow rate of bank j in [m^3/h], A_T is the cross-sectional area of the flotation in [m^2], $G_{ST_{j-1}}$is tailing mass flow rate of tank $j-1$ in [t/h], and $L_{T_{i(j-1)}}$is the tailing grade of the mineralogical class i in the bank $j-1$ in [%].

On the other hand, the volumetric balance for the bank j is:

$$A_T \frac{dh_{P_j}}{dt} = Q_{F_j} - Q_{T_j} - Q_{C_j} \qquad [m^3] \quad (4)$$

where Q_{F_j} and Q_{C_j} are the feeding volumetric flow rate of bank j and concentrate volumetric flow rate leaving the bank j respectively.

Rearranging terms we have the following state space representation:

$$\frac{dM_{SP_{ij}}}{dt} = -[\widetilde{k}_{ij} + \frac{Q_{Tj}}{A_T h_{P_j}}] M_{SP_{ij}} + G_{ST_{j-1}} L_{T_{i(j-1)}} \quad (5)$$

$$A_T \frac{dh_{P_j}}{dt} \qquad = Q_{F_j} - Q_{T_j} - Q_{C_j} \quad (6)$$

$i = \{1, \ldots, N_e\}$ and $j = \{1, \ldots, N\}$ where N_e is the number of mineralogical classes and N is the number of banks in the flotation circuit.

The feeding and tailing volumetric flow rates depend on the pulp levels of the adjacent tanks. The concentrate volumetric flow rate, however, is a linear function of the froth depth h_{E_j} of the same tank, which it is assumed to be the difference between the total height of the bank and the pulp level;i.e. $h_{E_j} = H_C - h_{P_j}$.

$$\begin{aligned} Q_{F_j} &= K_{v_{j-1}} v_{j-1} \sqrt{h_{P_{j-1}} - h_{P_j} + \delta h_P}, \\ & \quad j = \{2, \ldots, N\} \\ Q_{T_j} &= K_{v_j} v_j \sqrt{h_{P_j} - h_{P_{j+1}} + \delta h_P}, \\ & \quad j = \{1, \ldots, N\} \\ Q_{C_j} &= c_{0j} - c_{1j} \cdot h_{E_j}, \quad j = \{1, \ldots, N\} \end{aligned} \quad (7)$$

where v_j is the percentage of the valve opening i the tail of bank j, K_{v_j} is the valve gain, δh_P is the height difference between banks, and c_{0j}, c_{1j} are empirical constants.

The flotation constant of the mineralogical classes in each bank are modelled as :

$$\tilde{k}_{ij} = a_{ij} + b_{ij} \frac{Q_{F_j} L_{T_{i(j-1)}}}{h_{E_j}} \qquad [\mathrm{h}^{-1}] \tag{8}$$

where a_{ij} and b_{ij} are empirical constants.

The concentrate grades $L_{C_{ij}}$ and tailing grade $L_{T_{ij}}$ of the mineralogical class i in the bank j are given by the following expressions:

$$L_{C_{ij}} = \frac{\tilde{k}_{ij} M_{SP_{ij}}}{\sum_{i=1}^{N_e} \tilde{k}_{ij} M_{SP_{ij}}} \qquad [\%] \tag{9}$$

$$L_{T_{ij}} = \frac{M_{SP_{ij}}}{\sum_{i=1}^{Ne} M_{SP_{ij}}} \qquad [\%] \tag{10}$$

The cumulative copper concentrate grade L_C and the copper tail grade for each cell $L_{T_{cu,j}}$ are given by the following expressions:

$$L_C = \frac{\sum_{j=1}^{N} \tilde{k}_{i^*j} M_{SP_{i^*j}}}{\sum_{i=1}^{N_e} \sum_{j=1}^{N} \tilde{k}_{ij} M_{SP_{ij}}} \qquad [\%] \tag{11}$$

$$L_{T_{cu,j}} = \frac{M_{SP_{i^*j}}}{\sum_{i=1}^{N_e} M_{SP_{ij}}} \qquad [\%] \tag{12}$$

where the index i^* corresponds to copper. The model can be calibrated using the available physical information and the steady state information of any industrial flotation circuit.

4. A RECEDING HORIZON CONTROLLER AS REAL TIME OPTIMIZER

The optimizing level will consider the minimization of a future cost function with respect to the level set-points; i.e

$$I = P(kT + HT) \tag{13}$$

subject to constant future set points; i.e.

$$r_i(kT + T) = ... = r_i(kT + HT) \tag{14}$$

and constraints

$$r_1(kT + T) < ... < r_n(kT + HT) \tag{15}$$

The parameter H is also known as prediction horizon. Even though a more complex cost function can be designed, the extended horizon strategy has the property that as H tends to infinity the problem is transformed into a steady state optimization algorithm.

If the lower level stabilizes the system, then by properties of the receding horizon controller, it is always possible to find a finite H that renders the upper level stable.

In order to obtain reasonable estimates of the concentrate and tail grades without resorting to the full nonlinear model, the variation of the tail and concentrate grades are modelled as linear models with the level set-points as inputs; i.e.

$$L_C(k+1) = A_1(q^{-1}) L_C(k) + \sum_{j=1}^{N} B_{1,j}(q^{-1}) r_i(k) + d_1 \tag{16}$$

$$\begin{aligned} L_{T_{cu,j}}(k+1) = \; & A_2(q^{-1}) L_{T_{cu,j}}(k) + \\ & \sum_{j=1}^{N} B_{2,j}(q^{-1}) r_i(k) + d_2 \end{aligned} \tag{17}$$

where the polynomials $A_i(q^{-1})$ and $B_{ij}(q^{-1})$ are defined as:

$$\begin{aligned} A_i(q^{-1}) &= 1 + ... + a_n^i q^{-n}, \\ B_{ij}(q^{-1}) &= b_0^{ij} + ... + b_m^{ij} q^{-m}. \end{aligned} \tag{18}$$

where q is the shift operator.

The parameters of these models are calculated on-line using the least squares algorithm. For this application, however, it is worth using constrained identification algorithms as the one presented in Chia et al. (1991). This avoids convergence problems due to the transients of the identification algorithm.

The H-step ahead predictions are:

$$\begin{aligned} L_C(k+H) &= G_1(q^{-1}) L_C(k) + \\ \sum_{j=1}^{N} F_1(q^{-1}) & B_{1,j}(q^{-1}) r_j(k) + F_1(1) d_1 \end{aligned} \tag{19}$$

$$\begin{aligned} L_{T_{cu,j}}(k) &= G_2(q^{-1}) L_{T_{cu,j}}(k) + \\ \sum_{j=1}^{N} F_2(q^{-1}) & B_{2,j} r_j(k) + F_2(1) d_2 \end{aligned} \tag{20}$$

where F_i and G_i are polynomials of order $H-1$ and $n-1$ satisfying the Diophantine equation:

$$1 = q^{-H} G_i(q^{-1}) + F_i(q^{-1}) A_i(q^{-1}) \tag{21}$$

Equations (19) and (20) can be written in terms of current and future control signal as:

$$\begin{aligned} L_C(k+H) &= \sum_{j=i}^{n} \alpha_i^1 L_C(k-i+1) + \\ & \sum_{i=1}^{N} \sum_{j=1}^{m+H} \beta_{i,j}^1 r_j(k+T-j) + \delta^1 \end{aligned} \tag{22}$$

$$\begin{aligned} L_{T_{cu,j}}(k+H) &= \sum_{j=i}^{n} \alpha_i^2 L_{T_{cu,j}}(k-i+1) + \\ & \sum_{i=1}^{N} \sum_{j=1}^{m+H} \beta_{i,j}^2 r_j(k+T-j) + \delta^2 \end{aligned} \tag{23}$$

If all the future control set-point are assumed to be constant, as in (14), then

$$L_C(k+H) = \sum_{j=i}^{n} \alpha_i^1 L_C(k-i+1) + \sum_{i=1}^{N}\sum_{j=1}^{H} \beta_{i,j}^1 r_i(k) + \sum_{i=1}^{N}\sum_{j=H+1}^{m+H} \beta_{i,j}^1 r_i(k+T-j) + \delta^1 \quad (24)$$

$$L_{T_{cu,j}}(k+H) = \sum_{j=i}^{n} \alpha_i^2 L_{T_{cu,j}}(k-i+1) + \sum_{i=1}^{N}\sum_{j=1}^{H} \beta_{i,j}^2 r_i(k) + \sum_{i=1}^{N}\sum_{j=H+1}^{m+H} \beta_{i,j}^2 r_i(k+T-j) + \delta^2 . \quad (25)$$

Thus the optimization problem can be expressed as:

$$\min I = \eta + \sum_{i=1}^{N}\sum_{j=1}^{H} \beta_{i,j}^2 r_i(k) \quad (26)$$

subject to

$$\begin{aligned}
L_C^* - \sum_{j=i}^{n} \alpha_i^1 L_C(k-i+1) + \sum_{i=1}^{N}\sum_{j=1}^{H} \beta_{i,j}^1 r_i(k) + \\
\sum_{i=1}^{N}\sum_{j=H+1}^{m+H} \beta_{i,j}^1 r_i(k+T-j) + \delta^1 < \eta \\
\sum_{j=i}^{n} \alpha_i^1 L_C(k-i+1) - L_C^* - \sum_{i=1}^{N}\sum_{j=1}^{H} \beta_{i,j}^1 r_i(k) + \\
\sum_{i=1}^{N}\sum_{j=H+1}^{m+H} \beta_{i,j}^1 r_i(k+T-j) + \delta^1 < \eta \\
r_i(k-1) - r_i(k) < \mu \\
r_i(k) - r_{k-i}(k) < 0 \\
r_i(k) > r_{min} \\
r_N(k) < r_{max}
\end{aligned} \quad (27)$$

where μ defines the step change at each iteration. This restriction avoids performing big step changes, which can lead to a lost of precision in the linear estimates.

As the control horizon tends to infinity the prediction tends to

$$\bar{L}_C = \frac{\sum_{j=i}^{N} B_{1,i}(1)\bar{r}_i}{1 - A_1(1)} + \frac{d_2}{1 - A_1(1)} \quad (28)$$

$$\bar{L}_{T_{cu,j}} = \frac{\sum_{j=i}^{N} B_{2,i}(1)\bar{r}_i}{1 - A_2(1)} + \frac{d_2}{1 - A_2(1)} \quad (29)$$

and the problem defined by (26) is transformed in a static optimization problem.

5. SIMULATIONS

The simulations consider a model with five cells and three mineralogical classes; i.e copper, iron and gangue. The parameters were adjusted by using physical parameters and data from an industrial flotation circuit. Figure 2 shows the steady state validation results for the proposed model.

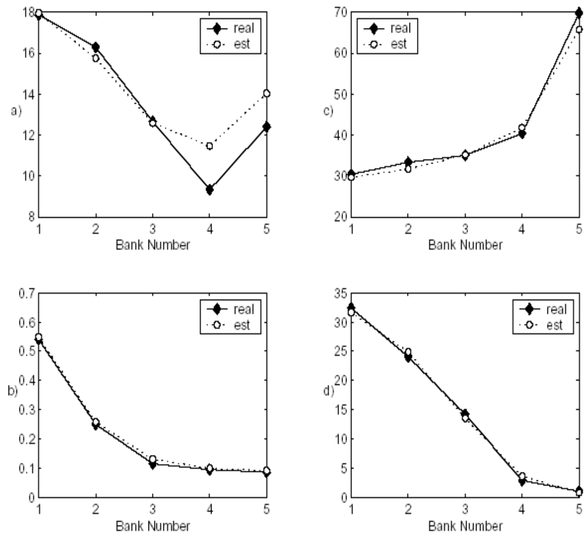

Fig. 2. Steady state model validation a) Concentrate solid mass flow rate $[t/h]$. b) Copper tailing grade $L_{T_{ij}}[\%Cu]$. c) Concentrate volumetric flow rate $[m^3/h]$. d) Copper concentrate grade $L_{C_{1j}}[\%Cu]$

The supervisor controller considers an Horizon $H = 2$ and $\lambda = .1$. Previous results, Maldonado et al. (2007) have shown that the optimal solutions mainly considered movements in the first three cells. Thus, in this work the optimization problem will only consider the first three set-points as independent variables.

A constrained identification algorithm was implemented in order to identify the linear models. The step size changes for the setpoint signals was $\delta = 5$. The incoming feed flow rate into the first cell has a continuous variations as shown in figure 3.

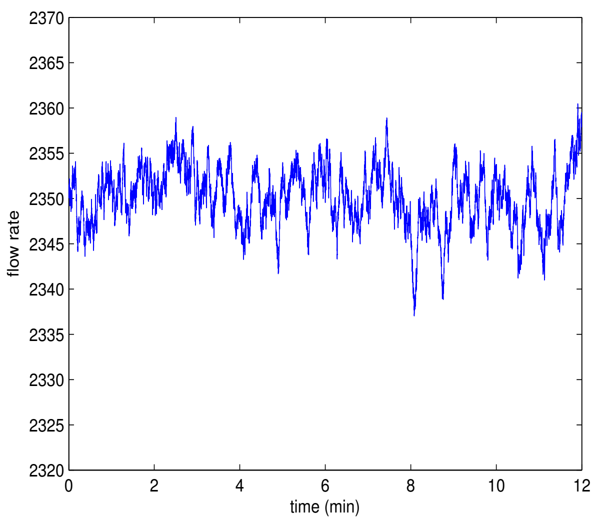

Fig. 3. Flowrate into the first cell

In the simulations, the optimizer starts working at time .5min, and at time 6 minutes a sudden reduction in the copper head grade occurs. As seen in figure 4, the supervisory system adjusts the set-points in order to minimize the cost function. If the head grade decreases the set-points are increased to maintain the concentrate grade around the desired value. Figure 5 shows the concentrate and tailing copper grade.

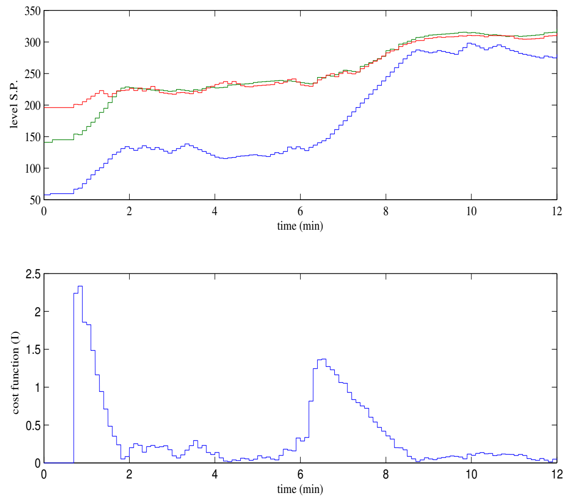

Fig. 4. Set points and Cost function

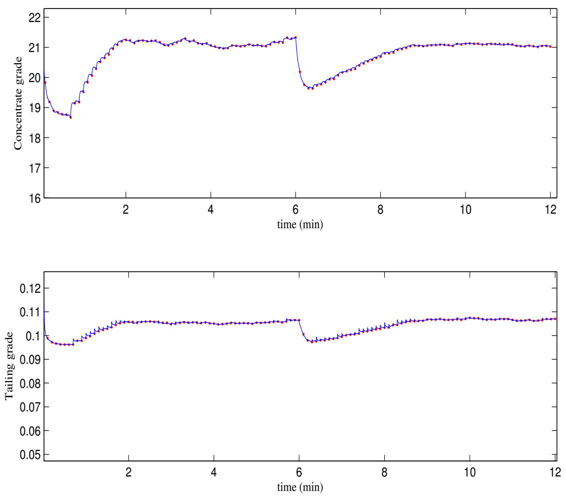

Fig. 5. Concentrate and tailing grade

6. CONCLUSIONS

This paper has proposed an optimizing strategy, based on a two hierarchical level structure, to deal with the optimal control of flotation circuits. The supervisory level is based on a constrained receding horizon control strategy, which can be seen as a generalization of a steady-state optimizer. The use of identified models requires the implementation of constrained identification algorithms in order to add some degree of robustness and avoid parameters inconsistencies. The modular approach offers benefits in terms of their implementation in industrial plants, since it can be developed over the existing regulatory systems. The simulation results obtained with a calibrated simulator demonstrate the feasibility of the strategy in optimizing a rougher flotation circuit controlled by a set of PI level controllers with decoupling.

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