Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

State Feedback Controller Design for a Class of Nonlinear Systems with General Criteria

Chung Seop Jeong*. Edwin Engin Yaz.*Yvonne Ilke Yaz **

** Marquette University, Department of Electrical and Computer Engineering, Milwaukee, Wisconsin 53201,USA (Tel: 414-288-6820; e-mail: chung.jeong@mu.edu, edwin.yaz@mu.edu).*
*** Milwaukee School of Engineering, Math Department, Milwaukee, Wisconsin 53202, USA (e-mail: yaz@msoe.edu)*

Abstract: In this work, we provide a framework for the design of linear state feedback controllers for a class of continuous-time conic nonlinear systems driven by finite energy disturbances. This controller design is presented for various performance criteria in a unified framework using linear matrix inequalities in the formulation. Illustrative examples are included.

1. INTRODUCTION

In this work, the problem of linear state feedback controller design is addressed using linear matrix inequality (LMI) for a class of continuous-time nonlinear systems with conic type nonlinearities and driven by finite energy disturbances. Various performance criteria are utilized in designing controllers leading to a common LMI framework. The possible utilization of efficient numerical schemes for solving LMI (Boyd, et al., 1994) is the reason to choose this approach. In (Jacobson, 1974), a similar nonlinear model with point-wise quadratic constraints is introduced in a stochastic discrete-time context and the finite-horizon quadratic optimal controller is derived. Mean-square optimal state estimator designs can be found in (Yaz, 1988). Reference (Yaz and Yaz, 2001) is on the extension of (Yaz, 1988) to generalized performance criteria. Infinite and receding horizon controllers with quadratic criteria are discussed together with their robustness properties in (Yaz, 1989a) and (Yaz, 1989b), respectively. System theoretic properties of such systems are investigated using the LMI approach in (Yaz and Yaz , 1999). In the present work, various control problems including guaranteed-cost suboptimal versions of H_2 , H_∞, etc. are tackled within a common deterministic framework using LMIs. So, in that sense, we can view the present work as an extension of mean –square optimal control results in (Yaz, 1989a) to the continuous-time deterministic case with generalized performance criteria and conic rather than point-wise quadratic constraints in time. A similar representation can be found in dissipative systems literature. Dissipative systems giving rise to integral quadratic constraints have a long history. Much of the groundwork in this area is laid in references (Willems, 1972) and (Fradkov and Yakubovich, 1973). More recent results and especially with applications to robust control and filtering include (Megretsky, 1992; Savkin and Petersen, 1995; Savkin and Petersen, 1996; James and Petersen, 1996) among others. The present work is also an extension of discrete-time nonlinear observer designs to the continuous -time control case (Yaz, et al., 2007).

In the next section, the system model is introduced. Then the performance criteria are presented and optimization possibilities are pointed out. Next, the solution of the control problem is given. Applications to various control criteria are illustrated in simulation examples.

The following notation is used in this work: $x \in R^n$ denotes an n-dimensional vector with real elements and with the associated Euclidean norm $\|x\| = (x^T x)^{1/2}$ where $(\cdot)^T$ represents the transpose. $A \in R^{m\times n}$ denotes an $m \times n$ matrix with real elements. A^{-1} is the inverse of matrix A, $A > 0\,(A < 0)$ means A is a positive (negative) definite matrix, and I_m is an identity matrix of dimension m. $\lambda_{\min}(A)(\lambda_{\max}(A))$ denotes the minimum(maximum) eigenvalue of the symmetric matrix A. L_2 is the space of all real-valued vectors with finite energy.

2. PROBLEM FORMULATION

Let us assume that the system equation is as follows**:**

$$\dot{x} = f(x,u,w) \tag{1}$$

with the linear state feedback control

$$u = Kx \tag{2}$$

where $x \in R^n$ is the state, u is the input and w is an L_2 disturbance input.

Let us also assume the following description for the nonlinear dynamics

$$\begin{aligned} &\|f(x,u,w) - (Ax + Bu + Fw)\|^2 \\ &\le (C_f x + D_f u + E_f w)^T (C_f x + D_f u + E_f w) \end{aligned} \tag{3}$$

This inequality can be viewed as describing the hypersphere in which the nonlinearity “f” resides. The center of the hypersphere is defined by a linear system

$$\dot{x} = Ax + Bu + Fw$$

and the square of its radius is bounded above by the quadratic term on the right hand side of (3). So "f" can deviate from its associated linear system by at most the quantity specified by this quadratic term.

Consider the controlled nonlinear model (1)-(3), performance output

$$z = C_z x + D_z u + E_z w \tag{4}$$

and consider the general performance objective

$$\dot{V} + \delta\|z\|^2 + \in\|w\|^2 - \beta z^T w \le 0 \tag{5}$$

for an energy function $V = x^T P x$ where $P > 0$.

Notice that upon integration, inequality (5) yields

$$\begin{aligned} x(t)^T Px(t) &\le x_0^T P x_0 \\ &-\int_0^t (\delta\|z(\tau)\|^2 + \in\|w(\tau)\|^2 - \beta z(\tau)^T w(\tau))d\tau \end{aligned} \tag{6}$$

or by using Rayleigh's inequalities, ($\lambda_{\min}(P)\|x\|^2 \le x^T Px \le \lambda_{\max}(P)\|x\|^2$), we obtain

$$\begin{aligned} \lambda_{\min}(P)\|x(t)\|^2 &\le \lambda_{\max}(P)\|x_0\|^2 \\ &-\int_0^t (\delta\|z(\tau)\|^2 + \in\|w(\tau)\|^2 - \beta z(\tau)^T w(\tau))d\tau \end{aligned} \tag{7}$$

that allows several design criteria to be addressed in a unified eigenvalue problem (Boyd, et al., 1994) framework. We can design different controllers for a variety of performance criteria for this system.

First of all, in the absence of noise $w(t) \equiv 0,\ t \ge 0$, by taking $\delta > 0$, $\beta = 0$, and $\in = 0$, (7) will yield a bound on the energy of the performance output in terms of the initial estimation error x_0

$$\int_0^t \|z(\tau)\|^2 d\tau \le \frac{1}{\delta}\lambda_{\max}(P)\|x_0\|^2 \tag{8}$$

Minimizing $\lambda_{\max}(P)$ and maximizing δ will give us a smaller bound on the energy of the performance output. This is sub-optimal H_2 control (Boyd, et al., 1994).

In the noisy case, by setting $\delta = 1$, $\beta = 0$, and, $\in < 0$ for $x_0 = 0$, gives the result

$$\int_0^t \|z(\tau)\|^2 d\tau \le -\in \int_0^t \|w(\tau)\|^2 d\tau \tag{9}$$

which means a bound on the L_2 to L_2 gain of the controlled system or a suboptimal H_∞ result.

When x_0 =0, if we use this formulation, we can design several dissipative controllers by using different values of δ, β, and $\in$.

If we set $\delta > 0$, $\beta = 1$, and $\in = 0$, we get output strict passivity:

$$\int_0^t z(\tau)^T w(\tau)d\tau \ge \delta\int_0^t \|z(\tau)\|^2 d\tau \tag{10}$$

Very strict passivity, which is the strict passivity both in the terms of the input and the output, can be obtained if we set $\delta > 0$, $\beta = 1$, and $\in > 0$:

$$\int_0^t z(\tau)^T w(\tau)d\tau \ge \in \int_0^t \|w(\tau)\|^2 d\tau + \delta\int_0^t \|z(\tau)\|^2 d\tau^{\,2} \tag{11}$$

As described above, this LMI formulation enables us to design various controllers according to different performance criteria in a common framework.

3. MAIN RESULT

The following is the main result of this paper:

Theorem 1. Consider the controlled nonlinear system model (1)-(3), and the performance output (4).

For $w(t) \equiv 0,\ t \ge 0$ and given $\delta > 0, \beta > 0$ and $\in$, if

$$G = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{12}^T & g_{22} & g_{23} \\ g_{13}^T & g_{23}^T & g_{33} \end{pmatrix} \ge 0 \tag{12}$$

where

$$g_{11} = -QA^T - AQ - BY - Y^T B^T - \alpha I$$

$$g_{12} = QC_f^T + Y^T D_f^T,\ g_{13} = QC_z + Y^T D_z$$

$$g_{22} = \alpha I,\ g_{23} = 0,\ g_{33} = \frac{1}{\delta} I$$

holds, for some $Y, Q > 0$ and $\alpha > 0$ then the controlled system will satisfy (8) with the gain being found by $K = YQ^{-1}$, for $t \ge 0$.

For $w(t) \ne 0,\ t \ge 0$, for given δ, β, $\in$ and other design parameters, if

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12}^T & s_{22} & s_{23} & s_{24} \\ s_{13}^T & s_{23}^T & s_{33} & s_{34} \\ s_{14}^T & s_{24}^T & s_{34}^T & s_{44} \end{pmatrix} \ge 0 \tag{13}$$

where

$$s_{11} = -QA^T - AQ - BY - Y^T B^T - \alpha I$$

$$s_{12} = \frac{\beta}{2}(QC_f + Y^T D_z) - F,\ s_{13} = QC_z + Y^T D_z$$

$$s_{14} = QC_f^T + Y^T D_f^T,\ s_{22} = -\in I + \frac{\beta}{2}(E_z + E_z^T)$$

$$s_{23} = E_z, s_{24} = E_f,\ s_{33} = \frac{1}{\delta} I, s_{34} = 0,\ s_{44} = \alpha I$$

holds, for some Y, $Q > 0$ and $\alpha > 0$ then the controlled system will satisfy (9), (10) or (11) with the controller gain being found by $K = YQ^{-1}$, for $t \ge 0$.

Sketch of Proof:

By using the energy function $V = x^T Px$ where $P > 0$, consider (5) along the motion of (1)-(4). Adding and subtracting $2x^T P[(A+BK)x + Fw]$ in (5), using $2a^T b \le \alpha a^T a + \alpha^{-1} b^T b,$ for any $a, b \in R^n, \alpha > 0$, lead to

$$(x^T \; w^T) \; H \begin{pmatrix} x \\ w \end{pmatrix} \le 0 \tag{14}$$

where

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{12}{}^T & h_{22} \end{pmatrix} \tag{15}$$

with

$$h_{11} = -P(A+BK) - (A+BK)^T P - \alpha P^2$$
$$-\alpha^{-1}(C_f + D_f K)^T (C_f + D_f K) - \delta(C_z + D_z K)^T (C_z + D_z K)$$
$$h_{12} = \frac{\beta}{2}(C_z + D_z K)^T - PF$$
$$-\alpha^{-1}(C_f + D_f)^T E_f - \delta(C_z + D_z K) E_z$$
$$h_{22} = -\alpha^{-1} E_f{}^T E_f - \in I + \frac{\beta}{2}(E_z + E_z^T)$$

After using Schur's complement (Boyd, et al., 1994) to convert this nonlinear matrix inequality to an LMI, substituting inverse of P for Q, pre-and post-multiplying by the block diagonal matrix (Q, I) and rearranging, we obtain LMI (12) for the case of no noise, and LMI (13) for the case of additive noise.

4. SIMULATION RESULTS

This example is provided to present some initial simulation results on the time responses of various controller designs proposed in this work. The following system model is considered.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.3 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ -\sin x_1 \end{bmatrix} + F \cdot w(t)$$

for both the no - noise case and the noisy case.

Note that the system linearized about the origin is unstable. In the following example, the sub-optimal H_2 observer ($w(t) \equiv 0, \; t \ge 0$), output strict passivity, and very strict passivity were chosen to show the verification of the proposed design methodology.

The values of design parameters used in the simulations are given in Table 1.

	Suboptimal H_2	Output Strict Passivity	Very Strict Passivity
β	0	1	1
$\in$	0	0	0.01
δ	1	0.5	0.5
C_z	0.1 I_2	0.1 I_2	0.1 I_2
D_z	[.1;.1]	[.1;.1]	[.1;.1]
E_z	0	0.1	0.1
C_f	0.1 I_2	0.1 I_2	0.1 I_2
D_f	[0.1;0.1]	[.1;.1]	[.1;.1]
E_f	0	0.1	0.1
F	[0;0]	[.5;.5]	[.5;.5]

Table 1. Design Parameter Values

For the simulation of the sub-optimal H_2 controller, the controller gain is found to be $K = \begin{bmatrix} -0.3286 \\ -2.3538 \end{bmatrix}$. For the case of the output strict passivity, K is found to be $K = \begin{bmatrix} -0.1202 \\ -3.4079 \end{bmatrix}$, and the gain K for the very strict passivity case is found to be $K = \begin{bmatrix} 0.2200 \\ -3.3202 \end{bmatrix}$.

The state variable plots for each case are given in Fig.s 1-3.

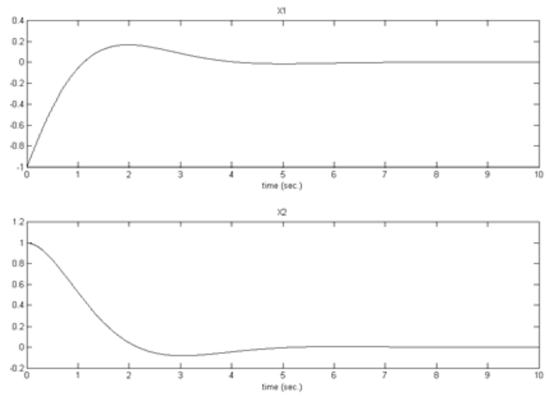

Fig. 1. Plots of x_1 and x_2 for the Sub-optimal H_2 case

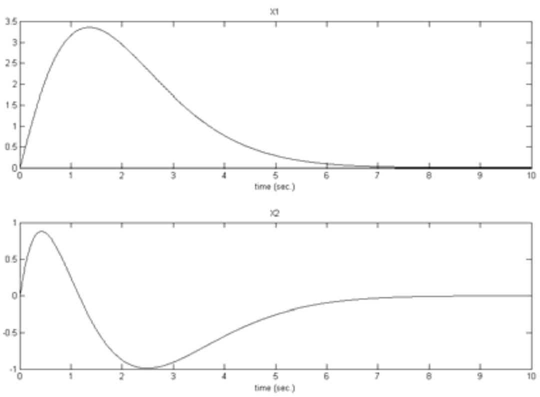

Fig. 2. Plots of x_1 and x_2 for the Output Strict Passivity

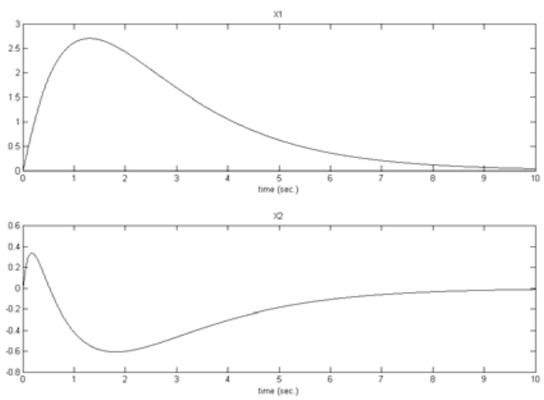

Fig. 3. Plots of x_1 and x_2 for the Very Strict Passivity

The co-plots of state variables for each case are given in Fig. 4. Notice the change in the peak value, shape and speed of the transient response for different criteria.

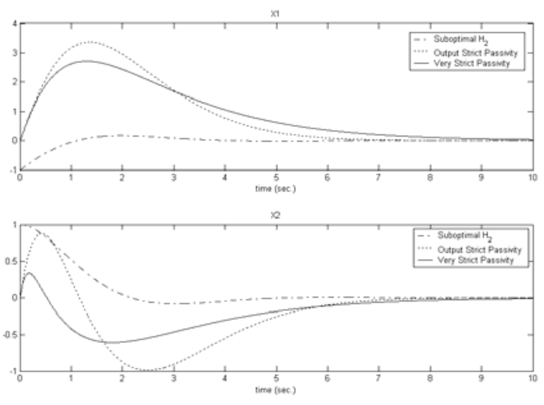

Fig. 4. Co-plots of x_1 and x_2 for the selected criteria

5. CONCLUSION

In this work, we have considered linear state feedback controller designs for a class of continuous-time nonlinear systems with general performance criteria. We have shown that a common framework using a linear matrix inequality formulation can be used to solve controller design problems for various performance criteria. One can see from the simulation examples that the use of different criteria will give rise to different time response performance characteristics such as response speed, overshoot, etc.

REFERENCES

Boyd, S., L. ElGhaoui, E. Feron, and V. Balakrishnan (1994). Linear Matrix Inequalities In System and Control Theory, SIAM Studies in Applied Mathematics. SIAM, Philadelphia.

Fradkov, A. L. and V. A. Yakubovich (1973). Necessary and sufficient conditions for absolute stability of systems with two integral quadratic constraints, Soviet Math. Doklady, vol. 14, pp. 1812 – 1815.

Jacobson, D. H. (1974). A general result in stochastic optimal control of nonlinear discrete time systems with quadratic performance criteria. J. Math. Anal. Applic., vol. 47, pp.153 – 161.

James, M.R. and I. R. Petersen (1996). Nonlinear state estimation for uncertain systems with an integral constraint. Proceediings of Conference on Decision and Control, pp. 560 – 561.

Megretsky, A. (1992). Power distribution approach in robust control. Tech. Rep. TRITA / MAT – 92 –0027, Dept. of Math, Royal Inst. of Tech., S – 10044, Stokholm, Sweden.

Savkin, A. V. and I. R. Petersen (1995). Structured dissipativeness and absolute stability of nonlinear systems. Int. J. Contr., vol. 62, pp. 443 – 460.

Savkin, A. V. and I. R. Petersen (1996). Robust state estimation for uncertain systems with averaged integral quadratic constraints. Int. J. Contr., vol. 64, pp. 923 – 939.

Willems, J. C. (1972). Dissipative dynamical systems I: General theory, II: Linear systems with quadratic supply rates. Archive for Rational Mechanics and Analysis, vol. 45, pp. 321 – 343.

Yakubovich, V. A. (1992). Nonconvex optimization problem: the infinite horizon linear quadratic control problem with quadratic constraints. Syst. Contr. Lett.s, vol. 19, pp. 13 –22.

Yaz, E. (1988). Linear state estimators for nonlinear stochastic systems with noisy nonlinear observations. Int. J. Control, vol. 48, pp. 2465 – 2475.

Yaz, E. (1989a) Robust design of stochastic controllers for nonlinear systems. IEEE Trans. Autom. Control, vol. 34, pp. 349 –353

Yaz, E (1989b). Infinite horizon quadratic optimal control of a class of nonlinear stochastic systems. IEEE Trans. Autom. Control, vol. 34, pp. 1176 – 1180.

Yaz, E. E., C. S. Jeong, Y. I. Yaz, and A. Bahakeem (2007). Discrete-time Observer Design with General Criteria. J. of Franklin Institute, vol. 344, pp. 918-928

Yaz, Y. I. and E. E. Yaz (1999). On LMI formulations of some problems arising in nonlinear stochastic system analysis, IEEE Trans. Autom. Control, vol. 44, pp. 813 – 816.

Yaz, Y. I. and E. E. Yaz (2001). State estimation of uncertain nonlinear stochastic systems with general criteria, Applied Mathematics Letters, vol. 14, pp. 605-610.