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Passivity Based Control of Power Plants ⋆

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Abstract: In this paper,we show that the state space spanned by the intensive variables such as temperature and pressure is isomorphic to that spanned by extensive variables such as the mass and energy inventories for a single component process system. Then we propose a state space model for power plants, which uses the mass and energy inventories as state variables, and has an affine structure in the control variables. In addition, a passivity based inventory controllers are developed. Numerical simulations suggest the performance and efficiency of the inventory controllers in power plant systems.

Keywords: Mathematic Modeling, Passivity Based Control, Power Plant, Dynamic Simulation, Inventory Control.

1. INTRODUCTION

Power plant models have been developed for steady state analysis, dynamic prediction and training simulator design during the last decades, Chien et al. [1958], Kwan & Anderson [1970], Tysso [1981], Astrom & Bell [2000], Flynn & Malley [1999]. Most of the modeling approaches use the intensive variables such as temperature and pressure as their point of departure for writing energy balances. The complex thermodynamic relationships are inevitably introduced into the differential equations. The nonlinear differential equations involve many nonlinear terms, have high order and are not easy to use for controller design. Some of the simulation models are so detailed that they can not be used in real time, Lu [1999].

Passivity theory provides an effective method to control a wide range of process systems. The main advantage of passivity is that it allows to develop controllers without detailed process modeling, Jillson & Ydstie [2007], Farschman, Viswanath & Ydstie [1998], Ruszkowski, Garcia-Osoria & Ydstie [2005]. In this paper, we show that the passivity can also be used to guide in process modeling. We choose the inventories for modeling and control. It is shown that a complex power plant system can be modeled as networks of unit operations, whose dynamics are described by balances of inventories, interconnected by material and energy flows. The inventory equations maintain the affine structures of the mass and energy balance equations, and the synthetic passive input-output pair can be deployed. They often produce a set of differential functions with a simple and rather fixed structure. In addition, according to the inventory control theory, an asymptotic stability of the closed-loop system is guaranteed when we use the strictly input passive feedbacks .

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2. SINGLE COMPONENT PROCESS SYSTEMS

We consider the modeling and control of a nonlinear system

$$\frac{dx}{dt} = f(x) + g(x, d, m) \tag{1}$$

where x is called the microscope state, d the disturbances, m the control variables. Following Farschman, Viswanath & Ydstie [1998], we define the inventories $Z_i(x), i = 1, \cdots, n$ to be any C^1 function, so that $Z_i(x) \geq 0$ for any x. From continuity, we can write

$$\frac{dZ_i}{dt} = \frac{\partial Z_i}{\partial x}\frac{dx}{dt} = p_i(x) + \phi_i(x, d, m) \tag{2}$$

where $p_i(x) = \frac{\partial Z_i}{\partial x} f(x)$ and $\phi_i(m, x, d) = \frac{\partial Z_i}{\partial x} g(x, d, m)$. An inventory is said to be invariant if the drift $p(x) = 0$. If $p(x) \geq 0$, the inventory satisfies the Clausius-Planck inequality, and if $p(x) \leq 0$, it satisfies the dissipative property.

The state Z of a single component thermodynamic system is defined by the vector of extensive variables, i.e. the internal energy, volume and mass.

$$Z = [U, V, M] \tag{3}$$

The inventories U, V, M are invariant so that $p_i(x) = 0$ in (2) holds for $i = 1, 2, 3$. A fundamental result in thermodynamics states that there exist an inventory $S(Z)$, which satisfies the Clausius-Planck property. It follows that $\frac{\partial S}{\partial x} f(x) \geq 0$ holds for all x, so that we have

$$p_i(x) \geq 0 \tag{4}$$

in (2). Inequality (4) is called the second law of thermodynamics. By using the fact that the state is determined by the vector Z defined in (3), we can define a vector of dual variables called potentials, so that $w = \frac{\partial S}{\partial Z}$. The potentials w are functions of the temperature, pressure and chemical potential, i.e. $w = [\frac{1}{T}, \frac{P}{T}, \frac{\mu}{T}]$. Sometimes the

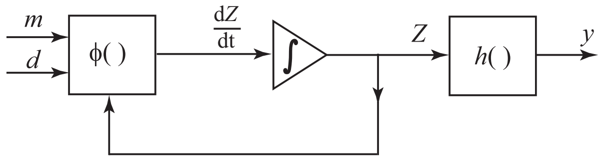

Fig. 1. Block diagram of passivity based process model.

potentials are also called observables, since they can be measured directly, whereas the extensive variables often has to be inferred indirectly. However, there exists a one-to-one map between the extensive and intensive variables. Following Sandler [1999], the Gibbs phase rule states

$$n_I = 2 + n_c - n_p - n_r \tag{5}$$

where n_p is the number of phases, n_c the number of components, n_r the number of reactions at equilibrium and n_I the number of intensive variables that must be fixed to identify the state. The number of extensive variable that must be fixed is equal to

$$n_E = 2 + n_c - n_I = n_p + n_r \tag{6}$$

Thus, for a single component system with a single phase ($n_r = 0, n_p = 1$), we have $n_E = 1$ and $n_I = 2$. Thus we need to specify at least one extensive variable, e.g. V or M, and two intensive variables, e.g. T and P, to specify the system state Z uniquely. For a two phase system, we have $n_E = 2$ and $n_I = 1$. Then we need to specify two extensive variables, e.g. V and M, and one intensive variable, e.g. T or P to specify the state uniquely. Therefore, we find that the mappings $(U, V, M) \mapsto (T, V, P)$ for a single phase system and $(U, V, M) \mapsto (T, V, M)$ for a two phase system are invertible. The mapping $(U, V, M) \mapsto (T, V, P)$ is not invertible for a two phase system, since we must fix two extensive variables to specify the state.

A generic macroscopic model of a single component process system with the microscopic model defined by (1) can be written in terms of its invariant as follows:

$$\begin{cases} \dfrac{dZ}{dt} = \phi(Z, d, m) \\ y = h(Z) \end{cases}$$

The net transport $\phi(Z, d, m)$ can be decomposed into n flows so that $\phi = \sum f_i$, where f_i denote the flows of mass, energy and volume with $i = 1, \cdots, n$. We note that the outputs y are intensive variables, e.g. the temperature, pressure and water level. The function $h(Z)$ defines the measurement strategy. The measurement have to be chosen so that the state Z is observable from y. In a two phase system, there needs at least two extensive variables that we must measure or infer. The developments in the previous section show that $h(Z)$ is invertible, provided suitable measurements are chosen, so that state estimation is not needed. Figure 1 shows a block diagram of the general process model.

3. POWER PLANT MODEL

Figure 2 presents a simple flowsheet of the most important units in a coal fired power plant. High pressure feed water flows through the drum boiler, primary and secondary superheater, where the water receives heat transferred from the combustion gases and the state changes from water to the superheated steam. In the inlet of the secondary superheater, water is sprayed into the steam to control the temperature before going to the turbine. The throttle valve controls the steam flow rate to the turbine, which is directly proportional to the power generated by the turbine. The feedwater flow $\dot{m}_{fw}$, coal flow $\dot{m}_{coal}$ and spray water flow $\dot{m}_{sw}$ control the process, while the mass flow rate $\dot{m}_{ssh}$ set by the power demand, the heat flow rates in the boiler, primary and secondary superheater Q_1, Q_2, Q_3 disturb the process.

We denote by $\dot{m}$ the mass flow rate, and Q the heat released by combustion in the furnace. Let the subscripts fw, b, psh, ssh, sw refer to the feedwater, drum boiler, primary superheater, secondary superheater and spray water, respectively.

Drum Boiler

In the boiler, water exists in two phases: saturated liquid and steam. It is assumed that both phases are in thermodynamic equilibrium. The conservation of mass and energy yield

$$\frac{dM_b}{dt} = \dot{m}_{fw} - \dot{m}_b \tag{7}$$

$$\frac{dU_b}{dt} = \dot{m}_{fw} h_{fw} - \dot{m}_b h_b + Q_b \tag{8}$$

Primary Superheater

The mass and energy equations for the primary superheaters are written

$$\frac{dM_{psh}}{dt} = \dot{m}_b - \dot{m}_{psh} \tag{9}$$

$$\frac{dU_{psh}}{dt} = \dot{m}_b h_b - \dot{m}_{psh} h_{psh} + Q_{psh} \tag{10}$$

Secondary Superheater

The conservation of mass and energy for the secondary superheater are written

$$\frac{dM_{ssh}}{dt} = \dot{m}_{psh} + \dot{m}_{sw} - \dot{m}_{ssh} \tag{11}$$

$$\frac{dU_{ssh}}{dt} = \dot{m}_{psh} h_{sh} + \dot{m}_{sw} h_{sw} - \dot{m}_{ssh} h_{ssh} + Q_{ssh} \tag{12}$$

Furnace

The heat released in the furnace is calculated as $Q = \dot{m}_{coal} h_{coal}$, where h_{coal} is the heat value of the coal power burned. The airflow is adjusted so that $\dot{m}_{air} = k_{air} \dot{m}_{coal}$, where k_{air} is adjusted to get complete combustion. The heat values are often obtained from experiments. In the simple model, we assume that the heat absorbed by the drum boiler, the primary and secondary superheater can be represented by $Q_b = k_1 Q, Q_{psh} = k_2 Q, Q_{ssh} = k_3 Q$, where $k_1, k_2, k_3 \in R^+$ and $k_1 + k_2 + k_3 = 1$.
The objective of the control system is to adjust control variables to keep the energy in the boiler, the mass in the boiler and the energy in the secondary superheater at given references.

Equations (7-12) can be combined into a state space model

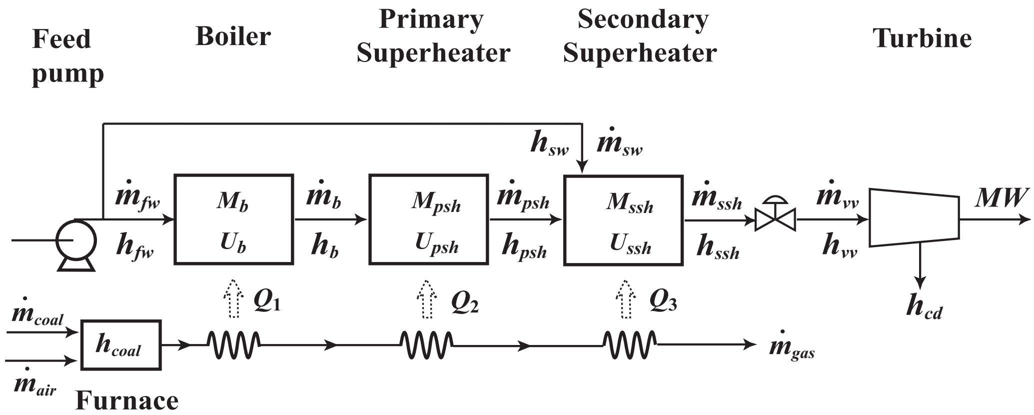

Fig. 2. Schematic representation of the power plant model.

$$\begin{bmatrix} dM_b/dt \\ dU_b/dt \\ dM_{psh}/dt \\ dU_{psh}/dt \\ dM_{ssh}/dt \\ dU_{ssh}/dt \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ h_{fw} & k_1 & 0 & -h_b & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & k_2 & 0 & h_b & -h_{psh} & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & k_3 & h_{sw} & 0 & h_{psh} & -h_{ssh} \end{bmatrix} \begin{bmatrix} \dot{m}_{fw} \\ Q \\ \dot{m}_{sw} \\ \dot{m}_b \\ \dot{m}_{psh} \\ \dot{m}_{ssh} \end{bmatrix} \tag{13}$$

We therefore decompose the state space into the controlled and uncontrolled components. In this case, the controlled variables are $Z_1 = [M_b, U_b, U_{ssh}]^T$ and the uncontrolled ones $Z_2 = [M_{psh}, U_{psh}, M_{ssh}]^T$. The control variable are $m = [\dot{m}_{fw}, \dot{m}_{coal}, \dot{m}_{sw}]^T$ and $d = [\dot{m}_b, \dot{m}_{psh}, \dot{m}_{ssh}]^T$. Then (13) can be rewritten as follows

$$\begin{cases} \dfrac{dZ_1}{dt} = B_1 m + F_1 d \\ \dfrac{dZ_2}{dt} = B_2 m + F_2 d \end{cases} \tag{14}$$

where

$$B_1 = \begin{bmatrix} 1 & 0 & 0 \\ h_{fw} & k_1 & 0 \\ 0 & k_3 & h_{sw} \end{bmatrix} \quad F_1 = \begin{bmatrix} -1 & 0 & 0 \\ -h_b & 0 & 0 \\ 0 & h_{psh} & -h_{ssh} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad F_2 = \begin{bmatrix} 1 & -1 & 0 \\ h_b & -h_{psh} & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

The mass flow rate $\dot{m}_b$ and $\dot{m}_{psh}$ is modeled using the state variables. Now we simply set $\dot{m}_{psh} = \dot{m}_{ssh} - \dot{m}_{sw} - K_{ssh}(M_{ssh} - M^*_{ssh})$ and $\dot{m}_b = \dot{m}_{psh} - K_{psh}(M_{psh} - M^*_{psh})$,where M^*_{psh}, M^*_{ssh} is calculated using the steady states, and K_{psh}, K_{ssh} are chosen so that the time constants are both less than 1 second.

Steam from the secondary superheater flows through the throttle valve. By omitting the energy loss and compressibility, we have $\dot{m}_{vv} = \dot{m}_{ssh}$ and $h_{vv} = h_{ssh}$. In the turbine, the enthalpy drop from the valve outlet h_{vv} to the condenser h_{cd}. This energy drop causes work to be done on the turbine

$$MW = K_{tb}\dot{m}_{vv}(h_{vv} - h_{cd}) \tag{15}$$

where $K_{tb} \in [0.6, 0.9]$ is the turbine efficiency, and MW denotes the electrical power generated, which is the output of this state space model.

Recalling that two extensive variables, e.g. mass and energy inventories, can specify the state of a single component system. The enthalpy $h_b, h_{psh}, h_{at}, h_{ssh}$ in the power plant are functions of the state variables Z_1, Z_2, i.e. $h_b = f(M_b, U_b)$, $h_{psh} = U_{psh}/M_{psh}$ and $h_{ssh} = U_{ssh}/M_{ssh}$. Therefore, we have the following state space model

$$\begin{cases} \dfrac{dZ_1}{dt} = B_1(Z_1, Z_2)m + F_1(Z_1, Z_2)d \\ \dfrac{dZ_2}{dt} = B_2(Z_1, Z_2)m + F_2(Z_1, Z_2)d \\ \quad y = h(Z_1, Z_2)m_4 \end{cases} \tag{16}$$

where $h(Z_1, Z_2) = K_{tb}(\frac{U_{ssh}}{M_{ssh}} - h_{cd})$ and $m_4 = \dot{m}_{vv}$.
The power plant model is an affine structure in the control variables m. This feature can be used to derive the inventory controllers.

4. PASSIVITY BASED INVENTORY CONTROL

It is proved in Farschman, Viswanath & Ydstie [1998] that the synthetic input and output pair (u, e_v) of the controlled part of system (16) is passive

$$\begin{cases} u = \phi(Z, d, m) + \dfrac{dZ_1^*}{dt} \\ e_Z = Z - Z_1^* \end{cases}$$

and that a control can be calculated if $\phi(Z, d, m)$ is invertible with respect to m. Note that m does not have to be unique. Z_1^* is the desired setpoint for Z_1. Therefore, the inventory control law can be written in the form:

$$u = -\mathbf{C}(e_Z) = \phi(Z, d, m) + \frac{dZ_1^*}{dt} \tag{17}$$

This control strategy ensures that the closed-loop system asymptotically tracks the desired set point. The operator $\mathbf{C}$, which maps errors into synthetic controls, should be strictly input passive, e.g. the PID controller, adaptive feedforward controllers, optimal controllers and many gain scheduling controllers Ruszkowski, Garcia-Osoria & Ydstie

[2005].

Combining (16) and (17), we get

$$\frac{dZ_1}{dt} = B_1(Z_1, Z_2)m + F_1(Z_1, Z_2)d - \mathbf{C}(Z_1 - Z_1^*) + \frac{dZ_1^*}{dt}$$

where $Z_1^* \in R^3$ and $dZ_1^*/dt = 0$. Notice that $B_1(Z_1, Z_2)$ is a lower diagonal matrix with nonzero diagonal elements i.e. $k_3, h_{sw} \neq 0$. The inequality $|B_1(Z_1, Z_2)| \neq 0$ holds for any Z_1 and Z_2. It immediately follows that $B_1(Z_1, Z_2)$ is invertible. Therefore, we design the following inventory controllers

$$m = B_1^{-1}(Z_1, Z_2)\left[-\mathbf{C}(Z_1 - Z_1^*) - F_1(Z_1, Z_2)d\right] \qquad (18)$$

This control structure is the classical combination of feedback and feed-forward control, i.e.

$$m = \underbrace{-B_1^{-1}(Z_1, Z_2)\mathbf{C}(Z_1 - Z_1^*)}_{Feedback} + \underbrace{B_1^{-1}(Z_1, Z_2)F_1(Z_1, Z_2)d}_{Feedforward}$$

Since the feedforward term cancels the nonlinearities, this method is therefore also referred to as input-output linearization.

5. STEP RESPONSE

In this example, the power plant model is subjected to a setpont change in the energy inventory of the secondary superheater equivalent to 10^5 kJ. The inventory controllers are used as stated in (18). The thermodynamic properties are calculated using the Xsteam package, Holmgren [2006]. This package is also used to bridge the inventories with the measured variables, e.g. the steam temperature and pressure.

Figure 3 shows the profiles of the inventories of the boiler, primary and secondary superheater. It is easy to see from Figure 3 (1)-(3) that all the mass and energy inventories are controlled around their setpoints. In particular, U_{ssh} tracks the new setpoint value after a small overshoot. This overshoot is caused by the PI controllers. The uncontrolled inventory U_{psh} is stable after the convergence of the controlled inventories. The control variables and the disturbance are plotted in Figure 4. Here the measured disturbance is kept constant.

Figure 5 demonstrates the step response of the net power output and some key state variables in traditional power plant control schemes. With an increased energy inventory and constant mass inventory, the steam temperature and pressure will increase in the secondary superheater. This leads to the increase of the outlet steam enthalpy h_{ssh}. Recalling that the mass flow rate $\dot{m}_{ssh}$ is constant, the net power output will increase, which requires an increased coal flow rate (See Figure 4(3)). For the drum boiler, the water level presents a typical swell and shrink phenomena. The water level increases initially due to the increased evaporation in the riser caused by the increasing heat flow rate. Then the evaporation in the drum dominates the dynamics of the water level. With the decreasing drum pressure, more water will be evaporated in the drum. This corresponds to the decrease of the water level and total

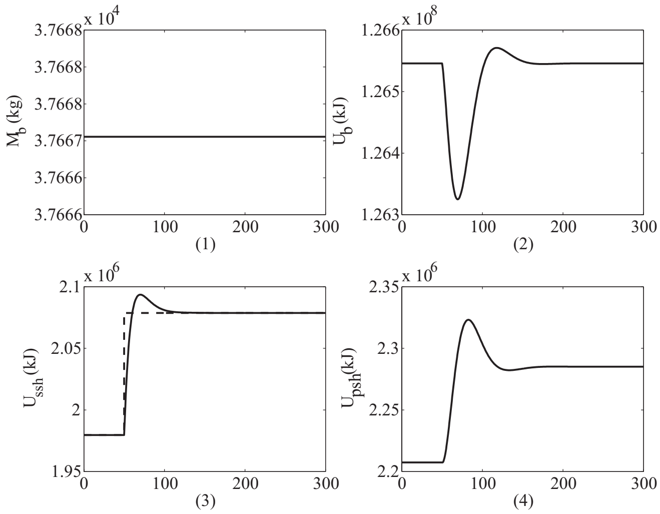

Fig. 3. Step responses of the mass and energy inventories.

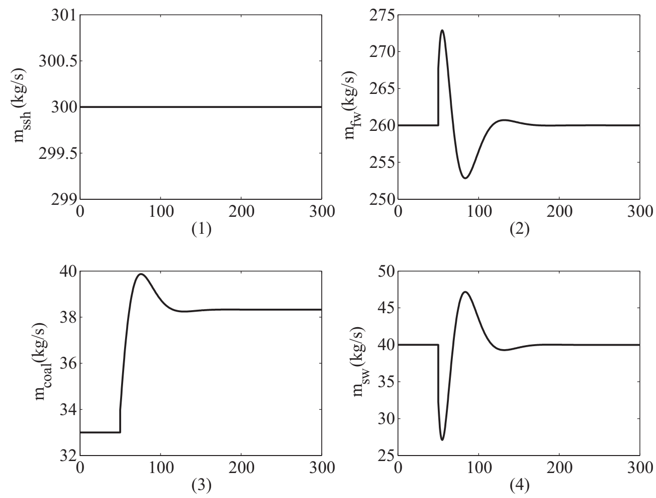

Fig. 4. The control variables and process disturbance.

water volume V_{wt}. Finally, less heat to the riser and increasing drum pressure cause the increasing condensation in both the riser and drum. The water volume presents the key contribution to the water level. Accordingly, the drum water level increases and decrease before it is stabilized at the initial value. Here we can see that the inventory controllers perform quite well in this example.

6. AREA REGULATION TESTS

In the next example, we simulate the dynamic responses to the Area Regulation (AR) test. The AR test signals consist of several ramps, which fluctuate around the base loading with a maximum deviation of $\pm 10\%$. The plot of the test signals are demonstrated in Figure 7 (1).

Figure 6 shows the dynamic responses of the mass and energy inventories of the boiler, primary and secondary superheaters. It is not hard to see from Figure 6 (1)-(3) that all the controlled inventories track their setpoints accurately with exception of several tiny fluctuations. The maximum deviation of these inventories is less than $\pm 0.5\%$ from the setpoints throughout the simulation time. According to Figure 6 (4), the uncontrolled inventory U_{psh}

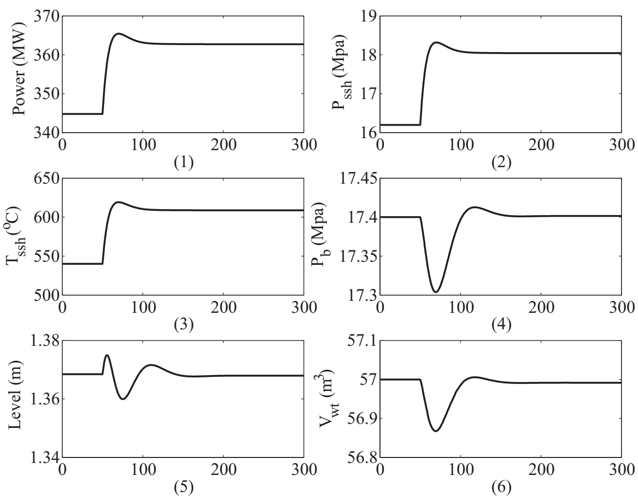

Fig. 5. Step responses of the output power, pressure, temperature and water level.

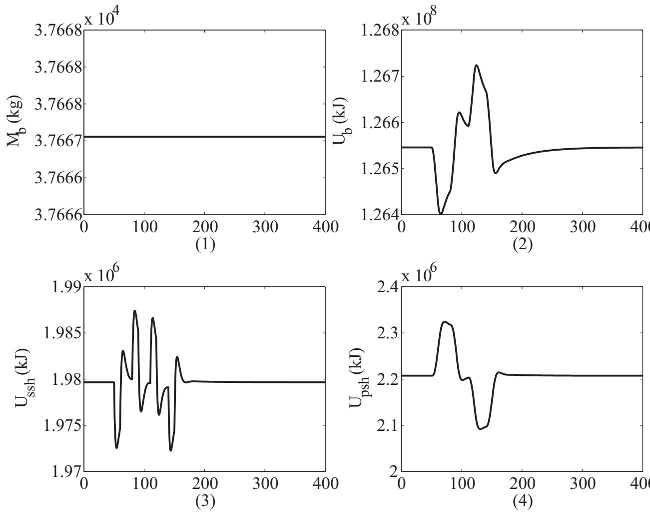

Fig. 6. Dynamic responses of the mass and energy inventories in AR test.

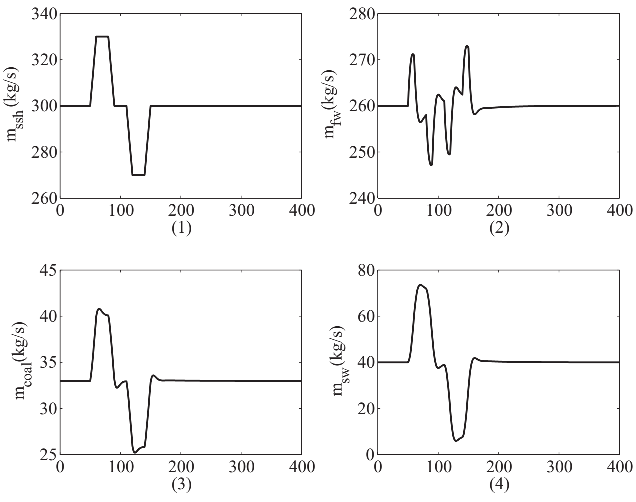

Fig. 7. The control variables and disturbance in AR test.

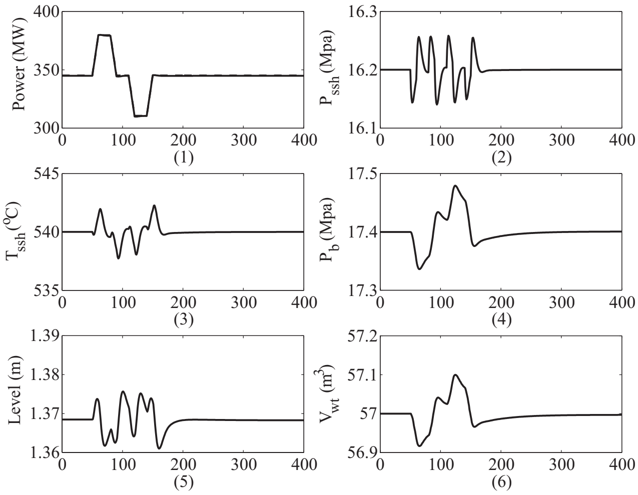

Fig. 8. Dynamic responses of the output power, pressure, temperature and water level in AR test.

also converge to stable value after an initial transient. Fig. 7 visualizes the profiles of the control variables and disturbance. Here the same control law is used as in the last example, and the AR test signal is treated as the process disturbances.

Figure 8 presents the dynamic responses of the power outputs, pressure, temperature, water level and volume. It is shown in Figure 8 (1) that the simulated power outputs are indistinguishable with the reference values. The maximum deviation is within $\pm 1.5\ MW$. The temperature and pressure of the steams from the secondary superheater returns to their initial values shortly after the convergence of the inventories. Similar result is also valid for the drum boiler. Using the theorem developed in Section 2, the drum pressure and water volume can be indirectly controlled, provided that we can control the total mass and energy inventories in the boiler. The pressure and water volume will further determine the drum water level. In this case, the maximum deviation of water level is less than ± 0.01 m from the normal value. Please refer to Astrom & Bell [2000] for the detail calculations of the pressure, water level and volume.

Note that both the inventories and the key states in traditional power plant models are controlled quite well. The power output tracks the reference values perfectly throughout the simulation time. The inventory controllers have a good perform in the AR test.

7. CONCLUSION

In this paper, we propose a state space model for a simplified power plant system. This model uses the mass and energy inventories as the state variables, and has an affine structure in the control variables. A passivity based inventory controllers are developed, which ensures the asymptotic stability of the closed-loop systems. Numerical simulations shows the performance and efficiency of the proposed control method. In addition, the affine structure in this model is derived directly from the mass, energy and momentum balance laws, the proposed modeling and passivity based control scheme are promising in a wide range of process systems.

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