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Bumpless Transfer for Adaptive Switching Controls

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Abstract:
In this paper, a new bumpless transfer method is introduced based on slow-fast decomposition of the controller. The method is especially well-suited to situations in which the plant model is poor or yet to be identified, as may be the case in adaptive switching control. Simulation results are presented.

Keywords: Bumpless Transfer; Switching Control; Adaptive Control; Slow-fast decomposition; controller state reset

1. INTRODUCTION

Controller switching has been found to be useful in both adaptive and non-adaptive feedback control systems. Non-adaptive applications include switching from a manual to automatic control and anti-windup compensation (e.g., Hanus et al. [1987], Kothare et al. [1994]). In the adaptive control setting, switching among a finite set of controllers has offered as an alternative to continuous parameter tuning methods (e.g., Morse et al. [1992]). Adaptive switching control has improved existing adaptive control system behavior in many ways, but it has also introduced a new problem not associated with earlier continuous adaptive control methods. The problem is that the controller output can have undesired transients, called 'bumps', when the currently active controller and the new controller to be switched have different outputs at the switching instant. To attenuate these bumps associated with controller switching, a variety of bumpless transfer methods have been suggested over the years since the 1980's (Hanus et al. [1987], Graebe and Ahlén [1996], Turner and Walker [2000], Zaccarian and Teel [2004]), some of which are better suited to adaptive switching problems than others.

In adaptive control, the plant is generally not precisely known at the outset, and the goal of adaptive control is to change the controllers to improve performance as plant data begins to reveal some information about the plant. Thus, in adaptive switching control an exact plant is generally unavailable at the time of switching. This implies that bumpless transfer methods that may be suitable for non-adaptive applications such like anti-windup or transfer from manual to automatic control where the true plant is well-known, may not be ideal for adaptive switching control applications. In particular, in adaptive switching applications where the true plant model may only be poorly known at controller switching times, it may be preferable to employ a bumpless transfer technique for adaptive control that does not depend on precise knowledge of the true plant model.

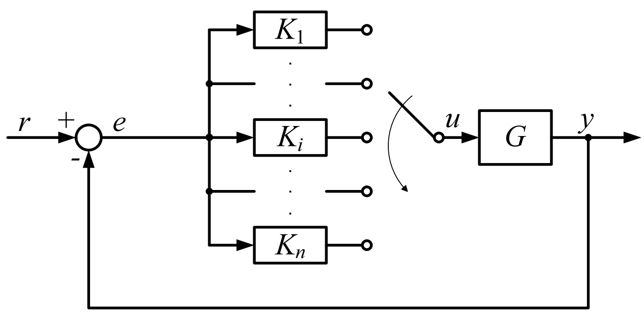

Fig. 1. Switching control system

While bumpless methods such as Graebe and Ahlén [1996] and Zaccarian and Teel [2004] require explicit knowledge of the true plant mode, other methods do not. For instance, the conditioning methods of Hanus et al. [1987], the continuous switching method of Arehart and Wolovich [1996], and linear quadratic optimal bumpless transfer method of Turner and Walker [2000] are examples of methods that do not require a plant model. Likewise, Arehart and Wolovich [1996] solved the problem how to ensure control signal continuity without precise plant knowledge, but did not consider transient effects that may follow immediately after controller switch.

In this paper, we present a new bumpless transfer method based on slow-fast decomposition of the controller. Like Arehart and Wolovich [1996], it is a method that can be implemented without precise knowledge of the true plant at switching times. Our slow-fast decomposition approach bumpless transfer is inspired by an adaptive PID controller in Jun and Safonov [1999]. A PID controller has a pole and a zero at origin. It is a special case of the controller which has fast modes (the differentiator) and slow modes (the integrator). Generalizing the PID controller case, the bumpless transfer suggested in this paper decomposes the original controllers into the fast modes controllers and the slow modes controllers. By appropriately re-initializing the states of the slow and fast modes at switching times, our methods can ensure that not only will the controller output be continuous, but also that it avoid fast transient bumps after switching.

The organization of the paper is as follows. Notation and a switching control system configuration are introduced in Section 2. The bumpless transfer problem formulation is presented in Section 3. The solution of the problem is suggested in Section 4. Section 5 shows simulation results and Section 6 concludes the paper.

2. PRELIMINARIES

2.1 Switching control system

We consider the switching control system as shown in Fig. 1. The system includes a plant and a set of controllers

$$\boldsymbol{K} = \{K_1, \cdots, K_i, \cdots, K_n\} \quad (i = 1, 2, \cdots, n). \tag{1}$$

The plant is a mapping $\mathbf{G} : C^0 \to C^0$ where C^0 denotes the signal to be continuous. The input of the plant is $u(t)$ and the output is $y(t)$. Plant input is directly connected to the controller output. Controller input is $e(t) = r(t) - y(t)$ where $r(t) \in C^0$ is a reference signal.

When controller K_i is in the feedback loop, then this controller is said to be *on-line*, and the other controllers are said to be *off-line*. The i-th controller K_i is assumed to have state-space realization

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i e \\ y_{Ki} &= C_i x_i + D_i e \end{aligned} \tag{2}$$

where e is the controller input and y_{Ki} is the output. Equivalently, we write

$$K_i(s) \overset{s}{=} \left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}\right]. \tag{3}$$

We are interested in the situation in which the on-line controller is switched from K_i to K_j at time t_s, so that

$$u = \begin{cases} y_{Ki} & \text{for } t < t_s \\ y_{Kj} & \text{for } t \geq t_s \end{cases}. \tag{4}$$

Since the controller output y_{Ki} is replaced by y_{Kj} at the switching instant t_s, the control signal u can have bumps in the neighborhood of $t = t_s$ if y_{Ki} and y_{Kj} have different values. Times immediately before and after t_s are denoted as t_s^- and t_s^+, respectively.

The objective of bumpless transfer is to ensure continuity in the control signal and to smooth 'bumpy' transients at, and immediately following, the switching instant.

2.2 Slow-fast decomposition

In this paper we consider controllers that can be additively decomposed into slow and fast parts as follows:

$$K(s) = K_{slow}(s) + K_{fast}(s) \tag{5}$$

with respective minimal realizations

$$K_{slow}(s) \overset{s}{=} \left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right] \text{ and } K_{fast}(s) \overset{s}{=} \left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right]. \tag{6}$$

The poles of $K_{slow}(s)$ are of smaller magnitude than the poles of $K_{fast}(s)$

$$|\lambda_i(A_s)| \leq |\lambda_j(A_f)| \text{ for all } i, j$$

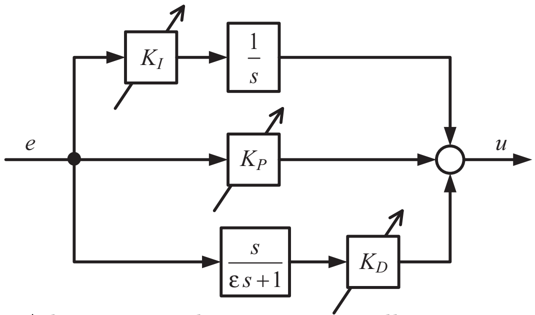

Fig. 2. Adaptive switching PID controller

where $\lambda_i(\cdot)$ denotes the i-th eigenvalue.

The $K_{slow}(s)$ and $K_{fast}(s)$ of the slow-fast decomposition may be computed by various means, e.g., the MATLAB `slowfast` algorithm, which is based on the stable-antistable decomposition algorithm described in Safonov et al. [1987].

The slow-fast decomposition of the i-th controller K_i in the set $\boldsymbol{K}$ is denoted with the subscript i as (6).

$$K_{islow}(s) \overset{s}{=} \left[\begin{array}{c|c} A_{is} & B_{is} \\ \hline C_{is} & D_{is} \end{array}\right] \text{ and } K_{ifast}(s) \overset{s}{=} \left[\begin{array}{c|c} A_{if} & B_{if} \\ \hline C_{if} & D_{if} \end{array}\right] \tag{7}$$

Further details on how the controller modes are divided as slow or fast will be described in a later section.

3. PROBLEM FORMULATION

In this paper, we define bumpless transfer as follows.

Definition 1. (Bumpless Transfer) A switching controller with slow-fast decomposition (5) is said to perform a *bumpless transfer* if, whenever controller is switched, the new controller state is reset so as to satisfy both of the following two conditions:

(a) The control input signal $u(t)$ is continuous at t_s whenever $r(t) \in C^0$, and
(b) the state of fast part of controller $K_{fast}(s)$ is reset to zero at t_s. ◇

Condition (a) in Definition 1 is the same definition used in Arehart and Wolovich [1996]. Condition (b) in Definition 1 consists of the state reset. This additional requirement for our bumpless transfer is needed to ensure that there are no rapid transients immediately following controller switching. How the state resets be performed to simultaneously satisfy both conditions will be described in the following section.

4. BUMPLESS TRANSFER IMPLEMENTATION

As mentioned before, our idea of using slow-fast decomposition of the controller as the basis for bumpless transfer generalizes a related idea introduced by Jun and Safonov [1999] for adaptive PID controller switching. To clarify this and to put our result in perspective, we begin by presenting a brief explanation of our slow-fast decomposition interpretation of the adaptive PID controller controller switching approach. Then, we will introduce our main result.

4.1 Bumpless transfer for a PID controller

A PID controller has three gains; K_P, K_I, and K_D. In PID adaptive switching control, each of these gains may be changed by switching three PID control gains as shown in Fig. 2, with the values of each of the three gains taking values in a discrete set.

A practical PID feedback controller implementation takes the form

$$u = K(s)e \tag{8}$$

where

$$K(s) = K_{slow}(s) + K_{fast}(s)$$

and

$$K_{slow}(s) = K_P + \frac{K_I}{s}$$
$$K_{fast}(s) = \frac{K_D s}{\epsilon s + 1}$$

and $\epsilon > 0$ is a small constant (with $\epsilon = 0$ for an ideal PID controller).

Location of switching gains Locating switching PID gains (K_P, K_I, K_D) plays an important role in determining the continuity of the controller output. As shown in Fig. 2, an integrator gain K_I may be located before an integrator in order that the controller output does not have a discontinuity when the integrator gain K_I is switched. However, a derivative gain K_D should be located after (i.e., at the output of) a differentiator because the change of K_D will respond an extreme overshoot or undershoot if a switching gain K_D occurs before (i.e., at the input of) the differentiator.

Controller state reset Locating K_I before an integrator as in Fig. 2 is sufficient to ensure that the output of the integrator remains both continuous and smooth when K_I switches, the problem of ensuring that the control signal response remains both continuous and smooth when switching K_D or K_P is more complicated. To deal with the later, states of the controller in Jun and Safonov [1999] use a PID controller realization that places the gains K_D and K_P to be switch directly at the input the plant, then reset the state in the integrator at the switching time to a value precisely calculated so as to ensure control signal continuity and thus to achieve the desired bumpless controller switch. The integrator has only one pole at $s = 0$, which is an infinitely slow mode, whereas the differentiator term $K_D s = \lim_{\epsilon \to 0} K_D s/(\epsilon s + 1)$ has an infinitely fast mode associated with a pole at $s = 1/\epsilon$ where $\epsilon \to 0$. Smoothness and continuity of the control signal at switching times is ensured by resetting only the state associated with the slow integrator mode of the PID controller and leaving the state of the infinitely fast differentiator mode alone. As we shall show, this approach to bumpless controller switching can be generalized to other types of controllers by restricting switching-time state resets to the states associated with the slow modes of the controllers.

Comment: In the PID controller implementation of Jun and Safonov [1999], a command signal $r(t)$ was also included in the control loop and to prevent step switches

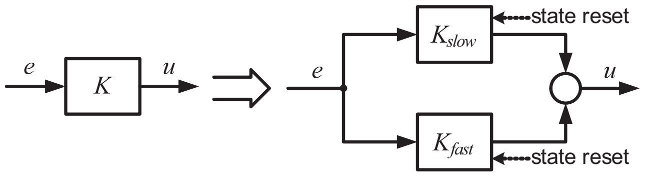

Fig. 3. Slow-fast decomposition and state reset of a controller

in external command signals from producing ‘bumps’ or discontinuities in the control signal the derivative term $(\frac{K_D s}{\epsilon s+1})$ of the switched PID controller was positioned in the feedback path ahead of the point where the command signal r enters so that step commands $r = 1/s$ could not produce a ‘bump’ by directly exciting the fast mode of the derivative term. Thus, the issue bumps excited by external step or other similarly ‘bumpy’ command signals, if present, can be always addressed this way, i.e., by putting the fast part of the controller $K_{fast}(s)$ in the feedback path so that command signal bumps cannot directly excite fast modes of the controller. In the present paper, we shall not explicitly consider the issue of command signal induced bumps, but simply note here that they can be always be handled by appropriately positioning the command signal input so that it does not directly excite the input to the fast part of the candidate controllers $K_{ifast}(s)$.

4.2 Bumpless transfer with slow-fast decomposition

Our bumpless transfer method which will be stated in this section requires the following assumption hold for each of the candidate controllers.

Assumption 1. For each candidate controller K_i, the slow part K_{islow} in (7) has at least $m = \dim(u)$ states. $\diamond$

The Assumption 1 is sufficient to allow the state of the slow controller $K_{islow}(s)$ to be reset at switching times to ensure both continuity and smoothness of the control signal $u(t)$, as we shall now explain.

In general, even if all the controllers have the same order and all share a common state vector, when the controller switching occurs, any or all of the slow and fast controller state-space matrices will be switched, which can lead to bumpy transients or discontinuity in the control signal $u(t)$ at switching times. However, if only A_{is} or B_{is} are switched and there is common state vector before and after the switch, then the control signal will be continuous and furthermore no ‘bumpy’ fast modes of the controller will be excited. Fast transient ‘bumps’ or discontinuities, when they occur, may arise from switching the D_{is} matrix of the slow controller or from switching any of the state-space matrices $(A_{if}, B_{if}, C_{if}, D_{if})$ of the fast controller. In the case of switches in the matrices A_{if}, B_{if} switches do actually not result in discontinuous jumps in $u(t)$, but nevertheless can result ‘bumpy’ fast transients in the control signal which, if very fast, may appear to be nearly discontinuous.

Our goal of bumpless transfer is to avoid both discontinuity and fast transients induced by switching. We would like our methods to work even when the order of the controller changes at switching times, and to allow for the possibility that the true plant may be imprecisely known, we would like our switching algorithm not to depend on precise knowledge of the true plant. In our method, we can do this by initializing the state of the slow part of the new controller $K_{jslow}(s)$ after each switch to a value computed to assure continuity, and setting the state of the fast part $K_{jfast}(s)$ to zero.

Theorem 1. (Main Result). Suppose that each of the candidate controllers have slow-fast decomposition (7) satisfying Assumption 1 and suppose that at time t_s the online controller is switched from controller K_i to controller K_j. At t_s, let the states of the slow and fast controllers be reset as follows

$$x_{fast}(t_s^+) = 0 \tag{9}$$
$$x_{slow}(t_s^+) = C_{js}^{\dagger}[u(t_s^-) - (D_{js} + D_{jf})e(t_s^-)] + \zeta \tag{10}$$

where $C_{js}^{\dagger}$ is the pseudoinverse matrix of C_{js} and ζ is any element of the null space of C_{js};

$$C_{js}\zeta = 0 \ . \tag{11}$$

Then, bumpless transfer is achieved at the switching time t_s. ♢

Proof: The control signal immediately after switching (time t_s^+) can be written, based on state space representation model (7) of the new controller $K_j(s)$, as

$$u(t_s^+) = C_{js}x_{slow}(t_s^+) + C_{jf}x_{fast}(t_s^+) + (D_{js} + D_{jf})e(t_s^+) \ .$$

By (9) – (10),

$$u(t_s^+) = C_{js}[C_{js}^{\dagger}\{u(t_s^-) - (D_{js} + D_{jf})e(t_s^-)\} + \zeta] + (D_{js} + D_{jf})e(t_s^+)$$

By Assumption 1, $C_{js}C_{js}^{\dagger} = I_{m\times m}$ where m is larger than or same to the number of states of K_j. This results in

$$u(t_s^+) = u(t_s^-) - (D_{js} + D_{jf})e(t_s^-) + (D_{js} + D_{jf})e(t_s^+) \ .$$

Since $e(t_s^-) = e(t_s^+)$, we finally have

$$u(t_s^+) = u(t_s^-) \ .$$

The result follows immediately from the Definition 1. *Q.E.D.*

Comment: Since C_{js} is a full rank matrix which consists of m linearly independent vectors, $C_{js}C_{js}^T$ is invertible and

$$C_{js}^{\dagger} = C_{js}^T(C_{js}C_{js}^T)^{-1} \ .$$

For details, see Ben-Israel and Greville [2003]. ♢

Equations (9) and (10) now define our *slow-fast bumpless transfer algorithm*. An example using this algorithm will be presented in the following section.

5. SIMULATION RESULTS

One of previously suggested slow-fast decomposition can be found in Balas et al. [2005]. Or, controllers can be decomposed by inspection if they has poles and zeros which clearly represent fast or slow modes. An example of this type of controller is a PID controller. Thus, the simulation shows how the method suggested above works with PID controllers.

5.1 Adaptive PID controller

A plant in this example is

$$G(s) = \frac{s^2 + s + 10}{s^3 + s^2 + 98s - 100}.$$

Two controllers having the structure in (4) were used to show the results. Each controllers have three gains to be switched, and the gains are as follows;

Controller 1; $K_{P1} = 80, \ K_{I1} = 50, \ K_{D1} = 0.5$
Controller 2; $K_{P2} = 5, \ \ K_{I2} = 2, \ \ K_{D2} = 0.6$

A small number ϵ is 0.01 and the reference input is $r = 1$. The ϵ prevents the differentiator not to make a infinite peak when a discontinuity comes into the controller. A PID controller is naturally decomposed into a slow and a fast part. Since a proportional gain is memoryless component, it can be added to either part. Therefore, the controllers were decomposed into

$$K_{slow}(s) \overset{s}{=} \left[\begin{array}{c|c} 0 & K_I \\ \hline 1 & K_P \end{array}\right] . \tag{12}$$

And, in the same way, K_{fast} can be written by

$$K_{fast}(s) \overset{s}{=} \left[\begin{array}{c|c} -1/\epsilon & 1/\epsilon \\ \hline -K_D/\epsilon & K_D/\epsilon \end{array}\right] . \tag{13}$$

Controller 1 and Controller 2 in this particular case were, respectively,

$$K_1(s) = K_{1slow} + K_{1fast} = 80 + \frac{50}{s} + \frac{0.5s}{0.01s + 1}$$

$$K_2(s) = K_{2slow} + K_{2fast} = 5 + \frac{2}{s} + \frac{0.6s}{0.01s + 1}$$

$K_1(s)$ was designed to stabilize the plant, while $K_2(s)$ cannot stabilize the plant. In this experiment, K_2 is the on-line controller at first. Thus, the plant was not stabilized at early stage. After 2 seconds, the on-line controller was switched into K_1.

The experiments were done twice for a comparison. One included the bumpless transfer method, but the other did not. The upper part of Fig. 4 shows the controller output of both cases. The output $u(t)$ with bumpless transfer had a smooth transient even at the switching instant. On the other hand, $u(t)$ without bumpless transfer had a large spike after $t = 2$.

Fig. 5 shows the controller output around the switching instant in detail. The solid line is continuous and changes moderately because of using bumpless transfer method. The dotted line is not continuous and has spikes. After the large overshoot, it changes abruptly.

The difference of controller output resulted in the different plant output as shown in the lower part of Fig. 4. An

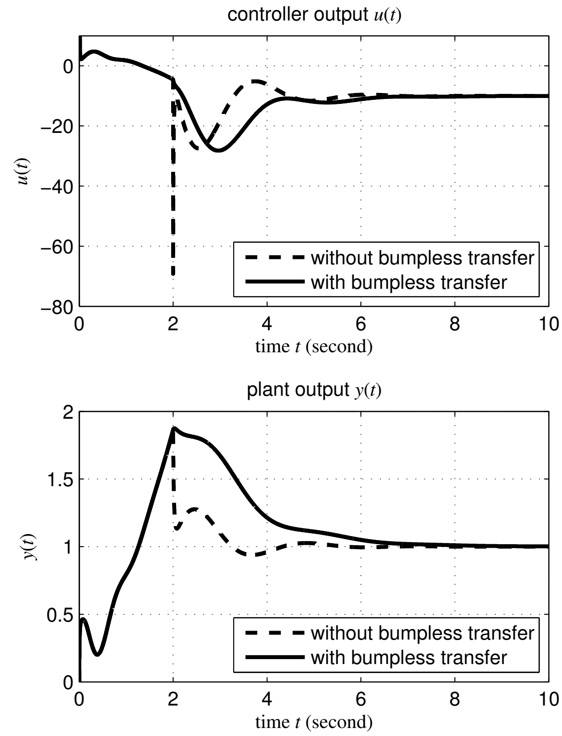

Fig. 4. Controller output $u(t)$ without bumpless transfer and with bumpless transfer (upper figure); Plant output $y(t)$ without bumpless transfer and with bumpless transfer (lower figure). Controller is switched at $t = 2$.

abrupt transient in the system without bumpless transfer caused a undesired jump in the plant output $y(t)$. This is too fast response to occurs in practice. Since generating this kind of response in the plant output needs infinitely large energy, this is not desirable and even not feasible in real world. Evidently, both the control signal and the plant output in the case are significantly smoother with bumpless transfer.

5.2 Bumpless transfer using slow-fast decomposition

The second example involves slow-fast decomposition of two controllers, each of which has two poles and two zeros. The plant is

$$G(s) = \frac{1}{s^3 + 15s^2 + 50s}\ .$$

Controller 1 and Controller 2 are, respectively,

$$K_1(s) = \frac{(s+4)(s-0.1)}{(s+7.28)(s+10)}$$

$$K_2(s) = \frac{(s+4)(s-0.1)}{(s+7.28)(s+0.01)}\ .$$

Now, slow-fast decomposition is applied to both of controllers.

$$K_1(s) = K_{1slow}(s) + K_{1fast}(s)$$

where

$$K_{1slow} \overset{s}{=} \left[\begin{array}{c|c} -7.28 & -51.7 \\ \hline -32.02 & 95.6 \end{array}\right];\ K_{1fast} \overset{s}{=} \left[\begin{array}{c|c} -10 & 39.19 \\ \hline -106.5 & 95.6 \end{array}\right] .(14)$$

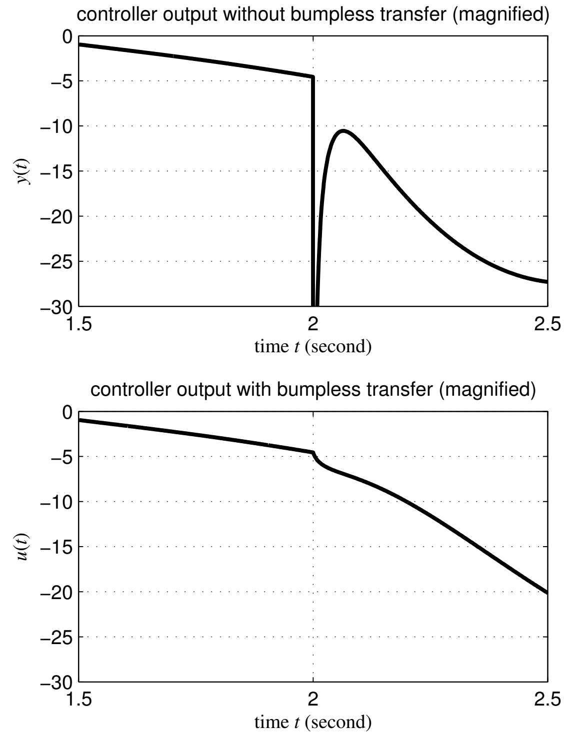

Fig. 5. Magnified $u(t)$ around the switching instant ($t = 2$).

In the same way,

$$K_{2slow} \overset{s}{=} \left[\begin{array}{c|c} -0.01 & 2.634 \\ \hline 3.585 & 95.6 \end{array}\right];\ K_{2fast} \overset{s}{=} \left[\begin{array}{c|c} -7.28 & 28.98 \\ \hline -21.37 & 95.6 \end{array}\right] \quad (15)$$

A function named `slowfast` in Balas et al. [2005] was used to have the results in (14) and (15). After the slow-fast decomposition, the transform in Arehart and Wolovich [1996] is used with $F = 1$ for the continuity of K_{slow}. Then, we have

$$\bar{K}_{1slow} \overset{s}{=} \left[\begin{array}{c|c} -7.28 & 1655.45 \\ \hline 1 & 95.6 \end{array}\right];\ \bar{K}_{2slow} \overset{s}{=} \left[\begin{array}{c|c} -0.01 & 9.44 \\ \hline 1 & 95.6 \end{array}\right] .$$

In this example, on-line controller is switched from K_1 to K_2 at $t = 5$. The results were shown in Fig. 6. Similarly to the earlier example, the switching controller with bumpless transfer method shows better transient in controller output and plant output.

6. CONCLUSION

After brief review of previously existing bumpless transfer methods, a new definition of bumpless transfer which adds in addition to the usual continuity requirement an additional requirement that there be no transients induced by controller switching. A simple new bumpless transfer method based on slow-fast decomposition and state reset has been introduced. Simulation results demonstrate the effectiveness of our bumpless transfer method. Because the method makes use only of knowledge of the switched controllers' state-space matrices and the value of the control signal just prior to switched, the method is particularly well-suited to adaptive switching control applications where the true plant model is imprecisely known or yet to be identified.

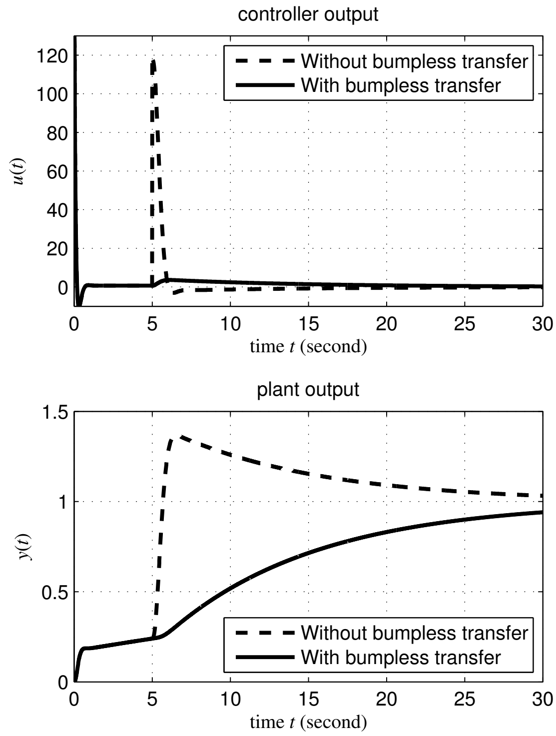

Fig. 6. Controller output $u(t)$ without bumpless transfer and with bumpless transfer (upper figure); Plant output $y(t)$ without bumpless transfer and with bumpless transfer (lower figure). Controller is switched at $t = 5$.

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