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Global Output Regulation of Nonlinear Time-delay Output Feedback Systems With Unknown Exosystems ⋆

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Abstract: The output regulation problem for nonlinear time-delay systems can be solved under the assumptions that certain integral regulator equations are solvable and the full information of exosystems is available. This paper shows that these two assumptions can be removed for a class of nonlinear time-delay output feedback systems by introducing a transfer matrix dependent on the system delays. Based on a filtered transformation and an adaptive control, a global output regulation method is developed in this paper for a class of nonlinear time-delay output feedback systems under disturbances generated from unknown exosystems.

1. INTRODUCTION

In this paper, we consider the global output regulation problem of a class of nonlinear time-delay systems described by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + \Phi(y(t), w(t)) + \Psi(\bar{y}(t-d), w(t)) + Bu(t)\\ y(t) &= Cx(t)\\ e(t) &= y(t) - q(w(t))\\ x(\theta) &= \delta(\theta), \theta \in \left[-\bar{d}, 0\right]\end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re$ the control input, $y(t) \in \Re$ the system output, and $e(t) \in \Re$ the measurement output. The nonlinear functions

$\Phi(y(t), w(t)) = \mathrm{col}(\phi_1(y(t), w(t)), \cdots, \phi_n(y(t), w(t))) \in \Re^n$, $\bar{y}(t-d) = \mathrm{col}(y(t-d_1), \cdots, y(t-d_n)) \in \Re^n$, $\Psi(\bar{y}(t-d), w(t)) = \mathrm{col}(\psi_1(y(t-d_1), w(t)), \cdots, \psi_n(y(t-d_n), w(t))) \in \Re^n$, and the smooth vector fields $\phi_i(\cdot,\cdot) : \Re \times \Re^m \to \Re$ and $\psi_i(\cdot,\cdot) : \Re \times \Re^m \to \Re$ for $i = 1, \cdots, n$ are polynomials of their variables and satisfy that $\phi_i(0,\cdot) = 0$ and $\psi_i(0,\cdot) = 0$. $d_i, i = 1, 2, \cdots, n$ are the constant but unknown time delays in the system output, $\bar{d}$ is the upper bound of time delays, and $\delta(\theta)$ is the initial condition of the system. The system matrices are given as

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 0\end{bmatrix}, B = \begin{bmatrix} 0\\ \vdots\\ b_r\\ \vdots\\ b_n\end{bmatrix}, C = \begin{bmatrix} 1\\ \vdots\\ 0\end{bmatrix}^T,$$

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and the function $q(w) \in \Re$ is the disturbance to be rejected or the desirable trajectory to be tracked. It is an unknown polynomial of $w(t) \in \Re^{2m}$ which is generated from an exosystem

$$\dot{w}(t) = S(\sigma)w(t), \tag{2}$$

where $S(\cdot) \in \Re^{2m \times 2m}$ is diagonalizable, and $\sigma \in \Re^{n_s}$ is unknown.

It is observed that the system described by (1) and (2) without time delays reduces to the class of output feedback systems (see e.g. Ding [2001], Huang [2004], and Liu and Huang [2006]). For simplicity, we also call this kind of time-delay system as time-delay output feedback system. The stabilization problem of this class of time-delay systems has been studied in Hua et al. [2005].

The global output regulation problem concerns with stabilization of dynamic systems as well as rejecting the disturbances or tracking the desired trajectories within any compact set. The measurement or the tracking error $e(t)$ converges to zero asymptotically. Recently, some global adaptive output regulation approaches of output feedback systems are reported in Ding [2001], and Chen and Huang [2005]. In Ding [2003], an adaptive global output regulation method is proposed for the output feedback systems with completely unknown parameters, including the sign of high frequency gain. This problem is also solved in Liu and Huang [2006] for a class of uncertain nonlinear systems with unknown high-frequency gain sign. However, to the best knowledge of the authors, the global output regulation method for nonlinear time-delay output feedback systems has not been reported in the existing literatures.

As far as the time-delay systems is concerned, a solution of output regulation problem was introduced in Gilliam et al. [2002] for linear state delay system, where a pair of finite dimensional regulator equations were proposed in the infinite dimensional state space. The same equations

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are also discussed in Byrnes et al. [2002]. The solutions for such equations are dependent on the matrix S of (2) and the transfer function of the linear system. Therefore, it is very difficult to extend this method into nonlinear time-delay systems. More recently, based on center manifold theory, it was reported in Fridman [2003] that the output regulation problem is solvable for a special class of time-delay nonlinear systems if and only if the integral regulator equations are solvable. It should be noted here that it is also difficult to solve the regulator equations even though there exists an invariant manifold $\pi(t)$ on which the exosystem is immersed into the dynamical delay system. Moreover, the feedback controller design in Fridman [2003] depends on the full information of $\pi(t)$. Thus this approach cannot be applied to solve the output regulation problem if there are unknown parameters in the dynamic systems and/or in the exosystems.

In order to overcome the difficulties caused by regulator equations and the full information of invariant manifold $\pi(t)$, we extend the internal model design and parameterization technique (see Ding [2001, 2003]) into the time-delay systems (1) and (2) by introducing a transfer matrix $T_d(\sigma, d)$. Most importantly, this matrix does not need to be known. Based on an adaptive internal model, a measurement feedback control method is proposed to solve the global output regulation problem.

2. PROBLEM FORMULATION AND PRELIMINARIES

The objective of this paper is to design a feedback controller $u(t)$ capable of forcing the regulated variable $e(t)$ to zero while keeping all variables of (1) bounded.

In the following, we list two standing assumptions.

Assumption 1. The system (1) is of minimum phase, i.e. the polynomial $\mathcal{B}(s) \triangleq \sum_{i=r}^{n} b_i s^{n-i}$ is Hurwitz.

Assumption 2. The eigenvalues of S are distinct and of zero real parts.

For the simplicity of regulator design, two kinds of coordinate transformation are to be conducted to the system (1). One is so-called filtered transformation which is used to deal with the high relative degree of (1). The resulting system is then dealt with by the other transformation to extract the zero dynamics.

For the system (1) with relative degree $r > 1$, we firstly apply the following filtered transformation (see Marino and Tomei [1993], and Ding [2003]):

$$\dot{\xi}(t) = A_\xi \xi(t) + B_\xi u(t) \tag{3}$$

where $\xi(t) = \mathrm{col}(\xi_1(t), \cdots, \xi_{r-1}(t))$,

$$A_\xi = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & -\lambda_{r-1} \end{bmatrix}, B_\xi = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \Re^{r-1}$$

and the positive scalars λ_i for $i = 1, 2, \cdots, r-1$ are the adjusting parameters.

Now, define a new variable vector as $\bar{z}(t) = x(t) - \bar{h}\xi(t)$, where $\bar{h} = \left[\, \bar{h}_1 \ \cdots \ \bar{h}_{r-1} \,\right], \bar{h}_{r-1} = B, \bar{h}_i = (A + \lambda_{i+1} I)\bar{h}_{i+1}, i = 1, 2, \cdots, r-2$.

Then the system (1) is transformed into

$$\begin{aligned} \dot{\bar{z}}(t) &= A\bar{z}(t) + \Phi(y(t), w(t)) + \Psi(\bar{y}(t-d), w(t)) + h\xi_1(t), \\ y(t) &= C\bar{z}(t), \end{aligned} \tag{4}$$

where $h = \mathrm{col}\,(h_1, \cdots, h_n) \triangleq [A + \lambda_1 I]\bar{h}_1$. It can be easily seen that $h_1 = b_r$ and

$$\mathcal{H}(s) \triangleq \sum_{i=1}^{n} h_i s^{n-i} = \mathcal{B}(s) \prod_{i=1}^{r-1} (s + \lambda_i). \tag{5}$$

Thus, it follows from Assumption 1 that this polynomial is Hurwitz. Moreover, it has been shown that the system (4) is of relative degree one and minimum phase with respect to the input $\xi_1(t)$.

Secondly, considering the relation of

$$\xi_1(t) = (\dot{\bar{z}}_1 - \bar{z}_2 - \phi_1(y(t), w) - \psi_1(y(t-d_1), w(t)))/h_1$$

from (4) and substituting it into the other equations of (4) yields that

$$\begin{aligned} \dot{z}(t) &= Hz(t) + \Theta(y(t), \bar{y}(t-d), w(t)) \\ \dot{y}(t) &= z_1(t) + \Xi(y(t), \bar{y}(t-d), w(t)) + \xi_1(t), \end{aligned} \tag{6}$$

where the new variable $z(t)$ is defined as $z(t) = \bar{z}_{2:n}(t) - \frac{1}{h_1} h_{2:n} y(t)$. The notation $(\cdot)_{2:n}$ refers to the extracted vector or matrix formed by the 2nd row to the nth row. The other notations in (6)are given by

$$H = \begin{bmatrix} -h_2/h_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -h_{n-1}/h_1 & 0 & \cdots & 1 \\ -h_n/h_1 & 0 & \cdots & 0 \end{bmatrix},$$

$$\begin{aligned} \Theta(y(t), \bar{y}(t-\tau), w(t)) = {} & H\frac{1}{h_1} h_{2:n} y + \phi_{2:n}(y(t), w) \\ & + \psi_{2:n}(\bar{y}(t-\tau), w(t)) \\ & - \frac{1}{h_1} h_{2:n}(\phi_1(y(t), w) \\ & + \psi_1(\bar{y}(t-\tau), w(t)), \\ \Xi(y(t), \bar{y}(t-\tau), w(t)) = {} & \frac{h_2}{h_1} y + \phi_1(y(t), w(t)) \\ & + \psi_1(\bar{y}(t-\tau), w(t)). \end{aligned}$$

Note that $\Theta(0, 0, w(t)) = 0$ and $\Xi(0, 0, w(t)) = 0$. The subsequent discussion is based on the transformed system (6).

3. INTERNAL MODEL DESIGN

It is well known that the crucial step for solving output regulation problem is the internal model design. The internal model can produce the desired steady state output. In order to design an internal model for the time-delay system (1), we need the following lemma and assumption.

Lemma 1. Given the exosystem (2) and a constant delay d, there exists a constant matrix $T_d(\sigma, d)$ such that $w(t - d) = \bar{T}_d(\sigma, d)w(t)$.

Proof. Given the exosystem (2), there exists a state transformation $\bar{w}(t) = Dw(t)$ with an invertible constant matrix $D \in \Re^{2m\times 2m}$ such that $\dot{\bar{w}}(t) = \bar{S}(\sigma)\bar{w}(t)$, where $\bar{S}(\sigma) = DSD^{-1} = \text{diagblock}\{\bar{S}_1, \cdots, \bar{S}_m\}$ and

$$\bar{S}_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix} = T \begin{bmatrix} j\omega_i & 0 \\ 0 & -j\omega_i \end{bmatrix} T^{-1}, T = \begin{bmatrix} -j & j \\ 1 & 1 \end{bmatrix},$$

ω_i for $i = 1, \cdots, m$ are the frequencies of the signal $w(t)$.

The time response of $w(t)$ is given by

$$w(t) = D^{-1}e^{\bar{S}(\sigma)t}Dw(0) = D^{-1}\bar{T}\bar{T}_e(t)\bar{T}^{-1}Dw(0),$$

where $\bar{T} = \text{diagblock}\{T_1, \cdots, T_m\}$, $T_i = T$, $\bar{T}_e(t) = \text{diagblock}\{T_e(\omega_1 t), \cdots, T_e(\omega_m t)\}$, $T_e(\omega_i t) = \text{diag}\{e^{j\omega_i t}, e^{-j\omega_i t}\}$ for $i = 1, \cdots, m$, and $w(0)$ is the initial condition of (2).

Moreover, we have

$$\begin{aligned} w(t-d) &= D^{-1}\bar{T}\bar{T}_e(t-d)\bar{T}^{-1}Dw(0) \\ &= D^{-1}\bar{T}\bar{T}_e(-d)\bar{T}^{-1}DD^{-1}\bar{T}\bar{T}_e(t)\bar{T}^{-1}Dw(0) \\ &= \bar{T}_d(\sigma, d)w(t) \end{aligned}$$

with the transfer matrix

$$\bar{T}_d(\sigma, d) = D^{-1}\text{diagblock}\{T_d(\omega_1, d), \cdots, T_d(\omega_m, d)\}D,$$

$$T_d(\omega_i, d) = TT_e(\omega_i d)T^{-1} = \begin{bmatrix} \cos\omega_i d & -\sin\omega_i d \\ \sin\omega_i d & \cos\omega_i d \end{bmatrix}, \quad (7)$$

for $i = 1, \cdots, m$. This completes the proof. ■

Based on Lemma 1, we have $q(w(t-d_i)) = q(\bar{T}_d(\sigma, d_i)w(t))$ for $i = 1, \cdots, n$. Since $\text{col}(q(w(t-d_1)), \cdots, q(w(t-d_n)))$ is a vector function of $w(t)$, we can define it as $\bar{p}(w(t))$. Thus we are ready to present the following assumption.

Assumption 3. There exists an invariant manifold

$$\pi(w(t), \sigma) \triangleq \text{col}(\pi_1, \cdots, \pi_{n-1}) \in \Re^{n-1}$$

satisfying

$$\frac{\partial \pi(w(t), \sigma)}{\partial w(t)} S(\sigma)w(t) = H\pi(w(t), \sigma) + \Theta(q(w(t)), \bar{p}(w(t)), w(t)). \quad (8)$$

Remark 1. With the assistance of $\bar{T}_d(\sigma, d)$, the delay (functional) differential equations defined in Lemma 2 of Fridman [2003] reduce to the standard differential equation (8) which is similar to the one in Ding [2001, 2003] for non-delay systems. Most importantly, it will be shown in a moment that Assumption 3 facilitates us to apply the reformulation technique of internal model design. It thus avoids solving the finite dimensional equations in Byrnes et al. [2002] and Gilliam et al. [2002], and the integral equation in Fridman [2003].

Based on Assumption 3, it follows from (6) that there exists a function $\alpha(w(t), \sigma)$ such that

$$\frac{\partial q(w(t))}{\partial w(t)} S(\sigma)w(t) = \pi_1 + \Xi(q(w(t)), \bar{p}(w(t)), w(t)) + b_r\alpha(w(t), \sigma).$$

Here, $\alpha(w(t), \sigma)$ is the desirable feedforward term for output regulation to tackle $q(w(t))$.

Set an error state vector as $\tilde{z}(t) = z(t) - \pi(w(t), \sigma)$, and then we have the following error model for internal model design,

$$\begin{aligned} \dot{\tilde{z}}(t) &= H\tilde{z} + \tilde{\Theta}(t), \\ \dot{e}(t) &= \tilde{z}_1(t) + \tilde{\Xi}(t) + b_r(\xi_1(t) - \alpha(w(t), \sigma)), \end{aligned} \quad (9)$$

with

$$\begin{aligned} \tilde{\Theta}(t) &= \Theta(y(t), \bar{y}(t-d), w(t)) - \Theta(q(w(t)), \bar{p}(w(t)), w(t)), \\ \tilde{\Xi}(t) &= \Xi(y(t), \bar{y}(t-d), w(t)) - \Xi(q(w(t)), \bar{p}(w(t)), w(t)). \end{aligned}$$

It is easily seen that $\tilde{\Theta}(t)|_{e(t)=0} = 0$ and $\tilde{\Xi}(t)|_{e(t)=0} = 0$.

Since all the nonlinear functions of the system (1) are polynomials of their variables, it is known from Byrnes et al. [1997] that there exists a mapping $\zeta(w(t), \sigma)$ satisfying the immersion conditions

$$\begin{aligned} \frac{\partial \zeta(w(t), \sigma)}{\partial w(t)} S(\sigma)w(t) &= \Omega(\sigma)\zeta(w(t), \sigma), \\ \alpha(w(t), \sigma) &= \Gamma\zeta(w(t), \sigma), \end{aligned} \quad (10)$$

where the pair $(\Omega(\sigma), \Gamma)$ is observable. The matrices $\Omega(\sigma)$ and Γ are constructed by the internal model principle (see, e.g., Byrnes et al. [1997]). The spectrum of $\Omega(\sigma)$ contains the distinct eigenvalues of the exosystem (2) and their certain multiples. These multiples represent higher order sinusoidal harmonics generated by the nonlinearities of the system (6).

Choose a controllable pair $\{F, G\}$ with compatible dimensions such that there exists an invertible real matrix $M(\sigma)$ satisfying the Sylvester equation $M(\sigma)\Omega(\sigma) - FM(\sigma) = G\Gamma$. Based on this equation, we can re-formulate (10) into

$$\begin{aligned} \dot{\eta}(t) &= F\eta(t) + G\alpha(w(t), \sigma), \\ \alpha(w(t), \sigma) &= L^T\eta(t), \end{aligned} \quad (11)$$

where $\eta(t) = M(\sigma)\zeta(w(t), \sigma) \in \Re^s$ and $L^T = \Gamma M^{-1}(\sigma) \in \Re^s$. Since $\eta(t)$ and L are both dependent on the unknown parameter σ, we introduce the following internal model,

$$\dot{\hat{\eta}}(t) = F\hat{\eta}(t) + G\xi_1(t). \quad (12)$$

Consider the mismatch of the states between the internal model (12) and the mapping (10), and define an auxiliary error as $\tilde{\eta}(t) = \eta(t) - \hat{\eta}(t) + b_r^{-1}Ge(t)$.

It follows that

$$\dot{\tilde{\eta}}(t) = F\tilde{\eta}(t) - b_r^{-1}FGe(t) + b_r^{-1}G(\tilde{z}_1(t) + \tilde{\Xi}(t)). \quad (13)$$

Then the controller design and stabilization for the overall system with combinations of (3), (9) and (13) is to be analyzed in the next section.

4. CONTROLLER DESIGN AND STABILIZATION ANALYSIS

If $r = 1$, i.e. the system (1) is of relative degree one, let $\xi_1(t)$ be the control input $u(t)$ for (9). For the case of $r > 1$, a kind of backstepping technique will be used to obtain an adaptive control law.

Introduce a new vector $\tilde{\xi}(t) = \xi(t) - \hat{\xi}(t)$, where $\hat{\xi} \in \Re^{r-1}$ is the virtual control vector. It can also be considered as the estimate of the desirable value of $\xi(t)$ with which the controller $u(t)$ stabilizes the overall system aforementioned.

Since the nonlinear functions involved in $\tilde{\Theta}(t)$ and $\tilde{\Xi}(t)$ are polynomials with $\tilde{\Theta}(t) = 0$ and $\tilde{\Xi}(t) = 0$ for $e(t) = 0$, and $w(t)$ is bounded with constant unknown vector σ, the following propositions hold,

$$\|\tilde{\Theta}(t)\|^2 \leq r_0(e^2(t)+e^{2p}(t)) + \sum_{i=1}^{p}\sum_{j=1}^{n} r_{ji}e^{2i}(t-d_j),$$
$$\|\tilde{\Xi}(t)\|^2 \leq s_0(e^2(t)+e^{2p}(t)) + \sum_{i=1}^{p}\sum_{j=1}^{n} s_{ji}e^{2i}(t-d_j) \quad (14)$$

where p is a known positive integer which depends on order of the polynomials $\tilde{\Theta}(t)$ and $\tilde{\Xi}(t)$. The positive scalars r_0, s_0, r_{ji} and s_{ji} for $i = 1, \cdots, p, j = 1, \cdots, n$ are unknown.

Now, design the virtual control vector $\hat{\xi}_1(t)$ as follows:

$$\hat{\xi}_1(t) = \hat{L}^T(t)\hat{\eta}(t) - b_r^{-1}\left[c_0e(t) + \hat{c}_1(t)(e(t)+e^{2p-1}(t))\right], \quad (15)$$

with $c_0 > 0$, where $\hat{L}(t)$ is the estimate of L, and the adaptive tuning law of the parameter $\hat{c}_1$ is

$$\dot{\hat{c}}_1(t) = k(e^2(t)+e^{2p}(t)), k > 0. \quad (16)$$

Thus the resulting error dynamics of (9) can be shown as

$$\dot{e}(t) = \tilde{z}_1(t) + \tilde{\Xi}(e(t),\bar{e}(t-d),w(t)) + b_r(\tilde{\xi}_1(t) - L^T\eta(t) + \hat{L}^T\hat{\eta}(t)) - c_0e(t) - \hat{c}_1(t)(e(t)+e^{2p-1}(t)). \quad (17)$$

For the case of $r > 1$, we present the other virtual control variables as follows:

$$\hat{\xi}_2 = -b_re(t) - c_2\tilde{\xi}_1(t) + \lambda_1\hat{\xi}_1(t) - k_1\left(\frac{\partial\hat{\xi}_1(t)}{\partial e(t)}\right)^2\tilde{\xi}_1(t) + \frac{\partial\hat{\xi}_1(t)}{\partial\hat{L}(t)}l_1(t) + b_r\frac{\partial\hat{\xi}_1(t)}{\partial e(t)}(\xi_1(t) - \hat{L}^T\hat{\eta}) + \frac{\partial\hat{\xi}_1(t)}{\partial\hat{\eta}(t)}\dot{\hat{\eta}}(t) + \frac{\partial\hat{\xi}_1(t)}{\partial\hat{c}_1(t)}\dot{\hat{c}}_1(t), \quad (18)$$

$$\hat{\xi}_\rho = -\tilde{\xi}_{\rho-2}(t) - c_\rho\tilde{\xi}_{\rho-1}(t) + \lambda_{\rho-1}\hat{\xi}_{\rho-1}(t) - k_{\rho-1}\left(\frac{\partial\hat{\xi}_{\rho-1}(t)}{\partial e(t)}\right)^2\tilde{\xi}_{\rho-1}(t) + \frac{\partial\hat{\xi}_{\rho-1}(t)}{\partial\hat{L}(t)}l_{\rho-1}(t) + b_r\frac{\partial\hat{\xi}_{\rho-1}(t)}{\partial e(t)}(\xi_1(t) - \hat{L}^T\hat{\eta}(t)) + \gamma_{\rho-2}(t) + \frac{\partial\hat{\xi}_{\rho-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}(t) + \frac{\partial\hat{\xi}_1(t)}{\partial\hat{c}_1(t)}\dot{\hat{c}}_1(t), 2 < \rho \leq r \quad (19)$$

where $l_{\rho-1}(t) = b_rQ\left[\sum_{i=1}^{\rho-1}\frac{\partial\hat{\xi}_i(t)}{\partial e(t)}\hat{\eta}(t)\tilde{\xi}_i(t) - \hat{\eta}(t)e(t)\right]$, $\gamma_{\rho-2}(t) = b_rQ\sum_{i=1}^{\rho-2}\frac{\partial\hat{\xi}_{\rho-1}(t)}{\partial e(t)}\frac{\partial\hat{\xi}_i(t)}{\partial\hat{L}(t)}\hat{\eta}\tilde{\xi}_i(t)$, $c_\rho > 0, k_{\rho-1} > 1$ for $2 < \rho \leq r$, and $Q \in \Re^{s\times s}$ is a positive definite matrix.

Now we are ready to design the control law as

$$u(t) = \hat{\xi}_r(t). \quad (20)$$

In order to analysis the stability of the closed system with (20), define the following Lyapunov-Krasovskii functional candidate

$$V(t) = \alpha_1\tilde{z}^T(t)P\tilde{z}(t) + \alpha_2\tilde{\eta}^T(t)P_\eta\tilde{\eta}(t) + \frac{1}{2}\left(k^{-1}\tilde{c}_1^2(t) + \tilde{L}^T(t)Q^{-1}\tilde{L}(t)\right) + V_e(t) + V_\xi(t) + V_d(t),$$
$$V_e(t) = \frac{1}{2}e^2(t), V_\xi(t) = \frac{1}{2}\sum_{i=1}^{r-1}\tilde{\xi}_i^2(t),$$
$$V_d(t) = \alpha_3\sum_{i=1}^{p}\sum_{j=1}^{n}\int_{t-d_j}^{t}(r_{ji}+s_{ji})e^{2i}(\theta)d\theta$$

with positive scalars $\alpha_i, i = 1, \cdots, 3$, where P and P_η are positive definite matrices satisfying $PH + H^TP = -I$, and $P_\eta F + F^TP_\eta = -I$, $\tilde{L}(t) = L - \hat{L}(t)$, $\tilde{c}_1(t) = c_1 - \hat{c}_1(t)$, and c_1 is an unknown constant.

The time derivative of $V(t)$ along the transformed overall system can be obtained as follows:

$$\begin{aligned}\dot{V}(t) &= \alpha_1\left[-\tilde{z}^T(t)\tilde{z}(t) + 2\tilde{z}^T(t)P\tilde{\Theta}(t)\right] + \alpha_2\left[-\tilde{\eta}^T(t)\tilde{\eta}(t) + 2b_r^{-1}\tilde{\eta}^TP_\eta(-FGe(t) + G(\tilde{z}_1(t)+\tilde{\Xi}(t)))\right] - \tilde{c}_1(t)\dot{\hat{c}}_1(t) - \tilde{L}^T(t)Q^{-1}\dot{\hat{L}}(t) \\ &\quad + \dot{V}_e(t) + \dot{V}_\xi(t) + \dot{V}_d(t) \\ &\leq -\frac{2}{3}\alpha_1\tilde{z}^T(t)\tilde{z}(t) + 3\alpha_1\|P\tilde{\Theta}(t)\|^2 - \frac{2}{3}\alpha_2\tilde{\eta}^T(t)\tilde{\eta}(t) + 9\alpha_2b_r^{-2}\left[\|P_\eta FG\|^2e^2(t) + \|G\|^2(\tilde{z}_1^2(t)+\tilde{\Xi}^2(t))\right] \\ &\quad - \tilde{L}^T(t)Q^{-1}\dot{\hat{L}}(t) - \tilde{c}_1(t)(e^2(t)+e^{2p}(t)) + \dot{V}_e(t) + \dot{V}_\xi(t) + \dot{V}_d(t). \end{aligned} \quad (21)$$

For simplicity, we first obtain the derivative of $V_e(t)$,

$$\dot{V}_e(t) \leq -c_0e^2(t) - \hat{c}_1(t)(e^2(t)+e^{2p}(t)) + (\frac{3}{2}\alpha_1^{-1} + L^TG + \frac{3}{4}\alpha_2^{-1}b_r^2L^TL)e^2(t) + \frac{\alpha_2}{3}\tilde{\eta}^T(t)\tilde{\eta}(t)) + \frac{\alpha_1}{3}\tilde{z}_1^2(t) + \frac{\alpha_1}{3}\tilde{\Xi}^2(t) + b_re(t)(\tilde{\xi}_1(t) - \tilde{L}^T(t)\hat{\eta}(t)). \quad (22)$$

If the adaptive law of $\hat{L}$ is chosen as

$$\dot{\hat{L}} = l_{r-1}(t), \quad (23)$$

then we obtain from (15)-(19) that

$$\begin{aligned}\dot{V}_\xi(t) &= -b_r\tilde{\xi}_1(t)e(t) - \sum_{i=1}^{r-1}(c_{i+1}+\lambda_i)\tilde{\xi}_i^2(t) - \sum_{i=1}^{r-1}k_i\left(\frac{\partial\hat{\xi}_i(t)}{\partial e(t)}\right)^2\tilde{\xi}_i^2(t) + \sum_{i=1}^{r-1}\tilde{\xi}_i(t)\frac{\partial\hat{\xi}_i(t)}{\partial e(t)}\left[-\tilde{z}_1(t) - \tilde{\Xi}(t) + b_rL^T\tilde{\eta}(t) - L^TGe(t)\right] \\ &\quad + \tilde{L}^T(Q^{-1}\dot{\hat{L}} + b_r\hat{\eta}(t)e(t)) \\ &\leq -b_r\tilde{\xi}_1(t)e(t) - \sum_{i=1}^{r-1}(c_{i+1}+\lambda_i)\tilde{\xi}_i^2(t) - \sum_{i=1}^{r-1}(k_i-1)\left(\frac{\partial\hat{\xi}_i(t)}{\partial e(t)}\right)^2\tilde{\xi}_i^2(t) + (r-1)\times\end{aligned}$$

$$\left[\tilde{z}_1^2(t) + \tilde{\Xi}^2(t) + b_r^2 \tilde{\eta}^T(t) L L^T \tilde{\eta}(t) + |L^T G|^2 e^2(t)\right] + \tilde{L}^T (Q^{-1} \dot{\hat{L}} + b_r \hat{\eta}(t) e(t)). \quad (24)$$

Next, the derivative of V_τ is shown as

$$\dot{V}_d(t) = \alpha_3 \sum_{i=1}^{p} \sum_{j=1}^{n} (r_{ji} + s_{ji}) \left[e^{2i}(t) - e^{2i}(t - d_j)\right]. \quad (25)$$

Finally, it follows from (21)-(25) that

$$\begin{aligned}\dot{V}(t) \le & -c_0 e^2(t) - \sum_{i=1}^{r-1} (c_{i+1} + \lambda_i) \tilde{\xi}_i^2(t) \\ & -(\frac{1}{3}\alpha_1 - \epsilon_1) \tilde{z}^T(t) \tilde{z}(t) - (\frac{1}{3}\alpha_2 - \epsilon_2) \tilde{\eta}^T(t) \tilde{\eta}(t) \\ & -(c_1 - \epsilon_3)(e^2(t) + e^{2p}(t)) \\ & +\epsilon_4 \sum_{i=1}^{p} \sum_{j=1}^{n} (r_{ji} + s_{ji}) e^{2i}(t - d_j) + \dot{V}_d(t) \quad (26)\end{aligned}$$

where

$$\begin{aligned}\epsilon_1 &= 9 b_r^{-2} \alpha_2 \|G\|^2 + r - 1, \epsilon_2 = (r-1) b_r^2 L^T L, \\ \epsilon_3 &= 3\alpha_1 \|P\|^2 r_0 + (\epsilon_1 + \frac{\alpha_1}{3}) s_0 - (9 b_r^{-2} \alpha_2 \|P_\eta F G\|^2 \\ & \quad + \frac{3}{2}\alpha_1^{-1} + L^T G + \frac{3}{4}\alpha_2^{-1} b_r^2 L^T L, \\ \epsilon_4 &= max\{3\alpha_1 \|P\|^2, \epsilon_1 + \frac{\alpha_1}{3}\}.\end{aligned}$$

Thus it can be shown that there exists a sufficiently big positive scalar α_2 satisfying $\alpha_2 > 6\epsilon_2$, and then sufficiently big positive scalars α_1 and α_3 satisfying $\alpha_1 > 6\epsilon_1, \alpha_3 > \epsilon_4$. Finally, there exists a sufficiently big c_1 satisfying $c_1 \ge \epsilon_3 + \alpha_3 \epsilon_5$, where $\sum_{i=1}^{p} \sum_{j=1}^{n} (r_{ji} + s_{ji}) e^{2i}(t) \le \epsilon_5 (e^2(t) + e^{2p}(t))$, so the following result holds

$$\dot{V}(t) \le -c_0 e^2(t) - \sum_{i=1}^{r-1} c_{i+1} \tilde{\xi}_i^2(t) - \frac{1}{6} \tilde{z}^T(t) \tilde{z}(t) - \frac{1}{6} \tilde{\eta}^T(t) \tilde{\eta}(t).$$

It thus implies that $e(t), \tilde{\xi}_i(t), \tilde{z}, \tilde{\eta}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ for $i = 1, 2, \cdots, r-1$ and $\hat{c}_1(t)$ and $\hat{L}(t)$ are bounded. Furthermore, it then implies the boundedness of $\hat{\xi}_1(t)$ and thus the boundedness of $\xi_1(t)$. It follows from the boundedness of $e(t), \tilde{\xi}_1(t), \xi_1(t), \hat{\xi}_1(t), \hat{\eta}(t), \hat{c}_1(t)$ and $\hat{L}(t)$ that $\hat{\xi}_2(t)$ is bounded. Together with the boundedness of $\tilde{\xi}_2(t)$, the boundedness of $\hat{\xi}_2(t)$ implies the boundedness of $\xi_2(t)$. Similarly, we can establish the boundedness of $\hat{\xi}_i, i = 1, 2, \cdots, r-1$. Therefore, all variables are bounded. It further implies the boundedness of $\dot{e}(t), \dot{\tilde{\xi}}_i(t), \dot{\tilde{z}}(t)$, and $\dot{\tilde{\eta}}(t)$, which implies, together with $e(t), \tilde{\xi}_i(t), \tilde{z}, \tilde{\eta}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\lim_{t\to\infty} e(t) = 0, \lim_{t\to\infty} \tilde{z}(t) = 0, \lim_{t\to\infty} \tilde{\eta}(t) = 0, \lim_{t\to\infty} \tilde{\xi}_i = 0, i = 1, 2, \cdots, r-1$ based on Barbalat's Lemma. In summary, we have established the following theorem.

Theorem 1. Under Assumptions 1-3, the feedback controller composed of (3), (12), (16), (20) and (23) solves the global output regulation problem for nonlinear time-delay system (1) with the unknown exosystem (2).

5. ILLUSTRATIVE EXAMPLE

In order to illustrate the proposed output regulation approach, consider a nonlinear system described by

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) + y^2(t - d_1) w_2(t), \\ \dot{x}_2(t) &= u, y(t) = x_1(t), e(t) = y(t) - w_1(t) \quad (27)\end{aligned}$$

and set $d_1 = 2$. The exosystem is described by $\dot{w}_1(t) = \sigma, \dot{w}_2(t) = -\sigma$, with an unknown frequency σ. So the transfer matrix in Lemma 1 is $\bar{T}_d(\sigma, 2)$.

Since the system (27) is of relative degree 2, we choose the filtered transformation as $\dot{\xi}_1(t) = -2\xi_2(t) + u$. The transformed model of (27) is thus obtained,

$$\begin{aligned}\dot{z}(t) &= -2z(t) + \Theta(y(t), \bar{y}(t-d), w(t)), \\ \dot{y}(t) &= z + \Xi(y(t), \bar{y}(t-d), w(t)) + \xi_1(t). \quad (28)\end{aligned}$$

with $\Theta(y(t), \bar{y}(t-d), w(t)) = -4y(t) - 2y^2(t-2) w_2(t)$ and $\Xi(y(t), \bar{y}(t-d), w(t)) = 2y(t) + y^2(t-2) w_2(t)$.

Observing the fact that the disturbance $w(\sigma) = [\, w_1 \ w_2 \,]$ is bounded and the following relations,

$$\begin{aligned}\tilde{\Theta} &= -4e(t) - 2e^2(t - d_1) w_2(t) \\ & \quad -4e(t - d_1) w_1(t - d_1) w_2(t), \\ \tilde{\Xi} &= 2e(t) + e^2(t - d_1) w_2(t) + 2e(t - d_1) w_1(t - d_1) w_2(t),\end{aligned}$$

we get $p = 2$. Moreover, there exist unknown constants r_0, s_0, r_{11}, r_{12}, s_{11} and s_{12} satisfying (14).

In order to show that Assumption 3 is satisfied with the invariant manifold $\pi(w(t), \sigma)$, we first consider the invariant manifold of (27) as $\pi_{x1}(w(t), \sigma) = q(w(t)) = w_1(t)$ and

$$\begin{aligned}\pi_{x2} &= \sigma w_2(t) - (\cos^2 2\sigma) w_1^2(t) w_2(t) \\ & \quad -2(\cos 2\sigma)(\sin 2\sigma) w_1(t) w_2^2(t) - (\sin^2 2\sigma) w_2^3(t).\end{aligned}$$

In this case the desirable input term is given by

$$\begin{aligned}\alpha(w(t), \sigma) &= \frac{\partial \pi_{x2}}{\partial w(t)} S(\sigma) w(t) \\ &= (-2\cos^2 2\sigma + 3\sin^2 2\sigma) \sigma w_1(t) w_2^2(t) \\ & \quad -\sigma^2 w_1(t) + (\sigma \cos^2 2\sigma) w_1^3(t) \\ & \quad -2\sigma(\cos 2\sigma)(\sin 2\sigma)(w_2^3(t) - 2w_1^2(t) w_2(t)).\end{aligned}$$

Hence there exists a mapping $\upsilon(w(t), \sigma)$ satisfying

$$\frac{\partial \upsilon(w(t), \sigma)}{\partial w(t)} S(\sigma) w(t) = -2\upsilon(w(t), \sigma) + \alpha(w(t), \sigma).$$

It should be noted that the existence of $\upsilon(w(t), \sigma)$ is established by observing the fact that it is the steady-state response of the first-order linear system $\dot{\xi}_1(t) = -2\xi_1(t) + \mu(w(t), \sigma)$. To this end, the invariant manifold is constructed as $\pi(w(t), \sigma) = \pi_{x2} - \upsilon(w(t), \sigma) - 2w_1(t)$. Thus Assumption 3 is satisfied.

Let

$$\Omega(\sigma) = \begin{bmatrix} 0 & \sigma & 0 & 0 \\ -\sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\sigma \\ 0 & 0 & -3\sigma & 0 \end{bmatrix}$$

such that the function $\alpha(w(t),\sigma)$ can be generated by (10). Furthermore, we can choose the controllable pair $\{F,G\}$ as

$$F = \begin{bmatrix} -3 & 1 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & -7 & 3 \\ 0 & 0 & -4 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Let $\sigma = 1, c_0 = 2, c_2 = k_1 = \lambda_1 = 1, k = 5$ and $Q = 5I$. Based on the design procedure developed in Section 4, the regulator for (27) can be constructed as follows,

$$\begin{aligned} u(t) &= -e(t) - \tilde{\xi}_1(t) + \hat{\xi}(t) - N^2\tilde{\xi}_1(t) \\ &\quad +5\hat{\eta}^T(-\hat{\eta}(t)N\tilde{\xi}_1(t) - \hat{\eta}(t)e(t)) \\ &\quad +\hat{L}^T\dot{\hat{\eta}}(t) + N(\xi_1(t) - \hat{L}^T\hat{\eta}) + (e+e^3)\dot{\hat{c}}_1(t), \\ N &= \frac{\partial \xi_1(t)}{\partial e(t)} = -2 - \hat{c}_1(t)(1+3e^2(t)), \\ \dot{\hat{\eta}}(t) &= F\hat{\eta}(t) + G\xi_1(t), \dot{\hat{c}}_1(t) = 5(e^2(t)+e^4(t)), \\ \dot{\hat{L}}(t) &= 5I * (N\hat{\eta}(t)\tilde{\xi}_1(t) - \hat{\eta}(t)e(t)). \end{aligned}$$

The control results are then shown in Figs. 1-3 where the initial conditions are set as $x(s) = [\,-0.7\ 1\,]^T, s \in [-2,0]$, $u = 0$, $\hat{L}(0) = \hat{\eta}(0) = [\,0\ 0\ 0\ 0\,]^T$, $\xi_1(0) = 0$ and $\hat{c}_1(0) = 0$.

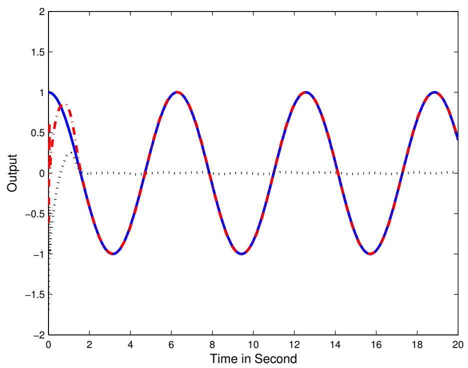

Fig. 1. Actual output and desired output (solid line: $q(t)$; dash-dotted line: $y(t)$); dotted line: $e(t)$)

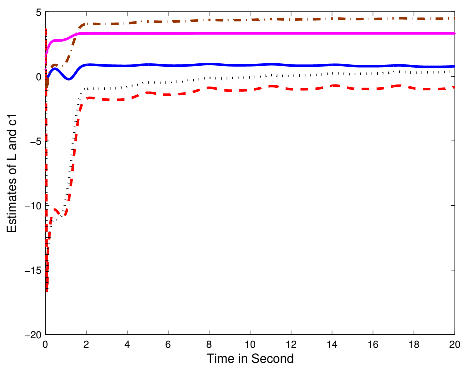

Fig. 2. Estimat variables $\hat{L}$ and $\hat{c}_1$(solid line: $\hat{L}_1$; dashed line: $\hat{L}_2$; dotted line: $\hat{L}_3$; dash-dotted line: $\hat{L}_4$; marker +: $\hat{c}_1$)

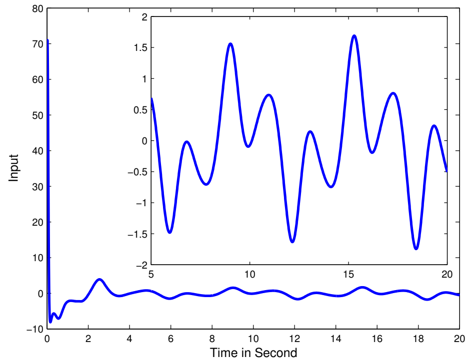

Fig. 3. The control input $u(t)$

6. CONCLUSION

An adaptive global output regulation method has been proposed for a class of nonlinear time-delay output feedback systems. The assumption on the invariant manifold of nonlinear delay systems is reduced to standard one for non-delay systems via a delay dependent transfer matrix. An adaptive internal model has been designed to deal with the unknown disturbances in the measurement.

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