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Statistic Tracking Control for Non-Gaussian Systems Using T-S Fuzzy Model

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Abstract: This paper studies a new type of control framework for dynamical stochastic systems, called statistic tracking control. Non-Gaussian systems are considered and the tracked objective is the statistical information of a given target probability density function (PDF). Following neural network approximation to the performance function, the concerned problem is transferred into the tracking of given weights. Different from the previous related works, the time delay T-S fuzzy models with the exogenous disturbances are applied to represent the nonlinear weighting dynamics. Meanwhile, the generalized PI controller structure and the improved convex LMI algorithms are proposed to fulfil the tracking problem. Furthermore, in order to enhance the robust performance, the peak-to-peak measure is applied to optimize the tracking performance.

1. INTRODUCTION

Non-Gaussian variables exist in many complex stochastic systems due to nonlinearity which may posses asymmetric and multiple-peak stochastic distributions (Wang, 2000). For non-Gaussian systems, mean and variance are insufficient to characterize the stochastic properties. On the other hand, motivated by several typical examples in practical systems, a group of control strategies that control the shape of output PDF for general stochastic systems have been developed (Wang, 2000; Yue&Wang 2003; Shalom&Li, 2000; Forbes, 2004; Guo&Wang, 2005). This novel control framework has been called as stochastic distribution control. In this paper, we present a new type of stochastic tracking control framework for non-Gaussian systems called statistic tracking control (STC). Different from both the conventional stochastic tracking (Astrom, 1970) and the output PDF tracking problems (Wang, 2000; Guo&Wang, 2005), the goal of control here is to ensure that the statistical information of the system output is made to follow that of a target PDF. In comparison with previous works, the main results obtained in this paper have two features. Firstly, since the mean and variance are two commonly used control objectives for Gaussian systems, our control objective is a reasonable generalization for non-Gaussian systems. Secondly, the obtained statistic tracking will eliminate the constraints widely seen in the B-spline approximation for the PDFs (Wang, 2000).

In recent years, fuzzy technique has been widely and successfully used in nonlinear system modeling and control. The well known T-S fuzzy model (Takagi&Sugeno, 1985; Tanaka, Ikeda & Wang, 1996) was recognized as a popular and powerful tool for approximating a complex nonlinear system. Recently, T-S fuzzy model has been applied to complex nonlinear models, for example, descriptor system (Taniguchi, Tanaka & Wang, 2000), time-delay nonlinear model (Cao &Frank, 2000; Lee, Jeung & Park; 2001) and stochastic system (Wang, Daniel & Liu, 2004). In the literature, various techniques have been developed for stability analysis of T-S fuzzy systems (Tanaka, Ikeda & Wang, 1996; Wang, Daniel& Liu, 2004; E.Kim, H. Lee, 2000). On the other hand, the T-S fuzzy model has also been studied for the nonlinear tracking control problem (Ying, 1999; Tseng, Chen & Uang, 2001; Zheng, Wang & Lee, 2002). In (Ying,1999), feedback linearization technique was proposed to design fuzzy tracking controller. Variable structure control approach has been applied to solve the tracking problems for T-S fuzzy systems (Zheng, Wang & Lee, 2002). A simple observer-based fuzzy controller was developed in (Tseng, Chen & Uang, 2001) to reduce the tracking errors in terms of LMI approach.

Similar to the PDF tracking control problem, after using NN approximation theory for the performance function, it is shown that the STC design problem can be transformed into a tracking problem for the weighting dynamics. Different from the previous works (Wang, 2000; Guo&Wang, 2005), the T-S fuzzy models are firstly utilized to describe the nonlinear weighting dynamics that can not be modeled exactly, which represents a significant extension to the previous results. In this paper, a robust tracking problem is studied for the T-S fuzzy weighting model where there exist non-zero equilibriums, time delay and exogenous disturbances. Meanwhile, a generalized PI controller can be obtained through improved LMI algorithms such that the stability, tracking performance for the target weighting vector or the dynamic reference model and robustness are guaranteed simultaneously. Furthermore, in order to enhance the robust performance, the peak-to-peak measure is applied to optimize the tracking performance which generalizes the corresponding result for linear systems with zero equilibrium (Scherer & Weiland, 2000).

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2. FORMULATION OF STC PROBLEM

For a dynamical stochastic system, denote $u(t) \in R^m$ as the control input, $\eta(t) \in [a,b]$ as the stochastic output, whose conditional PDF is denoted by $\gamma(y,u(t))$, where y is the variable in the sample interval $[\alpha,\beta]$. It is noted that $\gamma(y,u(t))$ is a dynamic functional of y along with the time variable t. In the previous works, the PDF tracking problem has been studied with some effective design algorithms (Wang, 2000; Shalom&Li, 2000; Forbes, 2004; Guo&Wang, 2005). B-spline expansions have been used to approximate $\gamma(y,u(t))$ or $\sqrt{\gamma(y,u(t))}$ and the control objective is to find $u(t)$ such that $\gamma(y,u(t))$ is convergent to the target PDF $g(y)$.

The idea results from a simple observation. It is well known that mean and variance can characterize the stochastic property of a Gaussian variable. Generally, the moments from the lower order to higher order can decide the shape of a non-Gaussian PDF. In addition, entropy has also been widely used in communication and control theories as a measure of the average information contained in a given PDF (Yue&Wang, 2003). Thus, the PDF tracking can be achieved via tracking the statistical information, which motivated the so-called STC problem.

To illustrate the design algorithm, in this paper we consider a special STC problem. The considered performance index is $\int_\alpha^\beta \delta(\gamma,u(t))dy$, where

$$\delta(\gamma,u(t)) = Q_1\gamma(y,u(t))\ln(\gamma(y,u(t))) + Q_2 y\gamma(y,u(t)) \tag{1}$$

where Q_1 and Q_2 are two parameters. In (1) the integral of the first term is the entropy and that of the second term is the mean of the output PDF respectively.

Based on the previous PDF control theory, we construct B-spline expansions to approximate $\delta(\gamma,u(t))$ as follows

$$\delta(\gamma,u(t)) = C(y)V_0(t) + \varepsilon(y,t) \tag{2}$$

$$C(y) = [B_1(y)\dots B_n(y)] \quad V_0(t) = [v_1(t)\dots v_n(t)]^T \tag{3}$$

$B_i(y)$ $(i = 1,2,\cdots,n)$ are pre-specified basis functions and $v_i(u(t)) := v_i(t)$, $(i = 1,2,\cdots,n)$ are the corresponding weights. $\varepsilon(y,t)$ represents the bounded modeling error. Assuming $\varepsilon(y,t)$ can be replaced by $\varepsilon(y,t) = C(y)w_0(t)$, where $w_0(t)$ can also be regarded as an unknown perturbation. Hence, (2) can be rewritten as

$$\delta(\gamma,u(t)) = C(y)V_0(t) + C(y)w_0(t) = C(y)V(t) \tag{4}$$

Based on (1) and (4), for the target PDF we can find the corresponding weights which can be denoted as $V_g(t)$. That is, $\delta(g,u(t)) = C(y)V_g(t)$. The tracking objective is to find $u(t)$ such that $\delta(\gamma,u(t)) - \delta(g,u(t)) = C(y)e(t)$ converges to 0, where $e(t) := V(t) - V_g(t)$.

3. PI CONTROLLER DESIGN BASED ON T-S FUZZY WEIGHTING MODEL

Once B-spline models have been made for the performance function, the next step is to find the dynamic relationships between the control input and the weight vector. However, most published results only concerned linear precise models, it is noted that a linear mapping cannot change the PDF shape of the stochastic output, which confines the practical applications. Recently, the T-S fuzzy model has been proved to be a very good representation for a certain class of nonlinear dynamic systems in control systems and signal processing. So we consider the nonlinear weighting dynamics which could be described by the following T-S fuzzy model with r plant rules:

Plant Rule i: If θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip}, then

$$\begin{aligned}\dot V(t) = A_{0i}V(t) + F_{0i}V_\tau(t) + B_{01i}u(t)\\ +B_{02i}u_\tau(t) + E_{0i}w(t)\end{aligned} \tag{5}$$

where $V(t) \in R^n$ is the independent weighting vectors, $u(t)$ and $w(t)$ represent the control input and the exogenous perturbation respectively. $V_\tau(t) = V(t-\tau(t))$ represents the time delay weighting vectors, and $u_\tau(t) = u(t-\tau(t))$ is the control input with time delay term. $A_{0i}, F_{0i}, B_{01i}, B_{02i}$ and E_{0i} are known coefficient matrices with compatible dimensions. $\theta_j(x)$ and μ_{ij} $(i = 1,\cdots,r, j = 1,\cdots,p)$ are respectively the premise variables and the fuzzy sets, r is the number of If-Then rules, and p is the number of the premise variables. The time-varying delays $\tau(t)$ satisfy $0 < \dot\tau(t) < \beta < 1$.

By fuzzy blending, the overall fuzzy model is inferred as

$$\begin{aligned}\dot V(t) = \sum_{i=1}^r h_i(\theta)(A_{0i}V(t) + F_{0i}V_\tau(t) + B_{01i}u(t)\\ +B_{02i}u_\tau(t) + E_{0i}w(t))\end{aligned} \tag{6}$$

where $\theta = [\theta_1,\cdots,\theta_p]$, $\omega_i : R^p \to [0,1], i = 1,\cdots,r$ is the membership function of the system with respect to plant rule i, and $h_i(\theta) = \omega_i(\theta)/\sum_{i=1}^r \omega_i(\theta)$. It is obvious

$$h_i(\theta) \ge 0, \quad \sum_{i=1}^r h_i(\theta) = 1$$

Based on the above mentioned T-S fuzzy model (6), we introduce a new state variable $x(t) := [V^T(t), \int_0^t e^T(\tau)d\tau]^T$. Then the following augmented system with disturbance $w(t)$ and reference input $V_g(t)$ can be established as

$$\begin{cases}\dot x(t) = \sum_{i=1}^r h_i(\theta)(A_i x(t) + F_i x_\tau(t) + B_{1i}u(t)\\ \qquad +B_{2i}u_\tau(t) + E_i w(t) + HV_g)\\ z(t) = \sum_{i=1}^r h_i(\theta)(C_i x(t) + D_i w(t))\\ x(t) = \phi(t), \qquad t \in [-\tau(t), 0]\end{cases} \tag{7}$$

where $z(t)$ is the controller output, and

$$A_i = \begin{bmatrix} A_{0i} & 0\\ I & 0\end{bmatrix} \quad F_i = \begin{bmatrix} F_{0i} & 0\\ 0 & 0\end{bmatrix} \quad B_{1i} = \begin{bmatrix} B_{01i}\\ 0\end{bmatrix}$$

$$B_{2i} = \begin{bmatrix} B_{02i}\\ 0\end{bmatrix} \quad E_i = \begin{bmatrix} E_{0i}\\ 0\end{bmatrix} \quad H = \begin{bmatrix} 0\\ -I\end{bmatrix}$$

To solve the tracking problem for weight vectors, a direct application of PI control strategy would lead to

Plant Rule j: If θ_1 is μ_{j1} and $\cdots$ and θ_p is μ_{jp}, then

$$u(t) = \sum_{j=1}^r h_j(\theta)(K_{Pj}V(t) + K_{Ij}\int_0^t e(\tau)d\tau) \tag{8}$$

where K_{Pj} and K_{Ij} are controller gains to be determined.

With such an augmented system (7), tracking problem can be further reduced to a stabilization framework because the PI controller can be formulated by

$$u(t) = \sum_{j=1}^r h_j(\theta)[K_j x(t)], \quad K_j = [K_{Pj} \;\; K_{Ij}] \tag{9}$$

Remark: It should be pointed out that although some nonlinear tracking approaches have been provided in the last decade, less results can be applied to the above T-S fuzzy model. They do not consider non-zero equilibrium and exogenous disturbance simultaneously. In the following, we will perform LMIs with convex algorithms to achieve the above control objectives. On the other hand, it is noted that B-spline expansions and weight modeling procedures result in modeling errors and uncertainties which were ignored previously (Wang, 2000; Guo&Wang, 2005). In this paper, the uncertain vector $w(t)$ comprises of two parts; perturbation within T-S fuzzy weighting model (5) and modeling error in PDF approximation denoted $w_0(t)$.

4. MAIN RESULTS

4.1 Peak-to-peak performance

L_1 performance index is the measure used to describe the level of disturbance attenuation, which is also called peak-to-peak performance(Scherer & Weiland, 2000).

Definition: The peak-to-peak control gain for a nonlinear system is defined by $sup_{\|w\|_\infty \le 1} \|z(t)\|_\infty$. The peak-to-peak control problem is to find controller $u(t)$ such that the peak-to-peak gain is minimized or satisfies

$$sup_{\|w\|_\infty \le 1} \|z(t)\|_\infty < \gamma^2 \tag{10}$$

or $sup_{0 \le \|w\| \le \infty} \frac{\|z(t)\|_\infty}{\|w(t)\|_\infty} < \gamma^2$, where γ is a given constant.

Since $V_g(t)$ is a known vector, we denote $y_d := \|V_g(t)\|^2$. The following result provides a criterion for the L_1 performance problem of the unforced system of (7), which also generalizes the corresponding result for linear systems with zero equilibrium (Scherer & Weiland, 2000).

Theorem 1 : For the known parameters $\mu_i (i = 1, 2, 3)$, $\alpha > 0$ and $\gamma > 0$, suppose that there exist $S, P > 0$ and for $i = 1, \cdots, r$, $j = 1, \cdots, r$ such that the following matrix inequalities

$$\begin{bmatrix} sym(A_i^T P) + \mu_1^2 P + S & PF_i & PE_i & PH \\ F_i^T P & -(1-\beta)S & 0 & 0 \\ E_i^T P & 0 & -\mu_2^2 I & 0 \\ H^T P & 0 & 0 & -\mu_3^2 I \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} \mu_1^2 P & 0 & \frac{1}{2}(C_i^T + C_j^T) \\ 0 & (\gamma - \mu_2^2 - \mu_3^2 y_d) I & \frac{1}{2}(D_i^T + D_j^T) \\ \frac{1}{2}(C_i + C_j) & \frac{1}{2}(D_i + D_j) & \gamma I \end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix} \alpha I & P \\ P & P \end{bmatrix} > 0$$

$$\begin{bmatrix} P & 0 & \frac{1}{2}(C_i^T + C_j^T) \\ 0 & (\gamma - \alpha x_m^T x_m) I & \frac{1}{2}(D_i^T + D_j^T) \\ \frac{1}{2}(C_i + C_j) & \frac{1}{2}(D_i + D_j) & \gamma I \end{bmatrix} > 0 \tag{13}$$

are solvable, then the unforced system (7) is stable, and $sup_{0 \le \|w\| \le \infty} \frac{\|z(t)\|_\infty}{\|w(t)\|_\infty} < \gamma^2$ holds.

4.2 Peak-to-peak tracking performance

Considering the state feedback control with PI controller structure, substituting $u(t) = \sum_{j=1}^r h_j(\theta)[K_j x(t)]$ into (7), we can get

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\theta) \sum_{j=1}^r h_j(\theta)[(A_i + B_{1i}K_j)x(t) \\ \quad + (F_i + B_{2i}K_j)x_\tau(t) + E_i w(t) + HV_g(t)] \\ z(t) = \sum_{i=1}^r h_i(\theta)[C_i x(t) + D_i w(t)] \end{cases} \tag{14}$$

The following results provide a solution for the considered nonlinear tracking control problem with disturbance attenuation performance.

Theorem 2: For the known parameters $\mu_i (i = 1, 2, 3)$, $\alpha > 0$ and $\gamma > 0$, suppose that there exist $S > 0$, $Q = P^{-1}$ and for $i = 1, \cdots, r, j = 1, \cdots, r$ such that the following matrix inequalities

$$\begin{bmatrix} \Pi_{ij} & F_i Q + B_{2i} R_j & E_i & H \\ QF_i^T + R_j^T B_{2i}^T & -(1-\beta)QSQ & 0 & 0 \\ E_i^T & 0 & -\mu_2^2 I & 0 \\ H^T & 0 & 0 & -\mu_3^2 I \end{bmatrix} < 0 \tag{15}$$

where

$$\Pi_{ij} = sym(A_i Q) + sym(B_{1i} R_j) + \mu_1^2 Q + QSQ$$

$$\begin{bmatrix} \mu_1^2 Q & 0 & \frac{Q}{2}(C_i^T + C_j^T) \\ 0 & (\gamma - \mu_2^2 - \mu_3^2 y_d) I & \frac{1}{2}(D_i^T + D_j^T) \\ \frac{1}{2}(C_i + C_j)Q & \frac{1}{2}(D_i + D_j) & \gamma I \end{bmatrix} > 0 \tag{16}$$

$$\begin{bmatrix} \alpha I & I \\ I & Q \end{bmatrix} > 0$$

$$\begin{bmatrix} Q & 0 & \frac{Q}{2}(C_i^T + C_j^T) \\ 0 & (\gamma - \alpha x_m^T x_m) I & \frac{1}{2}(D_i^T + D_j^T) \\ \frac{1}{2}(C_i + C_j)Q & \frac{1}{2}(D_i + D_j) & \gamma I \end{bmatrix} > 0 \tag{17}$$

are solvable, the closed-loop system (14) is stable and satisfies both $lim_{t \to \infty} V(t) = V_g$ and $sup_{0 \le \|w\| \le \infty} \frac{\|z(t)\|_\infty}{\|w(t)\|_\infty} < \gamma^2$. The PI control gain K_j can be solved via $R_j = K_j Q$.

4.3 Peak-to-peak tracking based on reference model

In this stage, a dynamic reference model is considered here as a part of the target model whose output $V_m(t)$ converges to $V_g(t)$ as $t \to \infty$. It is supposed that the reference vectors $V_m(t)$ satisfy

$$\dot{V}_m(t) = A_m V_m(t) + B_m r \tag{18}$$

where A_m, B_m are constant matrices of appropriate dimension and r is a constant input. The considered problem is transformed into a dynamic tracking for error vector

$$e_1(t) = V(t) - V_m(t) \tag{19}$$

Based on system (5, 18), we redefine a new state variable

$$\bar{x}(t) := [V^T(t), \int_0^t e_1^T(\tau) d\tau, V_m^T(t)]^T \tag{20}$$

Then the new T-S fuzzy weighting model can be transformed as

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{i=1}^{r} h_i(\theta)(\bar{A}_i\bar{x}(t) + \bar{F}_i\bar{x}_\tau(t) + \bar{B}_{1i}u(t) \\ \qquad +\bar{B}_{2i}u_\tau(t) + \bar{E}_iw(t) + \bar{B}_mr) \\ \bar{z}(t) = \sum_{i=1}^{r} h_i(\theta)(\bar{C}_i\bar{x}(t) + \bar{D}_iw(t)) \end{cases} \tag{21}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & H \\ 0 & A_m \end{bmatrix} \bar{F}_i = \begin{bmatrix} F_i & 0 \\ 0 & 0 \end{bmatrix} \bar{B}_{1i} = \begin{bmatrix} B_{1i} \\ 0 \end{bmatrix} \bar{B}_{2i} = \begin{bmatrix} B_{2i} \\ 0 \end{bmatrix}$$

$$\bar{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix} \quad \bar{B}_m = \begin{bmatrix} 0 \\ B_m \end{bmatrix} \quad \bar{C}_i = \begin{bmatrix} C_i \\ C_{1i} \end{bmatrix} \quad \bar{D}_i = \begin{bmatrix} D_i \\ D_{1i} \end{bmatrix}$$

Based on the T-S fuzzy model (21), the new PI control strategy can be formulated as

$$u(t) = \sum_{j=1}^{r} h_j(\theta)\bar{K}_j\bar{x}(t), \quad \bar{K}_j = [K_{Pj}, K_{Ij}, 0] \tag{22}$$

Substituting (22) into (21) yields

$$\dot{\bar{x}}(t) = \sum_{i=1}^{r} h_i(\theta) \sum_{j=1}^{r} h_j(\theta)[(\bar{A}_i + \bar{B}_{1i}\bar{K}_j)\bar{x}(t) \\ +(\bar{F}_i + \bar{B}_{2i}\bar{K}_j)\bar{x}_\tau(t) + \bar{E}_iw(t) + \bar{B}_mr] \tag{23}$$

Theorem 3: For the known parameters $\mu_i(i = 1, 2, 3)$, $\alpha > 0$ and $\gamma > 0$, suppose that there exist $\bar{S} = diag[S, S_1] > 0$, $\bar{Q} = diag[Q, Q_1] = diag[P^{-1}, P_1^{-1}] > 0$ and for $i = 1, \cdots, r, j = 1, \cdots, r$ such that the matrix inequalities (24-26) are solvable, then the T-S fuzzy model (21) is stable and satisfies both $lim_{t\to\infty}V(t) = V_m(t)$ and $sup_{0\le\|w\|\le\infty}\frac{\|z(t)\|_\infty}{\|w(t)\|_\infty} < \gamma^2$. In this case, the PI control gain K_j can be solved via $R_j = K_jQ$, and $\tilde{S} = QSQ$, $\tilde{S}_1 = Q_1S_1Q_1$.

5. AN ILLUSTRATIVE EXAMPLE

Suppose that the statistical information can be approximated using the square root B-spline models described by (2) with $n = 3$, $y \in [0, 1.5]$ and for $i = 1, 2, 3$

$$B_i(y) = \begin{cases} |\sin 2\pi y| & y \in [0.5(i-1); 0.5i] \\ 0 & y \in [0.5(j-1); 0.5j] \quad i \neq j \end{cases} \tag{27}$$

For simplicity, in the simulation it is supposed that the target PDF can be denoted as

$$\gamma_g(y) = \frac{1}{\sqrt{2\pi}b}exp(-\frac{(y-a)^2}{2b^2}) \tag{28}$$

where $a = 0.5 + \ln\sqrt{2\pi} + \frac{1.7}{\pi}$, $b = 1$. From (1), it can be obtained that

$$\Sigma_{i=1}^{3}(V_{gi}B_i(y)) = Q_1\gamma_g(y)ln(\gamma_g(y)) + Q_2y\gamma_g(y)$$

where $Q_1 = 1$, $Q_2 = 1$. As a result, we have

$$\Sigma_{i=1}^{3}(V_{gi}\int_{-\infty}^{\infty}(B_i(y)dy)) = (-\frac{1}{2})(1 + ln(2\pi b^2)) + a \tag{29}$$

Owing to $\int_{-\infty}^{\infty} B_i(y)dy = \frac{1}{\pi} \quad i = 1, 2, 3$, it can be shown that the reference weights corresponding to the target statistical information of the target PDF satisfy the condition $\Sigma_{i=1}^{3}(V_{gi}) = 1.7$. As such, the reference weights can be denoted as $V_g = [0.3 \quad 0.6 \quad 0.8]^T$. Consider a T-S fuzzy model with exogenous perturbation, together with the $i = 1, 2$. The model parameters are omitted here to save space.

$$\begin{bmatrix} sym(A_iQ) + sym(B_{1i}R_i) + \tilde{S} + \mu_1^2Q & HQ_1 & F_iQ + B_{2i}K_j & 0 & E_i & 0 \\ Q_1H^T & sym(A_mQ_1) + \tilde{S}_1 + \mu_1^2Q_1 & 0 & 0 & 0 & B_m \\ QF_i^T + K_j^TB_{2i}^T & 0 & -(1-\beta)\tilde{S} & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\beta)\tilde{S}_1 & 0 & 0 \\ E_i^T & 0 & 0 & 0 & -\mu_2^2I & 0 \\ 0 & B_m^T & 0 & 0 & 0 & -\mu_3^2I \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} \mu_1^2Q & 0 & 0 & \frac{1}{2}(QC_i^T + QC_j^T) \\ 0 & \mu_1^2Q_1 & 0 & \frac{1}{2}(Q_1C_{1i}^T + Q_1C_{1j}^T) \\ 0 & 0 & (\gamma - \mu_2^2 - \mu_3^2r)I & \frac{1}{2}(\bar{D}_i^T + \bar{D}_j^T) \\ \frac{1}{2}(C_iQ + C_jQ) & \frac{1}{2}(C_{1i}Q + C_{1j}Q) & \frac{1}{2}(\bar{D}_i + \bar{D}_j) & \gamma I \end{bmatrix} > 0 \tag{25}$$

$$\begin{bmatrix} \alpha I & I & I \\ I & Q & 0 \\ I & 0 & Q_1 \end{bmatrix} > 0, \quad \begin{bmatrix} Q & 0 & 0 & \frac{1}{2}(QC_i^T + QC_j^T) \\ 0 & Q_1 & 0 & \frac{1}{2}(Q_1C_{1i}^T + Q_1C_{1j}^T) \\ 0 & 0 & (\gamma - \alpha\bar{x}_m^T\bar{x}_m)I & \frac{1}{2}(\bar{D}_i^T + \bar{D}_j^T) \\ \frac{1}{2}(C_iQ + C_jQ) & \frac{1}{2}(C_{1i}Q + C_{1j}Q) & \frac{1}{2}(\bar{D}_i + \bar{D}_j) & \gamma I \end{bmatrix} > 0 \tag{26}$$

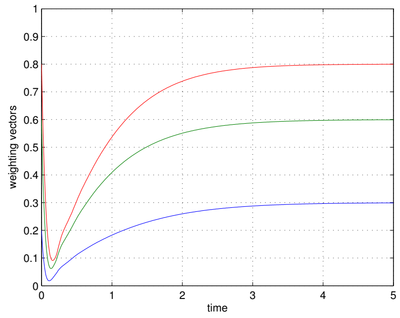

Fig. 1 Responses of the weighting vectors

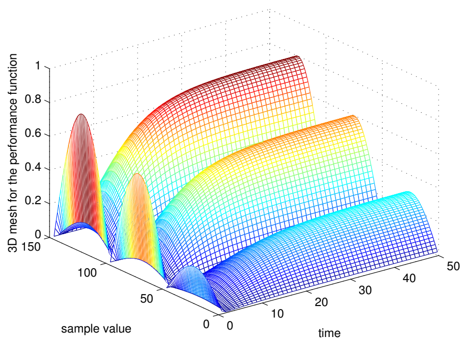

Fig. 2 3D mesh plot of the performance function

In this example, we choose Gaussian functions as our member functions, which are

$$M_i = \frac{exp(\frac{-(x_2 \pm 1)^2}{\sigma^2})}{exp(\frac{-(x_2+1)^2}{\sigma^2}) + exp(\frac{-(x_2-1)^2}{\sigma^2})}$$

Through solving the LMIs (19-23), the PI control gains can be computed as follows

$$K_{P1} = \begin{bmatrix} 36.5 & -4.2 & 3.4 \\ -2.2 & 42.7 & -2.3 \\ -1.6 & -5.7 & 35.5 \end{bmatrix}, K_{I1} = \begin{bmatrix} 47.9 & -5.2 & 4.5 \\ 2.5 & 52.0 & 1.1 \\ -4.2 & -5.3 & 45.8 \end{bmatrix}$$

$$K_{P2} = \begin{bmatrix} 17.1 & -1.9 & -0.2 \\ 1.2 & 16.9 & -1.5 \\ 2.5 & -0.5 & 18.6 \end{bmatrix}, K_{I2} = \begin{bmatrix} 23.0 & -3.5 & -2.0 \\ 3.9 & 22.1 & -1.1 \\ 3.3 & 0.9 & 22.5 \end{bmatrix}$$

Fig 1 is the responses of weighting vector and shows the performances of tracking. Fig 2 shows the 3-D mesh plot of the statistic performance function.

6. CONCLUSION

This paper considers the robust tracking problem for the statistic information of non-Gaussian processes by using generalized PI controller. B-spline neural networks and T-S fuzzy model are applied to formulate the tracking problem. Compared with the previous works, the main results here have three features: 1) The T-S fuzzy model, as a system identifier, is first applied into STC problem. 2) Exogenous disturbances, non-zero equilibrium and time delay are all considered in the T-S fuzzy model tracking control. 3) Using the LMI methods, multiple control objectives including stabilization, tracking performances and robustness can be guaranteed simultaneously.

REFERENCES

K. J. Astrom. *Introduction to Stochastic Control Theory*, Academic, New York, 1970.

C. Scherer, S. Weiland. *Lecture Notes DISC Course on Linear Matrix Inequalities in Control*. Dutch Institute of Systems and Control, Delft, The Netherlands, 2000.

H. Wang. *Bounded Dynamic Stochastic Systems: Modelling and Control*. Springer-Verlag, London, 2000.

H. Yue and H. Wang. Minimum entropy control of closed loop tracking errors for dynamic stochastic systems. *IEEE Trans. on Automatic Control*, 48,118-122, 2003.

Y.B. Shalom, X.R. Li. Nonlinear filter design using Fokker-Planck-Kolmogorov probability density evolutions. *IEEE Trans. Aerosp. Electron. Syst.*, 36(1): 309-315, 2000.

M.G. Forbes, J.F. Forbes and M. Guay. Control design for first-order processes: shaping the probability density of the process state. *Journal of Process Control*, 14: 399-410, 2004.

L. Guo, H. Wang. PID controller design for output PDFs of stochastic systems using linear matrix inequalities. *IEEE Trans. Systems, Man and Cybernetics- B*, 35(1): 65-71, 2005.

L. Guo, H. Wang. Generalized discrete-time PI control of output PDFs using square root b-spline expansion. *Automatica*, 41: 159-162, 2005.

T. Takagi, M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Systems, Man and Cybernatics*, 15(1): 116-132, 1985.

T. Taniguchi, K. Tanaka and H. O. Wang. Fuzzy descriptor systems and nonlinear model following control. *IEEE Trans. Fuzzy Systems*, 8: 442-452, 2000.

Y.Y. Cao, P.M. Frank. Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach. *IEEE Trans. Fuzzy Systems*, 8(2): 200-211, 2000.

E. Kim, H. Lee. New approaches to relaxed quadratic stability condition of fuzzy control systems. *IEEE Trans. Fuzzy Systems*, 8: 523-534, 2000.

F. Zheng, Q.G. Wang and T.H. Lee. Output tracking control of MIMO fuzzy nonlinear systems using variable structure control approach. *IEEE Trans. Fuzzy Systems*, 10: 686-697, 2002.

H. Ying. Analytical analysis and feedback linearization tracking control of the general Takagi-Sugeno fuzzy dynamic systems. *IEEE Trans. on Systems, Man and Cybernatics–C*, 29: 290-298, 1999.

C.S. Tseng, B.S. Chen and H.J. Uang. Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model. *IEEE Trans. Fuzzy Systems*, 9: 381-392, 2001.

K.R. Lee, E.T. Jeung and H.B. Park. Robust fuzzy H_∞ control for uncertain nonlinear systems via state feedback: An LMI approach. *Fuzzy Sets and Systems*, 120: 123-134, 2001.

Z. D. Wang, W. C. H. Daniel and X. H. Liu. A note on the Robust Stability of Uncertain Stochastic Fuzzy Systems With Time-Delays. *IEEE Trans. Systems, Man and Cybernatics–A*, 34(4): 570-576, 2004.

Appendix A. PROOF FOR THE THEOREM 1

Proof: Defining a Lyapunov function condition as

$$S_1(x(t),t) = x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(\beta)Sx(\beta)d\beta \quad (30)$$

Obviously it can be seen that

$$\begin{aligned}\frac{dS_1}{dt} &= 2x^T(t)P\dot{x}(t)+x^T(t)Sx(t)-(1-\dot{\tau}(t))x_\tau^T(t)Sx_\tau(t)\\ &= \sum_{i=1}^{r} h_i(\theta)x^T(t)\Upsilon_i x(t) - (1-\dot{\tau}(t))x_\tau^T(t)Sx_\tau(t)\\ &+2\sum_{i=1}^{r} h_i(\theta)x^T(t)PF_i x_\tau(t) + \|\mu_2 w(t)\|^2\\ &+\|\mu_3 V_g(t)\|^2 - \sum_{i=1}^{r} h_i \parallel \frac{1}{\mu_2}E_i^T Px(t) - \mu_2 w(t) \parallel^2\\ &- \parallel \frac{1}{\mu_3}H^T Px(t) - \mu_3 V_g(t) \parallel^2\\ &\le \sum_{i=1}^{r} h_i(\theta)\zeta^T(t)\Phi_i\zeta(t) + \|\mu_2 w(t)\|^2 + \mu_3^2 y_d \quad (31)\end{aligned}$$

where

$$\zeta = [x^T(t), \quad x_\tau^T(t)]^T, \qquad \Phi_i = \begin{bmatrix} \Upsilon_i & PF_i \\ F_i^T P & -(1-\beta)S \end{bmatrix}$$

$$\Upsilon_i = A_i^T P + PA_i + S + \frac{1}{\mu_2^2}PE_iE_i^TP + \frac{1}{\mu_3^2}PHH^TP \quad (32)$$

Based on Schur complement formula, (11) implies that for any $w(t)$ satisfying $\|w(t)\|_\infty \le 1$, $\Phi_i < -\mu_1^2 P$ hold. With (31), it can be seen that

$$\frac{dS_1(x(t),t)}{dt} \le -\mu_1^2 x^T(t)Px(t) + \mu_2^2 + \mu_3^2 y_d \quad (33)$$

where $\mu_3^2 y_d$ can be considered as a known parameter. Thus, $\frac{dS_1(x(t),t)}{dt} < 0$, if $x^T(t)Px(t) > \mu_1^{-2}(\mu_2^2 + \mu_3^2 y_d)$ holds. So for any $x(t)$, it can be verified that

$$\begin{aligned} x^T(t)Px(t) &\le max\{x_m^T Px_m, \mu_1^{-2}(\mu_2^2 + \mu_3^2 y_d)\}\\ \|x_m\| &= \sup_{-\tau(t)\le t\le 0} \|x(t)\| \end{aligned} \quad (34)$$

which also implies that the unforced system (7) is stable.

From(34), we get $x^T(t)Px(t) \le x_m^T Px_m$ or $x^T(t)Px(t) \le \mu_1^{-2}(\mu_2^2 + \mu_3^2 y_d)$. Defining $\eta(t) = [x^T(t), w^T(t)]^T$, $H_i = [C_i, D_i]$ and $H_j = [C_j, D_j]$, we have

$$\begin{aligned}\|z(t)\|^2 &= \Sigma_{i=1}^r\Sigma_{j=1}^r h_i(\theta)h_j(\theta)\eta^T(t)H_i^T H_j\eta(t)\\ &\le \frac{1}{4}\Sigma_{i=1}^r\Sigma_{j=1}^r h_ih_j\eta^T(t)(H_i+H_j)^T(H_i+H_j)\eta(t)\end{aligned}$$

Combining with (12), it can be seen that

$$\begin{bmatrix} \mu_1^2 P & 0 \\ 0 & (\gamma - \mu_2^2 - \mu_3^2 y_d)I \end{bmatrix} - \frac{1}{4\gamma}\begin{bmatrix} C_i^T + C_j^T \\ D_i^T + D_j^T \end{bmatrix}[\, C_i + C_j \;\; D_i + D_j \,] > 0$$

which guarantees under $x^T(t)Px(t) \le \mu_1^{-2}(\mu_2^2+\mu_3^2 y_d)$ and $\|w(t)\|_\infty \le 1$ that

$$\frac{1}{\gamma}\|z(t)\|^2 < (\mu_2^2 + \mu_3^2 y_d) + (\gamma - \mu_2^2 - \mu_3^2 y_d)w^T(t)w(t) = \gamma \quad (35)$$

On the other hand, from (13), it can also be shown that

$$\begin{bmatrix} P & 0 \\ 0 & (\gamma - \alpha x_m^T x_m)I \end{bmatrix} - \frac{1}{4\gamma}\begin{bmatrix} C_i^T + C_j^T \\ D_i^T + D_j^T \end{bmatrix}[\, C_i + C_j \;\; D_i + D_j \,] > 0$$

Similarly to the above proof, under $x^T(t)Px(t) \le x_m^T Px_m$ and $\|w(t)\|_\infty \le 1$, we can get

$$\frac{1}{\gamma}\|z(t)\|^2 < \alpha x_m^T x_m + (\gamma - \alpha x_m^T x_m)w^T(t)w(t) = \gamma \quad (36)$$

Hence, the L_1 norm of the unforced system is less than γ^2.

Appendix B. PROOF FOR THE THEOREM 2

Proof: Based on Theorem 1 and Lyapunov-Krasovskii function condition (30), we can get

$$\begin{aligned}\frac{dS_1}{dt} &= 2x^T(t)P\dot{x}(t) + x^T(t)Sx(t) - (1-\dot{\tau})x_\tau^T(t)Sx_\tau(t)\\ &\le \sum_{i=1}^{r} h_i \sum_{j=1}^{r} h_j\zeta^T(t)\Omega_{ij}\zeta(t) + \|\mu_2 w(t)\|^2 + \mu_3^2 y_d \quad (37)\end{aligned}$$

where $\zeta = [x^T(t), \quad x_\tau^T(t)]^T$ and

$$\Omega_{ij} = \begin{bmatrix} \Xi_{ij} & PF_i + PB_{2i}K_j \\ F_i^TP + K_j^TB_{2i}^TP & -(1-\beta)S \end{bmatrix}$$

$$\begin{aligned}\Xi_{ij} &= sym(A_i^TP) + sym(K_j^TB_{1i}^TP) + S\\ &+\frac{1}{\mu_2^2}PE_iE_i^TP + \frac{1}{\mu_3^2}PHH^TP \end{aligned} \quad (38)$$

Based on Schur complement formula, we can get

$$\Omega_{ij} \le \begin{bmatrix} -\mu_1^2 P & 0 \\ 0 & 0 \end{bmatrix}$$

So for any $w(t)$ satisfying $\|w(t)\|_\infty \le 1$, it can be seen that

$$\frac{dS_1(x(t),t)}{dt} \le -\mu_1^2 x^T(t)Px(t) + \mu_2^2 + \mu_3^2 y_d \quad (39)$$

Similarly to the proof of Theorem 1, it can be seen that (34) still holds for the closed-loop system, which implies that (14) is still stable in the presence of $w(t)$ and V_g. Similar to Theorem 1, it can be claimed that the closed-loop system satisfies the peak-to-peak disturbance attenuation performance from LMIs (16-17).

For a couple of $w(t)$ and V_g, we suppose that $\vartheta_1(t)$ and $\vartheta_2(t)$ are two trajectories of the closed-loop system corresponding to a fixed initial condition. Define $\sigma(t) := \vartheta_1(t) - \vartheta_2(t)$, the dynamics for $\sigma(t)$ can be described as

$$\dot{\sigma}(t) = \sum_{i,j=1}^{r} h_ih_j[(A_i + B_{1i}K_j)\sigma(t) + (F_i + B_{2i}K_j)\sigma_\tau(t)] \quad (40)$$

Similar to (30), Lyapunov function can be constructed as

$$S_2(\sigma(t),t) = \sigma^T(t)P\sigma(t) + \int_{t-\tau(t)}^{t} \sigma_\tau^T(\beta)S\sigma_\tau(\beta)d\beta \quad (41)$$

Hence it can be seen

$$\frac{dS_2(\sigma(t),t)}{dt} \le -\mu_1^2\sigma^T(t)P\sigma(t) \le -\mu_1^2\lambda_{min}(P)\|\sigma(t)\|^2$$

where $\lambda_{min}(P)$ is denoted as the minimal eigenvalue about P. It can be verified that $\sigma = 0$ is the unique asymptotically stable equilibrium point of the system (40). It means that the closed-loop system (14) also has a unique asymptotically stable equilibrium point. It can be concluded that $lim_{t\to\infty}\frac{d}{dt}(\int_0^t e(\tau)d\tau) = 0$, which shows that $lim_{t\to\infty}V(t) = V_g(t)$.