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Coordination of Decentralized Large-Scale Process Optimal Control Problems

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Abstract: A key issue in decentralized decision-making is ensuring that the decentralized optimal solution results in the overall optimum. In this paper, we present a method to achieve tractable coordination schemes for large-scale dynamic systems. We will focus on the Interaction Prediction Principle, first introduced by Msearovic et al. [1970b], and present an extension to achieve zero-offset. The proposed approach is illustrated using a forced-circulation evaporator system, in which we show how to decompose it and how to coordinate between its different units.

Keywords: Decentralization; Large-scale optimization; Optimal control theory.

1. INTRODUCTION

Many applications in manufacturing engineering and chemical process engineering are faced with computational difficulties due to the large number of decision variables or the complexity and high order of the underlining systems. Decomposing these optimal control problems into smaller sub-problems may be an adequate manner to deal with the computational difficulties, but we still have to implement efficient coordination algorithms in order to ensure that the overall optimum is achieved.

Decomposition-coordination literature often focuses on static problems. We refer the reader to the works of Dantzig and Wolfe [1961], Benders [1962], Cohen [1978], Lasdon et al. [2001]. The main contributions to the dynamic decomposition literature are the early works of Msearovic et al. [1970a,b] and Cohen [1977].

For static optimization problems, three decomposition coordination algorithms can be used. In Cohen [1978], a nice unification of these approaches is presented.

(1) The *price-driven* method consists on fixing an overall price for the resource consumptions: each decentralized unit optimizes its own assessment, while the coordinator fixes the price system.
(2) The *quantity-driven* method also known as the resource allocation method consists on fixing a consumption level for each decentralized unit: the coordinator allocates the available resources to the units.
(3) The *prediction-driven* method consists on fixing a price for the shared resources between the decentralized units and fixing a consumption level for the own resources of each sub-system: the coordinator fixes the price system and allocates the resources.

In this paper, we present an extension of the prediction-driven decomposition method to dynamic problems: the *Interaction Prediction Principle* (IPP) algorithm. The main contribution of this paper is set within the process control framework. To our knowledge, there is no published work about the applications of the prediction driven decomposition method to optimal control problems, in order to achieve zero-offset as it is often required in process control. For that sake, we focus on identifying the convergence issues associated with the IPP and we present a method to achieve a reasonable trade-off between algorithm convergence and zero-offset.

To highlight the advantages of the proposed decentralization scheme, a case study is presented. We will deal with a a forced-circulation evaporator system, decompose it into two sub-units and apply the modified IPP algorithm in order to achieve a zero-offset with a decentralized control scheme.

2. DECOMPOSING AN OPTIMAL CONTROL PROBLEM

There is a rich literature concerning decomposition-coordination methods for general static optimization problems. These methods can be successfully extended to the dynamic case, leading to different decentralization algorithms. In particular, if we apply the prediction-driven decomposition method to a linear quadratic optimal control problem, it results in the so called Interaction Prediction Principle (IPP).

In this paper, we will not present the IPP as an extension of static decomposition methods. Instead, we will use a *Pontryagin decomposition* scheme in order to present the IPP algorithm. We refer the reader to Cohen and Miara [1990] where a similar approach was taken.

2.1 The overall problem

We consider the following linear quadratic (LQ) optimal control problem with a finite time horizon:

$$\begin{aligned} \min_{x,u} \quad & \frac{1}{2}\int_0^{t_f} \left(x^\top(t)Qx(t) + u^\top(t)Ru(t)\right) dt, \\ \text{s.t.} \quad & \dot{x}(t) = Ax(t) + Bu(t), \\ & x(0) = \xi, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^m$ is the state vector, $u(t) \in \mathbb{R}^n$ is the input (control) vector and A, B, Q and R are matrices with appropriate dimensions (Q and R are symmetric). We assume that all the states are available for measurement.

The Pontryagin necessary optimality conditions of the optimal control problem (1) leads to the following equations:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ x(0) = \xi, \\ \dot{\mu}(t) = -A^\top \mu(t) - Qx(t), \\ \mu(t_f) = 0, \\ 0 = Ru(t) + B^\top \mu(t). \end{cases} \tag{2}$$

Remark 1. These conditions are also sufficient if Q is non-negative definite and R is positive definite. In that case, solving the problem (1) is equivalent to solving the set of equations (2). In the sequel, we will assume these conditions on the weighting matrices Q and R.

2.2 Decomposing the Pontryagin optimality conditions

We assume that the vector x is decomposable on N components $x^\top = (x_1^\top, \ldots, x_N^\top)^\top$, $x_i \in \mathbb{R}^{m_i}$, and $u^\top = (u_1^\top, \ldots, u_N^\top)^\top$, $u_i \in \mathbb{R}^{n_i}$. We consider a decomposition on N sub-units, following a block decomposition for the matrices defining the LQ problem. In order to have an additive cost, we also assume that Q and R are block diagonal ($Q_{ij} = 0$ and $R_{ij} = 0$ for all $i \neq j$).

The block diagonal elements of the matrices A and B represent the intra-unit dynamics and the off-diagonal elements represent the inter-units interactions. Therefore, the process dynamic of each sub-unit i ($i = 1, \ldots, N$) can be denoted as:

$$\begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) + v_i(t), \\ x_i(0) = \xi_i, \end{cases} \tag{3}$$

where v_i represents the interaction variable(s) between the dynamics of the different sub-units

$$v_i(t) = \sum_{j \neq i} \left(A_{ij}x_j(t) + B_{ij}u_j(t)\right). \tag{4}$$

Similarly, we can decompose the co-state dynamic. This leads to the following equations for each sub-unit i ($i = 1, \ldots, N$):

$$\begin{cases} \dot{\mu}_i(t) = -A_{ii}^\top \mu_i(t) - Q_{ii}x_i(t) - \eta_i(t), \\ \mu_i(t_f) = 0, \end{cases} \tag{5}$$

where η_i represents the interaction variable(s) between the co-state dynamics of the different sub-units

$$\eta_i(t) = \sum_{j \neq i} A_{ji}^\top \mu_j(t). \tag{6}$$

The gradient equation in the optimality conditions (2) can also be decomposed following the same principle. For all units $i = 1, \ldots, N$, we obtain:

$$0 = R_{ii}u_i(t) + B_{ii}^\top \mu_i(t) + \nu_i(t), \tag{7}$$

where ν_i is such that:

$$\nu_i(t) = \sum_{j \neq i} B_{ji}^\top \mu_j(t). \tag{8}$$

For each sub-unit $i = 1, \ldots, N$, the conditions (3), (4), (5), (6), (7) and (8) could be interpreted as the Pontryagin optimality conditions of the following optimal control problem:

$$\begin{aligned} \min_{x_i,u_i,\eta_i,\nu_i} \quad & \frac{1}{2}\int_0^{t_f} \Big(x_i^\top(t)Q_{ii}x_i(t) + u_i^\top(t)R_{ii}u_i(t) \\ & \qquad + 2\eta_i^\top(t)x_i(t) + 2\nu_i^\top(t)u_i(t) \Big) dt, \\ \text{s.t.} \quad & (3),\ (4),\ (6) \text{ and } (8). \end{aligned} \tag{9}$$

2.3 The IPP algorithm

If each unit solves its own sub-problem (9), we will eventually achieve the centralized optimal control trajectories associated with the overall problem (1). However, one challenge in adding *artificial variables* to the sub-problems is the introduction of non-convexities. Furthermore, it is more preferable to handle optimal control problems without algebraic constraints such as (4), (6) and (8).

To avoid these difficulties, we perform an iterative algorithm between the sub-units and a coordinator. The coordinator predicts at each step the value of the interaction variables given by the equations (4), (6) and (8). The sub-units will take these values as parameters and perform an *easy* convex control problem without algebraic constraints.

This algorithm, first introduced by Msearovic et al. [1970a,b], is called the Interaction Prediction Principle in reference to the fact that the interaction variables are predicted. We present hereafter the application of the IPP to the problem (1) (refer to Cohen [1977] for a deeper presentation of the IPP).

Algorithm 1.

- **At the first step [0]**
  The coordinator gives N initial predictions of the inputs $u_i^{[0]}, \quad i = 1, \ldots, N$.
- **At the step [k]**
  - The coordinator integrates the state dynamic (10) with the predicted input $u = u^{[k]}$ to obtain a predicted output $x^{[k]}$.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ x(0) = \xi, \end{cases} \tag{10}$$

  - The coordinator integrates the co-state dynamic (11) with the predicted output $x = x^{[k]}$ to obtain a predicted price $\mu = \mu^{[k]}$.

$$\begin{cases} \dot{\mu}(t) = -A^\top \mu(t) - Qx(t), \\ \mu(t_f) = 0, \end{cases} \tag{11}$$

  - Each sub-unit solves its own optimal control problem (12) with the predicted inputs, outputs and prices given by the coordinator.

$$\begin{aligned}\min_{x_i,u_i} \quad & \frac{1}{2}\int_0^{t_f}\Big(x_i^\top(t)Q_{ii}x_i(t)+u_i^\top(t)R_{ii}u_i(t)\\ & +\sum_{j\neq i}2\mu_j^{[k]\top}(t)(A_{ji}x_i(t)+B_{ji}u_i(t))\Big)dt,\\ \text{s.t.} \quad & \dot{x}_i(t)=A_{ii}x_i(t)+B_{ii}u_i(t)\\ & +\sum_{j\neq i}A_{ij}x_j^{[k]}(t)+B_{ij}u_j^{[k]}(t),\\ & x_i(0)=\xi_i.\end{aligned} \tag{12}$$

This leads to solving N quadratic Riccati equations:

$$\begin{aligned}&\dot{P}_i(t)+P_i(t)A_{ii}+A_{ii}^\top P_i(t)\\ &\quad -P_i(t)B_{ii}R_{ii}^{-1}B_{ii}^\top P_i(t)+Q_{ii}=0,\\ &\qquad\qquad P_i(t_f)=0,\end{aligned} \tag{13}$$

and N differential equations:

$$\begin{aligned}&\dot{p}_i(t)+\left(A_{ii}^\top-P_i(t)B_{ii}R_{ii}^{-1}B_{ii}^\top\right)p_i(t)\\ &\quad -P_i(t)B_{ii}R_{ii}^{-1}\left(\sum_{j\neq i}B_{ji}^\top\mu_j^{[k]}(t)\right)\\ &\quad +P_i(t)\left(\sum_{j\neq i}A_{ij}x_j^{[k]}(t)+B_{ij}u_j^{[k]}(t)\right)\\ &\quad +\sum_{j\neq i}A_{ji}^\top\mu_j^{[k]}(t)=0,\\ &\qquad\qquad p_i(t_f)=0.\end{aligned} \tag{14}$$

The feedback solution is:

$$\begin{aligned}u_i(t)=-R_{ii}^{-1}\Big(&B_{ii}^\top P_i(t)x_i(t)+B_{ii}^\top p_i(t)\\ &+\Big(\sum_{j\neq i}B_{ji}^\top\mu_j^{[k]}(t)\Big)\Big).\end{aligned} \tag{15}$$

- **At the step** $[\mathbf{k+1}]$
  Re-inject $u^{[k+1]}$ solution of (15) into (10).
- **At the step** $[\infty]$
  Stop when $\left\|u^{[k+1]}-u^{[k]}\right\|_{L^2([0,t_f];\mathbb{R}^m)}\leq\epsilon$.

Remark 2. This algorithm can be interpreted as a prediction driven decomposition where we decompose both optimization variables and resources. Each unit is responsible of the minimization of its own cost function while taking into account a valuation of the interaction with the other units ($A_{ji}x_i+B_{ji}u_i$). To value these interaction variables, a coordinator will compute a price system (μ).

3. TRADE-OFF BETWEEN CONVERGENCE AND ZERO-OFFSET

In process control, as in control of other apparatus, we often require that the optimal state trajectory reaches (or be as close as possible of) the equilibrium state within the given time horizon. In linear quadratic optimal control problems, we can always select the weighting matrices Q and R to achieve the desired controller performance. Therefore, we are able to tune the controller to attain zero-offset within the given time horizon.

The main concern and contribution of this paper is to ensure the convergence of the coordination algorithm between the different sub-units, while keeping the same performance as that of the overall system. For that purpose, we first focus on the convergence of the IPP algorithm.

3.1 Convergence of the IPP algorithm

The main contribution of the IPP is to tackle the non-convexity issues of problem (9) by iteratively predicting the interaction variables ($v_i,\eta_i,\nu_i,\quad i=1,\dots,N$). In addition, as any iterative optimization algorithm, it still requires convexity and coercivity assumptions on the other variables ($u_i,\quad i=1,\dots,N$). This is reflected by the assumptions of Theorem 1 (see Cohen [1978]).

Before presenting Theorem 1, we first introduce the so called integrated cost functions required for the convexity and coercivity assumptions. Let us denote by $x=S(u)$ the solution of the dynamic of the overall problem (1). We denote by $\mathcal{J}$ the overall cost function such that:

$$\mathcal{J}(u)=\frac{1}{2}\int_0^{t_f}\Big((S(u))^\top QS(u)+u^\top Ru\Big)\,dt.$$

For all $i=1,\dots,N$ and for a fixed initial condition ξ_i, the solution of the differential equation (3) introduces the following mapping:

$$x_i=S_i(u_i,v_i),\quad \forall i=1,\dots,N, \tag{16}$$

where $u_i,\ v_i,\ x_i$ are functions of the time $t\in[0,t_f]$. Using (16) in the right hand side of equation (4), we can rewrite (4) $\forall i=1,\dots,N$:

$$v_i=\sum_{j\neq i}H_{ij}(u_j,v_j)=\sum_{j\neq i}\left(A_{ij}S_j(u_j,v_j)+B_{ij}u_j\right). \tag{17}$$

In the same manner, we can use (16) to rewrite the cost function of (1) such that:

$$\begin{aligned}J(u,v)=\frac{1}{2}\sum_{i=1}^{N}\int_0^{t_f}\Big(&\left(S_i(u_i,v_i)(t)\right)^\top Q_{ii}\left(S_i(u_i,v_i)(t)\right)\\ &+u_i^\top(t)Q_{ii}u_i(t)\Big)dt.\end{aligned} \tag{18}$$

Theorem 1. We assume that the functionals J and $\mathcal{J}$ are strictly convex on u and that they are twice differentiable. We denote the matrices $\Gamma=\frac{d^2\mathcal{J}}{du^2}$, $\Delta_i=\frac{\partial^2 J}{\partial u_i}$ and Δ the block diagonal matrix generated from the $\Delta_i,\quad ,i=1,\dots,N$. If $\Delta-\frac{\Gamma}{2}$ is a coercive operator [1], then we have the strong convergence of the sequence $u^{[k]}$ solutions of (12) to u^* solution of (1) and the convergence of $\mathcal{J}(u^{[k]})$ to $\mathcal{J}(u^*)$.

Proof. See [Cohen, 1978, Theorem 1]

3.2 A proximal approach to ensure both zero-offset and convergence

For a given initial state condition and a time horizon, the decentralized control scheme presented by the IPP impose some conditions on the weighting matrices Q and R in order to fulfill the coercivity assumptions of the

[1] K is a coercive operator if and only if $\exists c>0,\quad \forall x,\quad \langle Kx,x\rangle\geq c\left\|x\right\|^2$.

convergence theorem 1. This may interfere with the zero-offset objective that often require tuning these matrices in order to achieve a desired control performance. The fact is that the problem may be not *coercive enough* to ensure the convergence of the IPP for the selected matrices Q and R.

Therefore, we are faced with a trade-off:

(1) The weighting matrices Q and R should be selected to achieve the required control objectives.
(2) The matrices Q and R have to satisfy the conditions given by theorem 1.

To realize this trade-off we have to make the sub-problems *more coercive* while keeping the same aggressiveness of the optimal control. This can be achieved by introducing an extra term in the objective of each sub-problem (12) of the IPP algorithm 1:

$$(x_i - x_i^{[k]})^\top \gamma Q_{ii}(x_i - x_i^{[k]}) + (u_i - u_i^{[k]})^\top \gamma R_{ii}(u_i - u_i^{[k]}), \quad (19)$$

where $\gamma > 0$.

This technique is not new in the static optimization literature. It is often called *proximal algorithm* and the added term (19) is called *the prox-term*. We refer to Rockafellar [1976] for a general presentation of this method and to Cohen [1977] for an application in the IPP framework. Nevertheless, to our knowledge, no literature was found concerning the application of proximal algorithms in order to achieve zero-offset for decentralized optimal state trajectories, which is the main interest on this paper.

We propose the following Proximal IPP algorithm:

Algorithm 2.

- **At the first step [0]**
  The coordinator gives N initial predictions of the inputs and fix a large enough value of γ.
- **At the step [k]**
  - The coordinator integrates the state dynamic (10).
  - The coordinator integrates the co-state dynamic (11).
  - Each sub-unit solves its own modified optimal control problem (12)+(19). This leads to solving N modified quadratic Riccati equations

$$\dot{P}_i(t) + P_i(t)A_{ii} + A_{ii}^\top P_i(t) - P_i(t)B_{ii}((1+\gamma)R_{ii})^{-1}B_{ii}^\top P_i(t) + (1+\gamma)Q_{ii} = 0, \quad P_i(t_f) = 0, \quad (20)$$

and N modified differential equations.

$$\dot{p}_i(t) + \left(A_{ii}^\top - P_i(t)B_{ii}((1+\gamma)R_{ii})^{-1}B_{ii}^\top\right)p_i(t) - P_i(t)B_{ii}((1+\gamma)R_{ii})^{-1}\left(-\gamma R_{ii}^\top u_i^{[k]} + \sum_{j\neq i} B_{ji}^\top \mu_j^{[k]}(t)\right) + P_i(t)\left(\sum_{j\neq i} A_{ij}x_j^{[k]}(t) + B_{ij}u_j^{[k]}(t)\right) - \gamma Q_{ii}^\top x_i^{[k]} + \sum_{j\neq i} A_{ji}^\top \mu_j^{[k]}(t) = 0, \quad p_i(t_f) = 0. \quad (21)$$

The feedback solution is:

$$u_i(t) = -((1+\gamma)R_{ii})^{-1}\left(B_{ii}^\top P_i(t)x_i(t) + B_{ii}^\top p_i(t) - \gamma R_{ii}^\top u_i^{[k]} + \sum_{j\neq i} B_{ji}^\top \mu_j^{[k]}(t)\right). \quad (22)$$

- **At the step [k + 1]**
  Re-inject $u^{[k+1]}$ solution of (22) into (10).
- **At the step [∞]**
  Stop when $\left\|u^{[k+1]} - u^{[k]}\right\|_{L^2([0,t_f];\mathbb{R}^m)} \leq \epsilon$.

4. CASE STUDY

4.1 Problem description

In this section, a case study is performed to illustrate the effectiveness of the proposed algorithm.

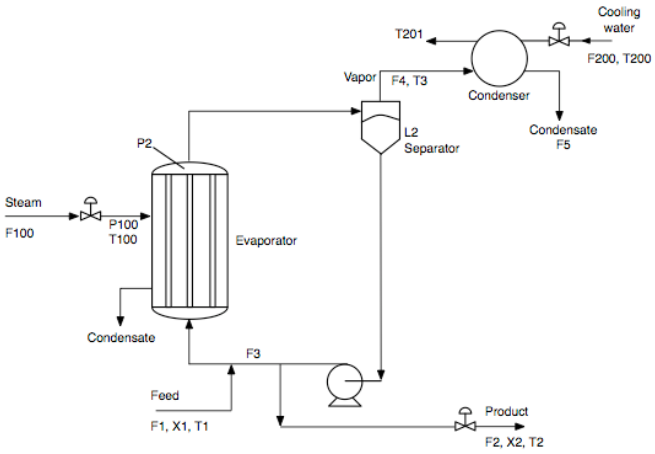

Fig. 1. Schematic of the forced-circulation evaporator (Newell and Lee [1989]).

We consider a forced-circulation evaporator system given in Newell and Lee [1989]. A schematic of the evaporator system is shown in Figure 1. In this process, a feed stream is mixed with a recycled liquor and this mixture is pumped through a vertical-tube heat exchanger. In this heat exchanger, heat supplied by a steam flow is used to boil the liquor mixture which is then sent to a separator. Next, the liquid and the vapor are separated. The vapor extracted from the separator vessel passes through a condenser where it condenses by exchanging heat with a cooling water stream. A fraction of the liquid extracted from the separator is obtained as product, while the rest of the liquid is recycled back to the vertical-tube heat exchanger.

The evaporator process model contains three measured states, three inputs and five disturbance variables. The name and description of the measured states and input variables, as well as their steady-state values and engineering units are given in Table 1. The disturbance variables are circulating flow rate F_3, feed flow rate F_1, feed concentration X_1, feed temperature T_1 and cooling water inlet temperature T_{200}. In this work, we consider these disturbance variables fixed.

The linearized model for the evaporator process given in p.12 in Newell and Lee [1989], can be arranged in a state

Table 1. Evaporator output and input variables

	Evaporator variables	Description	Equilibrium value
Output variables	L_2	Separator level	1 m
	X_2	Product composition	25 %
	P_2	Operating pressure	50.5 kPa
Input variables	F_2	Product flow rate	2 kg/min
	P_{100}	Steam pressure	194.7 kPa
	F_{200}	Cooling water flow rate	208 kg/min

space representation (with fixed disturbance variables) as following:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{23}$$

where

$$x^\top = (L_2 \;\; X_2 \;\; P_2)\,, \quad u^\top = (F_2 \;\; P_{100} \;\; F_{200})\,,$$

and

$$A = \begin{pmatrix} 0 & 0.10445 & 0.37935 \\ 0 & -0.1 & 0 \\ 0 & -0.10340 \times 10^{-1} & -0.54738 \times 10^{-1} \end{pmatrix},$$

$$B = \begin{pmatrix} -0.1 & 0.37266 & 0 \\ -0.1 & 0 & 0 \\ 0 & 0.36914 \times 10^{-1} & -0.75272 \times 10^{-2} \end{pmatrix}.$$

The dynamics of the system are expressed in normalized deviation variables (Newell and Lee [1989]).
The optimal control for the evaporator system is determined by minimizing the following cost function:

$$\frac{1}{2}\int_0^{t_f} (x^\top(t)Qx(t) + u^\top(t)Ru(t))dt. \tag{24}$$

4.2 Problem decomposition

We decompose the overall system into 2 subsystems. The first subsystem includes state variables L_2 and X_2 and input variables F_2 and P_{100}. The second subsystem involves state variable P_2 and input variable F_{200}. The optimization problem (24) is then divided into two subproblems, each with cost functions given by equation (12).

In the IPP algorithm, there is no limitation in the number of subsystems and this decomposition-coordination method can be successfully applied to subsystems with any combination of different variables. To decompose the evaporator system, we took advantage of the fact that cooling water flow rate (F_{200}) has no direct effect on the rate of change of the separator level (L_2) nor the rate of change of the product composition (X_2). In addition, there is no direct effect of the operating pressure (P_2) on the rate of change of variable X_2.

For each sub-unit $i = 1, 2$, the optimization problem is posed as in (9), with the following decomposition:

$$x = \left(\begin{array}{c|c} x_1^\top & x_2^\top \end{array}\right)^\top = \left(\begin{array}{cc|c} L_2 & X_2 & P_2 \end{array}\right)^\top,$$
$$u = \left(\begin{array}{c|c} u_1^\top & u_2^\top \end{array}\right)^\top = \left(\begin{array}{cc|c} F_2 & P_{100} & F_{200} \end{array}\right)^\top,$$

$$A = \left(\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{cc|c} 0 & 0.10445 & 0.37935 \\ 0 & -0.1 & 0 \\ \hline 0 & -0.10340 \times 10^{-1} & -0.54738 \times 10^{-1} \end{array}\right),$$

$$B = \left(\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right) = \left(\begin{array}{cc|c} -0.1 & 0.37266 & 0 \\ -0.1 & 0 & 0 \\ \hline 0 & 0.36914 \times 10^{-1} & -0.75272 \times 10^{-2} \end{array}\right),$$

$$\mu = \left(\begin{array}{c|c} \mu_1 & \mu_2 \end{array}\right)^\top = \left(\begin{array}{cc|c} \mu_{1a} & \mu_{1b} & \mu_2 \end{array}\right)^\top.$$

4.3 Simulation results

The decomposition-coordination algorithm was simulated for the evaporator process described in Section 4.1. The following parameters were used in the simulation: $\gamma = 1$, $Q = \mathbb{I}_3$; $R = 100\mathbb{I}_3$; $Q_{11} = \mathbb{I}_2$; $Q_{22} = 1$; $R_{11} = 100\mathbb{I}_2$; $R_{22} = 100$, where $\mathbb{I}_3$ and $\mathbb{I}_2$ denote the identity matrices of dimensions 3 and 2, respectively. Figures 2-4 show the results based on a 100 minutes closed-loop simulation with initial conditions $x(0) = \xi = (1\ 1\ 1)^\top$. The simulation results are shown in normalized deviation variables.

Figures (2a) and (2b) show the trajectory of the controlled outputs (L_2, X_2 and P_2) and the control inputs (F_2, P_{100} and F_{200}). It can be observed that the proposed scheme stabilizes the process (which is open-loop unstable) and drives the controlled variables to their desired targets.

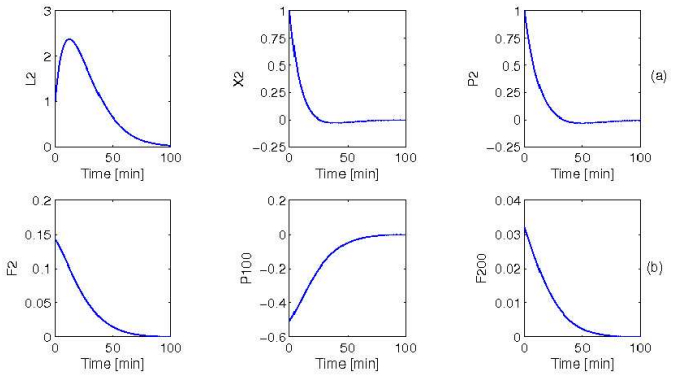

Fig. 2. (a) Trajectory of controlled outputs; (b)Trajectory of control inputs.

Figure (3) shows the dynamic price provided by the coordinator and updated at every iteration. As imposed by the final condition $\mu(t_f)$, the price vector converges to the origin at the end of the simulation time.

Finally, we show in Figure (4), the relative error on the inputs $\left(\left\|u^{[k]} - u^*\right\| / \|u^*\|\right)$, the relative error on the outputs $\left(\left\|x^{[k]} - x^*\right\| / \|x^*\|\right)$ and the error on the cost function $\left(\left\|\mathcal{J}(u^{[k]}) - \mathcal{J}(u^*)\right\| / \|\mathcal{J}(u^*)\|\right)$. The errors shown in Figure (4) are reported in logarithmic scale and are calculated as the normalized difference between the solution obtained with the IPP algorithm and the solution of overall

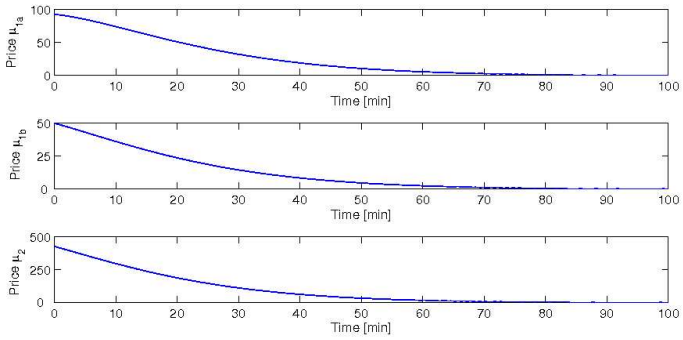

Fig. 3. Price provided by the coordinator.

problem. The solution of the overall problem is denoted with the superscript (*). It can be observed that after 29 iterations between the subsystems and the coordinator, the error is kept below a specified tolerance (10^{-3}). This indicates that, at convergence, the performance of the decomposed problem resulted in the optimal performance of the overall problem.

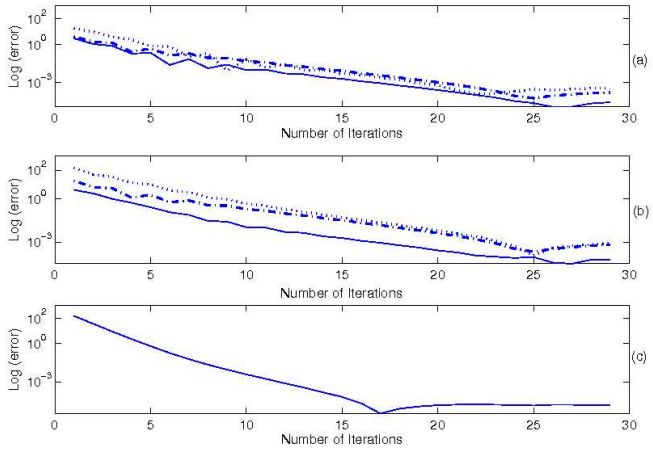

Fig. 4. (a) Logarithm of error on the output variables: error on L_2 (dotted line), error on X_2 (dash-dot line), error on P_2 (solid line); (b) Logarithm of error on the control input: error on F_2 (dash-dot line), error on P_{100} (solid line), error on F_{200} (dotted line); (c) Logarithm of error on the cost function.

In this simulation example, the prox-term (see equation (19)) was included in the objective function of each subsystem to ensure the convergence of the IPP algorithm. The effect of the parameter γ in the optimization problem was studied for the evaporator process example. For given matrices Q, R and final integration time t_f, there is a set of parameters $\gamma \geq \gamma_0$ which ensures that the convergence conditions (Theorem 1) are satisfied. The parameter γ_0 is a critical value below which the convergence is not guaranteed. As it is shown in Table 2, when the parameter γ is increased (for $\gamma_0 < \gamma$), it slows down the convergence of the algorithm, resulting in more iterations to achieve the overall optimum. In the example under study, the value of γ_0 is smaller than 0.5. This phenomenon can be interpreted as follows: the proximal term is made to penalize the deviation of the algorithm from the previous optimal results. At each step of the modified IPP algorithm, each sub-unit makes its actual decisions with small deviations with respect to the previous iteration. Therefore, the algorithm is slowed down to give to subunits and coordinator more time to interact in order to achieve the overall optimum. When designing the IPP algorithm, the parameter γ should be selected such that the computational load in solving the algorithm is appropriate.

Table 2. Effect of parameter γ in the evaporator process

Parameter γ	Number of Iterations
< 0.5	algorithm diverges
1	29
2	45
5	85
10	145

5. CONCLUSION

The objective of this paper was to highlight the main issues arising in the application of coordinated-decentralized schemes to process regulation problems. The coordinated-decentralized technique selected was the *Interaction Prediction* technique which is a decomposition-coordination method for dynamical systems.

The main contribution of this paper was to investigate the trade-off between the convergence of the IPP algorithm and the zero-offset objective for process control problems and show, inspired from the proximal algorithms, an efficient method to tackle this issue. We applied the modified algorithm to a chemical engineering example. The obtained results showed that the decentralized subunits satisfy the zero-offset objective and lead to the same optimal results than the overall process.

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