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# A modeling approach for engine dynamics based on electrical analogy

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**Abstract:** In this paper, a multi-cylinder, internal combustion engine model is presented. The paper is focused on a simple modular, physically based and lumped parameter (zero dimensional) approach, leading to a complete and coherent model structure. The model is conceived to be used with general purpose simulation software, as MATLAB/Simulink. The mean value outputs of the model (pressure, temperature, mass flow, torque) are compared with experimental data, collected by a Fiat 1.8 liter engine, in order to perform a standard identification and validation procedure.

Keywords: Modeling; Automotive system; SI engine model.

## GLOSSARY

| | |
|---|---|
| SI | Spark Ignition |
| DI | Direct Injection |
| CVCP | Continuously Varying Cam Phasers |
| WOT | Widely Open Throttle |
| $VVT$ | Variable Valve Timing |
| HCCI | Homogeneous Charge Compression Ignition |
| p | gas pressure $[N/m^2]$ |
| T | gas temperature $[K]$ |
| $T_{sl}$ | temperature of lateral surface of cylinder $[K]$ |
| $T_{sb}$ | temperature of basic surface of cylinder $[K]$ |
| $\dot{m}$ | air mass flow rate $[kg/s]$ |
| $\dot{Q}_{ext}$ | heat flow rate $[kJ/s]$ |
| V | volume $[m^3]$ |
| $\vartheta$ | crank angle $[deg]$ |
| $\theta$ | spark advance $[deg]$ |
| $c_x$ | specific heat capacities (x= p or v)$[J/(kgK)]$ |
| $\gamma$ | ratio of specific heats , $c_p/c_v$ $[-]$ |
| R | gas constant $[kJ/kgK]$ |
| $h_i$ | enthalpy $[kJ/kg]$ |
| $m_{fuel}$ | fuel mass $[kg]$ |
| $Q_{hv}$ | fuel lower heating value $[J/kg]$ |
| $\eta_{cb}$ | combustion efficiency $[-]$ |
| $\lambda$ | air/fuel ratio $[-]$ |
| S | heat release $[W/kg]$ |
| $\dot{m}_c$ | choked air mass flow rate $[kg/s]$ |
| $\dot{m}_{nc}$ | not choked air mass flow rate $[kg/s]$ |
| $C_D$ | discharge coefficient $[-]$ |
| $A_T$ | effective flow area $[m^2]$ |
| $\alpha$ | throttle angle $[deg]$ |

## 1. INTRODUCTION

The availability of complete software packages for the simulation of the internal combustion engine behavior is an useful tool for engine designers in order to have, in advance, a good insight of what their design will be able (or not) to perform. The complexity of the models, contained inside these software packages, involves the physical and chemical aspects of the combustion process (mono-zone or multi-zone), the air flow dynamics (wave effects) or the liquid deposition and vaporization dynamics (wall wetting).

Widely accepted method for modeling Naturally Aspirated (NA) SI engines is the Mean Value Engine Model (MVEM) [Muller et al., 1998] thanks to its great flexibility, good performance and low power computation. It describes dynamically the development of engine physical variables over time periods which are long compared to the dominant time constants of the engine, see among others [Hendricks, 1989], [Hendricks, 2001] and [Deur et al., 2003]. Mean value engine models are aimed at describing the average dynamic of key internal engine variables, such as crank shaft speed, manifold pressure and air/fuel ratio, or variables that cannot be measured directly, as thermal and volumetric efficiencies [Aquino, 1981], [Powell and Cook, 1987]. Nevertheless, MVEM does not include explicitly a description of the intake, exhaust or combustion processes, but simply represents the overall result.

In literature, particular interest is dedicated to engine modeling, both for analysis and control purpose. As an example, in [Karlsson and Fredriksson, 1999] a cylinder-by-cylinder model of experimental variable valve timing 4-cylinders engine has been developed. The model includes the cylinder and manifold air mass flow, temperature, pressure and valve actuator dynamics, but it doesn't describe the combustion effects and burned gas residual. In [Bengtsson et al., 2004] and [Rausen et al., 2004] mathematical engine models are developed to study the dynamics of the HCCI engines. It is an alternative piston-engine combustion process that could potentially reach higher efficiencies than direct-injection. The charge is optimally mixed resulting in a minimization of emissions. The proposed model structures consist of a zero-dimensional cylinder models combined with a reduced chemical systems to describe the ignition phase. In [Cook and Powell, 1988] and [Fiengo et al., 2002] are presented engine models aimed to control designs. The main features of these models are the simplicity (typically linear models are adopted) and the accuracy necessary to reach the control goals.

The focus of this paper is to introduce a simple approach to model all the components of the internal combustion engine. The authors, starting from an analogy with electrical systems, have tried to simplify the approach eliminating the space dynamics (multi-zone combustion and wave effects), while preserving the time dynamics (resonance and tuning phenomena). In this way they have obtained an engine description similar to an electrical (although not linear) circuit, with all the useful consequences in term of existence and numerical availability of the solution. The advantages are in the specific comparison that is found between the engine components and variables (as throttle valve, cylinder, inertial flows), with electrical counterparts (current, voltage, resistance).

The main benefits achievable with this methodology is the simplicity to compose the whole engine model and customize it including all the latest devices. So it is possible to easily implement both a baseline engine and a high complex automotive system. Moreover, in order to design a new engine model adopting this approach, it is not necessary to have a deep mechanical and electronical knowledge, but to possess a familiarity with the more simple electric circuits.

The application domain of the engine model, that is possible to obtain with this methodology, is the analysis of dynamical behavior of engine, as an example scavenging or exhaust gas recirculation, allowing to test the engine dynamics under different operative conditions with a good level of reliability and accuracy. The approach has been here tested on SI engine, but it is remarked that only few changes are necessary to extend the model to other kinds of engines, as Diesel, TurboCompressor, Tri-fuel and HCCI engines.

It is highlighted that the proposed modeling methodology is thought to be independent from the mathematical environment used for the simulation. In particular, in this work Matlab/Simulink platform has been used, but other similar software packages could be adopted, as Modelica and GT-Power [Schweiger et al., 2005], [Tiller, 2001], [Batteh et al., 2003], [Eriksson, 2003].

Table 1. Variables

| element | q | i | v |
|---|---|---|---|
| electric | charge<br>q $[C]$ | current<br>i $[A]$ | voltage<br>v $[V]$ |
| hydraulic | volume<br>V $m^3$ | flow rate<br>$\dot{m}$ $[kg/sec]$ | pressure<br>p $[N/m^2]$ |
| mechanic | angle<br>$\theta$ $[rad]$ | speed<br>$\omega$ $[rad/s]$ | torque<br>T $[Nm]$ |

Table 2. Parameters

| element | parameters | | |
|---|---|---|---|
| electric | resistance<br>$R = \frac{v}{i}$ | inductance<br>$L = \frac{v}{\frac{di}{dt}}$ | capacity<br>$C = \frac{i}{\frac{dv}{dt}}$ |
| hydraulic | pneumatic resistance<br>$R = \frac{p}{\dot{m}}$ | | pneumatic capacity<br>$C = \frac{\dot{m}}{\frac{dp}{dt}}$ |
| mechanical | friction<br>$B = \frac{T}{\omega}$ | inertia<br>$J = \frac{T}{\frac{d\omega}{dt}}$ | elasticity<br>$\frac{1}{K} = \frac{\omega}{\frac{dT}{dt}}$ |

## 2. SIMPLE APPROACH TO ENGINE MODEL

A physical system can be decomposed in elementary subsystems or elements. For each elementary subsystem is possible to define three variables named [Balestrino and Celentano, 1997]:

q quantity, i.e. variable of the element
i flow, equal to $dq/dt$, i.e. the variable that flows across the element
v forcing, i.e. the variable acting at the extremes of the element

As an example, table 1 reports these variables for the electric, hydraulic and mechanical elements. The main features is that one variable can be considered constant despite the others or, alternatively, can be considered constant the following ratios

$$\frac{v}{i}, \frac{v}{\frac{di}{dt}}, \frac{i}{\frac{dv}{dt}} \tag{1}$$

as shown in table 2. Moreover, it is possible to demonstrate that, for a network of elements and in particular conditions, the variables are related thought Kirchhoff laws, as follows [Feynman and Leighton, 2007]:

- for each node of the network, the algebraic sum of the flow variables is zero
$$\sum_h i_h = 0, \tag{2}$$
where $i_h$ is the flow across the h-th element.
- for each mesh of the network, the algebraic sum of the forcing variables is zero
$$\sum_h v_h = 0, \tag{3}$$
where $v_h$ is the forcing acting on the h-th element.

Now, considering the engine formed by mechanical components, as throttle valve, manifolds, cylinders and crank shaft, crossed by a gas, the approach proposed in this work is based on the analogy among the electric, simpler to model, and the mechanic and hydraulic elements.

Regarding the mechanic systems, an analogy can be found among speed and torque respectively with electric current and voltage, named "Maxwell's analogy" [Feynman and

Leighton, 2007]. It results in considering the mechanical friction B as an electric resistance R and, similarly, the inertia J as an electric inductance L and the inverse of the elasticity K as a capacitor C. The same considerations can be done for the hydraulic elements. Here the analogies are between the gas flow rate $\dot{m}$ with the current $i$, the pressure $p$ with the voltage $v$ and the volume $V$ with the charge $q$. Then, the electrical resistance corresponds to the pneumatic resistance, that is the resistance of gas flowing across an orifice, and the volume to an electric capacitor.

In this scenario, all the parts composing the engine can find an equivalent electrical circuit or element as detailed described in the following sections. Moreover, in order to exactly describe the operation of electrical circuit, it is necessary to use the Maxwell equations. These can capture both the dynamics of the electrical quantities, such as currents and voltages, and the related electromagnetical phenomena, as transmission and radiation. Fortunately, if the size of the circuit is small compared to the wavelength of the electrical variables (i.e. the ratio between the light speed and the frequency of the pulsating events), these electromagnetical phenomena can be neglected. As a consequence, the partial differential relationships of the Maxwell equations can be simplified to the widely used electrotechnic equations, that are the Kirchhoff laws and the current/voltage relationships of circuit components. Similarly, the same approach can be extended to the internal combustion engine, providing that the wavelength (in this case the ratio between the sound speed and the frequency of its pulsating events) is large enough compared to the length size of the engine. As an example, a four cylinder four stroke engine, running at 3000 rpm, generates intake pulses at 100 Hz, resulting in a of 3.4 meters. Considering the engine size of approximately 1 meter, the lumped parameter approach seems to be reasonable.

To conclude, the analogy between the mechanical and electrical components can be extended to the relationship governing the relative quantities. In fact, the first Kirchhoff equation has a specific counterpart in the mass conservation law and, similarly, the Bernoulli equation can correctly replace the second Kirchhoff equation.

## 3. COMPONENTS

The engine is seen as an array of cylinders, having common connections with an intake and an exhaust manifold. The connections are regulated by valves opening. According to the previous section, it is possible to distinguish separate subsystems interconnected each others, such as the intake manifold equipped with throttle valve, the exhaust manifold and cylinders. From the phenomenological point of view, the elements composing the engine can be classified in the following categories: volumes, orifices, inertial effects and combustion. In the following, each category is introduced and the relationship among the interested variables are reported.

### *3.1 Volumes*

Here are grouped the intake and exhaust manifold and cylinders, respectively as constant and variable volumes. The electric counterpart is the quantity of charge stored in a capacitor, as shown in Figure [1] 1, reporting the corresponding circuit.

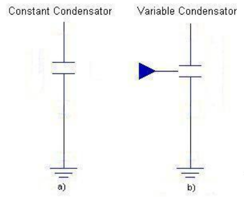


Fig. 1. Volume equivalent circuit: a) constant capacity; b) variable capacity.

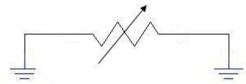

Fig. 2. Orifice equivalent circuit.

Applying the corresponding current/voltage relationship and considering the analogies with pressure and temperature inside the volume, it is possible to obtain the classical equations [Muller et al., 1998]. Starting from ideal gas equations

$$pV = mRT \tag{4}$$

where $R$ is the specific gas constant and $m$ is the mass of gas.

$$\dot{p} = \frac{R\gamma}{V}\left[\sum_i \dot{m}_i T_i - T\sum_j \dot{m}_j + \frac{\gamma-1}{R\gamma}\dot{Q}_{ext} - \frac{p\dot{V}}{R}\right] \tag{5}$$

$$\begin{aligned}\dot{T} =& \frac{R\gamma T}{pV}\left[\sum_i \dot{m}_i T_i\left(1-\frac{T}{\gamma T_i}\right) - T\sum_j \dot{m}_j\left(1-\frac{1}{\gamma T}\right)\right.\\ &\left. + \frac{\gamma-1}{R\gamma}\dot{Q}_{ext} - \frac{p\dot{V}}{R}\left(1-\frac{1}{\gamma T}\right)\right]\end{aligned} \tag{6}$$

where $i$ represents the entering mass flow and $j$ the outgoing mass flow. For sake of brevity, the details on how to obtain equations 5 and 6 are omitted.

It is remarked that, regarding the intake and exhaust manifolds, since the volume V is constant, the derivative terms in the equation disappear.

### *3.2 Orifices*

The orifices are responsible of the pressure drops along the gas path. They are modeled as variable resistances causing equivalent voltage drops, as illustrated in Figure 2. The size of the orifice is variable and regulated by valve opening, as throttle valve, air bypass, intake and exhaust valves [Heywood, 1988].

The electrical resistance is governed by a static relationship between voltage and current, corresponding to a static relationship between the analogue variables, i.e. pressure and flow rate, in according to the well known equations [Hendricks, 1989].

[1] The circuits have been drawn using Modelica.

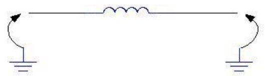

Fig. 3. Inertial effects equivalent circuit.

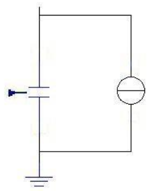

Fig. 4. Combustion equivalent circuit.

$$\begin{cases} \dot{m}_c = \frac{C_D A_T p_0}{\sqrt{RT}} \gamma_{\frac{1}{2}} (\frac{2}{\gamma+1})^{\frac{\gamma+1}{2(\gamma-1)}} & \frac{p_T}{p_0} > 1 \\ \dot{m}_{nc} = \frac{C_D A_T p_0}{\sqrt{RT}} (\frac{p_T}{p_0})^{\frac{1}{\gamma}} \{ \frac{2\gamma}{\gamma-1} [1 - (\frac{p_T}{p_0})^{\frac{\gamma-1}{\gamma}}] \} & \frac{p_T}{p_0} < 1 \end{cases} \tag{7}$$

where $p_T$ and $p_0$ are respectively the pressure upstream and downstream the orifice.

### 3.3 Inertial effects

The inertial phenomena can be considered as minor efforts but not complectly negligible. They describe the reduction or the increase of the pressure upstream the valve of a quantity proportional to the derivative of the mass flow through the same valve. Here, they are modeled as an linear inductance, see Figure 3, regulated by a differential relationship between voltage and current, corresponding to the following equation

$$p_{corr} = p - k\ddot{m} \tag{8}$$

where $p_{corr}$ is the manifold pressure and $k$ is a parameter to be set. It is remarked that this kind of relationship is not present in literature. In order to justify the adopted choice, both the analogy with the electrical circuit and the simulation results illustrated later on the paper can be adduced.

### 3.4 Combustion description

The combustion process constitutes the most meaningful and complex phenomenon occurring into the engine. In order to model the in-cylinder cycle pressure, an equivalent electric circuit has been adopted, as shown in Figure 4. The circuit is formed by a variable condensator, representing the cylinder volume according to section 3.1, equipped by an impulsive voltage generator. This causes an impulsive increase of the voltage at the condensator extremities and, consequently, generates a current flow thought the capacitor.

This phenomenon corresponds to the well known combustion process, i.e. an impulsive increase of the in-cylinder pressure caused by the combustion resulting in a torque generation and in mass flow through the exhaust valves. The equation regarding this process is described by the following relationship

$$\begin{aligned} \dot{Q}_{ext} = & h_i A_l(\theta)(T_{sl} - T) + h_2 A_b(T_{sb} - T) \\ & + m_{fuel}\eta_{cb}\eta_{burn}(\lambda, p)S(\theta, p)Q_{HV} \end{aligned} \tag{9}$$

where $h_1$ and $h_2$ are parameters to be set and $A_l$ and $A_b$ the lateral and base area respectively. It is remarked that equation 9 represents the heat power generated by the combustion affecting the pressure 5 and temperature 6. The cyclic variation has been implemented as a function of the operating conditions ($\eta_{burn}$ in 9) of engine and, partially, equipped with a random behavior needed to represent the combustion cycle-by-cycle and cylinder-by-cylinder irregularity [Hendricks, 2001] and [Deur et al., 2003].

## 4. ENGINE MODEL

### 4.1 Equivalent circuit

Based on the analogies depicted in the previous section, the entire engine can be represented by the circuit shown in Figure 5.

The model starts describing the dynamic of the air crossing the intake manifold, i.e. driven by the ambient pressure (a current generator), the air mass passes the filter (a resistance) and the throttle body (a variable resistance) and arrives into the cylinder through the intake valves (a new variable resistance). The cylinder are described by a parallel of "n" combustion equivalent circuit (see Figure 1), with "n" the number of cylinders composing the engine. Finally, the gas mixture is discharged into the exhaust manifold through the exhaust valves (a variable resistance) and ends into the ambient crossing the muffler (a resistance). For sake of completeness, it is possible to introduce the inductors that naturally represent the inertial effects of the current and are able to describe the analogous effects of the fluid columns. The validity of the modeling choices adopted in this work has been tested with experimental data of Fiat engine 1.8 liters with 8 valves and with a CVCP.

### 4.2 Simulation results

Two experiments are illustrated, the first at 1500 rpm and the second at 5500 rpm. In particular for the last experiment, the condition necessary to simplify the Maxwell equations, i.e. the wavelength of the electric variables must be much larger than the size of the circuit, is still valid but the approximation error is higher. In fact, a four cylinder pulses at 183 Hz, resulting in a wavelength at the fundamental frequency of 1.8 meter.

Figures 6 to 8 report the first experiment. In particular, Figures 6 and 7 show the simulated intake and exhaust mass flows for each cylinder and the pressure cycle and mass flows through the intake and exhaust valves, respectively. Figure 8 completes the experiment comparing the simulated in-cylinder pressure cycle with experimental data highlighting the good performance. The intake and exhaust valve lift ends the Figure 8.

Similarly, Figures 9 to 11 report the second experiment at 5500 rpm. Again, firstly are illustrated the simulated mass flows and pressure and finally the comparison with experimental data is shown in Figure 11. It is remarked that the performance are still high despite the lower accuracy of the condition at the base of the proposed approach.

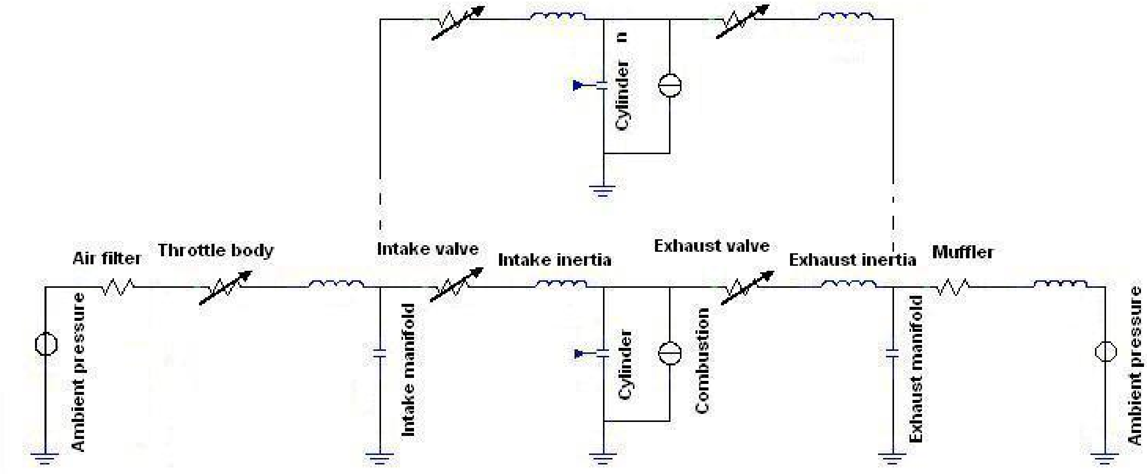


Fig. 5. Internal combustion engine equivalent circuit.

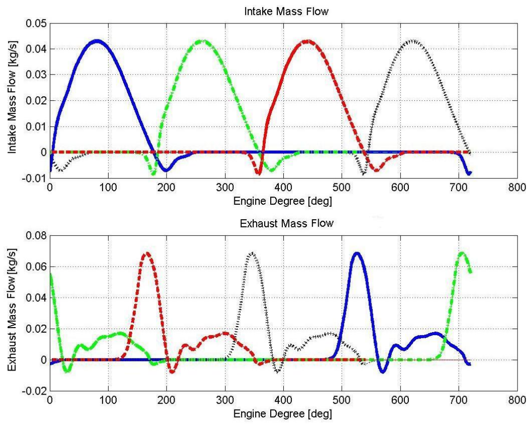


Fig. 6. Experiment 1 at 1500 rpm and WOT. Intake and exhaust mass flows for the four cylinders.

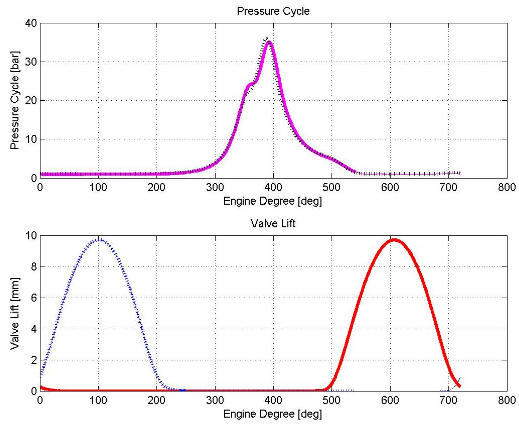


Fig. 8. Experiment 1 at 1500 rpm and WOT. The first plot compares experimental data (dotted-black line) of the pressure inside cylinder and simulated results (solid-magenta line); the second plot reproduces the intake and exhaust valves lift (experimental data).

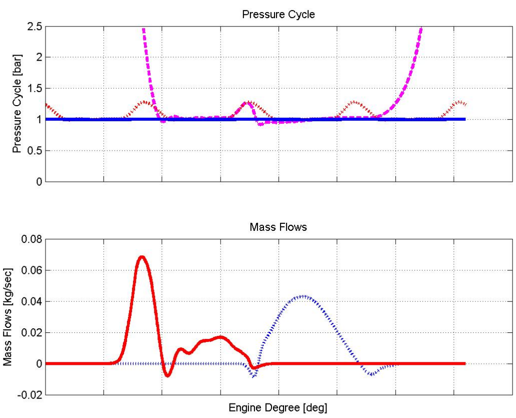


Fig. 7. Experiment 1 at 1500 rpm and WOT. The first plot depicts the pressure inside intake manifold (solid-blue line), exhaust manifold (dotted-red line) and cylinder (dashed-magenta line). The second plot describes the valves lift, showing the exhaust mass flow (solid-red line) and intake mass flow (dotted-blue line).

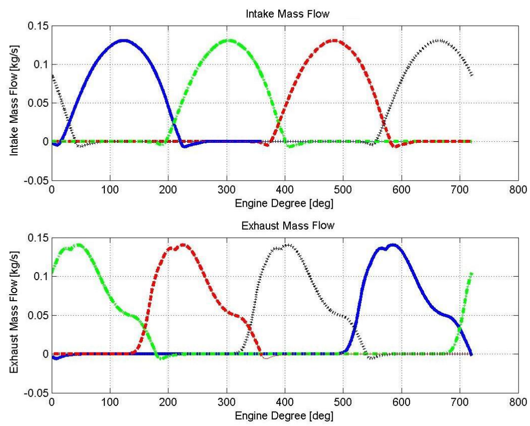


Fig. 9. Experiment 2 at 5500 rpm and WOT. Intake and exhaust mass flows for the four cylinders.

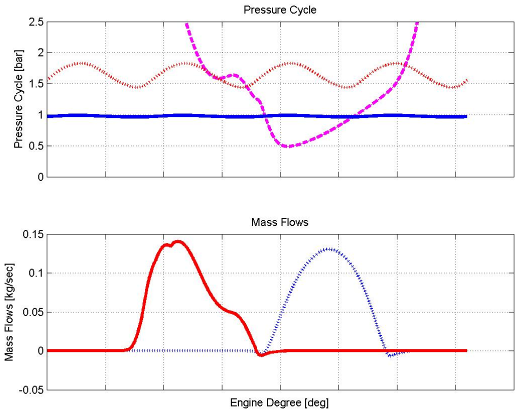


Fig. 10. Experiment 2 at 5500 rpm and WOT. The first plot depicts the pressure inside intake manifold (solid-blue line), exhaust manifold (dotted-red line) and cylinder (dashed-magenta line). The second plot describes the valves lift, showing the exhaust mass flow (solid-red line) and intake mass flow (dotted-blue line).

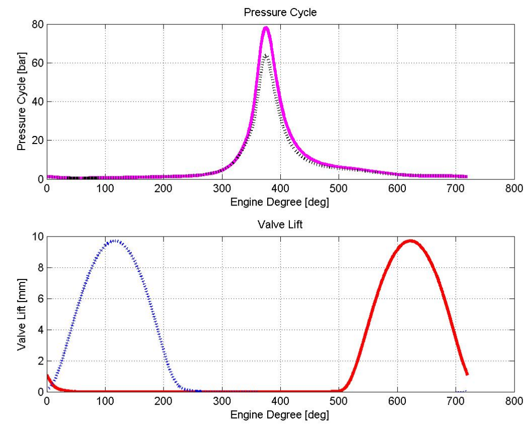


Fig. 11. Experiment 2 at 5500 rpm and WOT. The first plot compares experimental data (dotted-black line) of the pressure inside cylinder and simulated results (solid-magenta line); the second plot reproduces the intake and exhaust valves lift (experimental data).

## 5. CONCLUSION

A simple modeling approach for SI engine based on the equivalence with electric circuit has been introduced. The resulting mean value model has been tested by comparing with experimental data, showing a good level of reliability and accuracy.

Two points are been remarked, the calibration phase and the simulation complexity. The first aspect required the identification of the characteristic parameters. In particular, the model is based on known geometric constants and only few parameters need to be set. It results in a simple and quick calibration phase.

Conversely, the drawback is the high computational complexity necessary to simulate the model, causing the impossibility to run it for long test.

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