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Adaptive Fault Detection for a Class of Nonlinear Systems Based on Output Estimator Design ⋆

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Abstract:
This paper considers output estimator based fault detection problem for a class of nonlinear systems with unknown system parameters. Because observer design for such systems is extremely difficult if not impossible, output estimator design is used for the purpose of fault detection. In order to achieve output estimator design using adaptive approaches, a multi-input multi-output (MIMO) nonlinear system is first decomposed into a group of multi-input single-output (MISO) nonlinear systems. For each MISO nonlinear system, an output equation is derived through filtering the output, the inputs, and those nonlinear functions of the outputs, which depends linearly on all unknown system parameters. Based on the output equation and using adaptive approaches, an adaptive output estimator is designed for the corresponding output. By defining residuals using the adaptive output estimation errors resulting from the output estimators, a fault detection scheme is proposed. The efficacy of the proposed fault detection scheme is tested on a single-link flexible robot manipulator model thorough computer simulations.

1. INTRODUCTION

A common assumption for non-adaptive fault diagnosis schemes is that the system (or the nominal system) parameters are known (Saif and Guan (1993); Xiong and Saif (2000); Patton and Chen (2000); Saberi et al. (2000); Edelmayer et al. (2006); Gao et al. (2007)). However, in many real control systems (especially adaptive control systems), system parameters are often unknown. For such systems, adaptive approaches are needed to solve fault diagnosis problems.

Observer based strategies are commonly explored for fault diagnosis problems in systems with unknown parameters. However, a major difficulty in this approach is that adaptive observer design for multi-input multi-output (MIMO) systems is extremely difficult if not impossible. In the literature, adaptive observer design problem is only well solved for unknown systems with single output (Bastin and Gevers (1988); Kreisselmeier (1977); Marino (1990); Marino and Tomei (1992, 1995); Shafai et al. (2001)). For MIMO systems with unknown linear system part, the problem of adaptive observer design remains open because none of the existing adaptive observers (see for example, those reported in Farza et al. (2005); Yu et al. (2003); Rajamani and Hedrick (1995); Zhang and Delyon (2001); zhang (2002)) are applicable. This observation is even true for linear systems with all system matrices unknown. Because of the difficulty in adaptive observer design, adaptive observer based fault diagnosis has only had limited success. Existing fault diagnosis schemes have to put restrictions on the system structure and/or the unknown system parameters (Ding and Frank (1993); Yang and Saif (1997); Jiang et al. (2004); Shafai et al. (2001)) though they are not even applicable to unknown MIMO linear systems.

In order to remove the limitation of observer design and to deal with more challenging fault diagnosis problems, the idea of output estimator design has been proposed and used recently in Chen and Saif (2006, 2007a,b,c). In Chen and Saif (2006, 2007a), robust output estimators were designed to deal with unmatched unknown inputs under the condition that all the system parameters are known. For multi-input single-output linear systems with unknown system parameters, an adaptive output estimator based fault diagnosis scheme was proposed by Chen and Saif (2007b), which is able to solve the fault detection problem and the multiple constant actuator fault isolation problem. Using the idea of adaptive output estimator design, Chen and Saif (2007c) was able to solve sensor fault diagnosis problem for general unknown MIMO linear systems.

The purpose of this paper is to solve the fault detection problem for a class of MIMO nonlinear systems with unknown linear system part and with nonlinear functions of the outputs multiplied by unknown parameters. Because of the difficulty encountered in the adaptive observer design for MIMO systems with unknown system parameters, the idea of using adaptive observer is abandoned here, and instead, we propose to use the idea of adaptive output estimator design to accomplish fault detection.

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The remainder of the paper is arranged as follows: In Section 2, a MIMO nonlinear system model is introduced and the problem of interest is formulated. In Section 3, the MIMO system is decomposed into a group of MISO systems and a transfer function description for each MISO system is presented. In Section 4, in order to design output estimators, an output equation is derived for each MISO based on its transfer function. In Section 5, adaptive output estimators are designed based on the output equations derived, and an adaptive fault detection scheme is proposed based on the designed adaptive output estimators. The proposed adaptive fault detection scheme is tested on a single-link flexible robot manipulator model in Section 6 and simulation results are provided. Finally, concluding remarks are made in the last section.

2. SYSTEM OF INTEREST AND PROBLEM FORMULATION

Consider MIMO nonlinear systems described as below

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Df(y(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t)$, $y(t)$, $u(t)$ are the system state vector, output vector and input vector respectively, and $x(t) \in R^n$, $y(t) = (y_1(t) \ \cdots \ y_p(t))^T$, $u(t) = (u_1(t) \ \cdots \ u_m(t))^T$, $f(y(t)) = (f_1(y(t)) \ \cdots \ f_q(y(t)))^T$ is a vector of nonlinear functions, and the system matrices A, B, C and D are not assumed to be known.

- Assumption A1: n, m, and p are known.
- Assumption A2: $f_j(y(t)), j = 1, \cdots, q$ are known.

For system (1), the following problem is formulated.

Adaptive Fault Detection Problem:
Under the condition that assumptions A1 and A2 are satisfied, design a fault detection scheme such that it can detect faults adaptively.

If only assumptions A1 and A2 are assumed, adaptive observer for (1) is extremely difficult if not impossible to design since no adaptive observer has been found for such a system. Moreover, if systems given by (1) are not detectable, no observers can be designed to estimate all the states asymptotically. Because of these difficulties in observer design, the idea of adaptive observer based fault diagnosis is abandoned in this paper. Instead, the idea of adaptive output estimator design for fault diagnosis is applied because output estimator design is possible for systems given by (1) under assumptions A1 and A2 and output estimators are sufficient for the purpose of fault diagnosis. As will be shown later in this paper, it is the idea of designing output estimators rather than state observers that leads to an elegant solution to the Adaptive Fault Detection Problem.

3. SYSTEM DECOMPOSITION AND RELATED TRANSFER FUNCTION DESCRIPTION

It is found that trying to estimate all the outputs directly from the MIMO system given by (1) is very difficult. This observation motivates us to transform the difficult MIMO output estimator design problem into several simpler MISO output estimator design problems through decomposing (1) into a group of MISO systems, as will be shown in the sequel.

Let $C = (C_1^T \ \cdots \ C_p^T)^T$, $B = (B_1 \ \cdots \ B_m)$, and $D = (D_1 \ \cdots \ D_q)$. It is obvious that a MIMO system given by (1) can be decomposed into p MISO systems, where for $1 \leq j \leq p$, the jth MISO system is of the following form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Df(y(t)), \\ y_j(t) &= C_j x(t). \end{aligned} \tag{2}$$

Because (C_j, A) is not necessarily detectable even when (C, A) is, it is not appropriate to design any observer for the MISO system defined by (2). Therefore, the outputs may not be estimated through observer design.

In order to estimate the outputs without designing observers for the MISO systems defined by (2), the following input-output relation of $u(t)$ and $y_j(t)$ will be used.

$$y_j(t) = \sum_{l=1}^{m} G_{jl}(s) u_l(t) + \sum_{k=1}^{q} G_{jk}(s) f_k(y(t)) \tag{3}$$

where for $1 \leq j \leq p$, $1 \leq l \leq m$, and $1 \leq k \leq q$

$$\begin{aligned} G_{jl}(s) &= C_j(sI - A)^{-1} B_l \\ &= \frac{b_{jl,n-1}s^{n-1} + \cdots + b_{jl,1}s + b_{jl,0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \end{aligned} \tag{4}$$

and

$$\begin{aligned} G_{jk}(s) &= C_j(sI - A)^{-1} D_k \\ &= \frac{d_{jk,n-1}s^{n-1} + \cdots + d_{jk,1}s + d_{jk,0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \end{aligned} \tag{5}$$

and $s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = det(sI - A)$.

The expression in (3), though involves both frequency- and time- domain representations that may appear weird, is a common practice in adaptive control community. For convenience, define $a(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$, $b_{jl}(s) = b_{jl,n-1}s^{n-1} + \cdots + b_{jl,1}s + b_{jl,0}$, and $d_{jk}(s) = d_{jk,n-1}s^{n-1} + \cdots + d_{jk,1}s + d_{jk,0}$.

4. OUTPUT EQUATIONS FOR MISO SYSTEMS

In the remaining part of this paper, the dependence of variables on time t will be dropped for the sake of simplicity, for example, $u_j(t)$ will be simply written as u_j. For each $1 \leq j \leq p$, based on (3), (4), and (5), and inspired by Kreisselmeier (1977) and Krstic et al. (1994), the following state space realization can be given for (3).

$$\begin{aligned} \dot{x}_j &= \bar{A}x_j - ay_j + b_{j1}u_1 + \cdots + b_{jm}u_m \\ &\quad + d_{j1}f_1(y) + \cdots + d_{jq}f_q(y) \\ y_j &= x_{j,1} \end{aligned} \tag{6}$$

where $x_j = (x_{j,1} \ \cdots \ x_{j,n})^T$ and

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix},$$

$$b_{jl} = \begin{bmatrix} b_{jl,n-1} \\ \vdots \\ b_{jl,0} \end{bmatrix}, \ 1 \le l \le m,$$

$$d_{jk} = \begin{bmatrix} d_{jk,n-1} \\ \vdots \\ d_{jk,0} \end{bmatrix}, \ 1 \le k \le q. \tag{7}$$

It is important to note that (6) is an observable realization, which does not include the unobservable modes in the original system and thus is *not* the same as (2), which might not be observable. It is also crucial to know that the outputs for both (6) and (2) are the same. Therefore, (6) can always be used to estimate y_j regardless of whether (2) is observable or not. This implies that it is not necessary to require the original system (1) to be observable for the purpose of output estimation.

For each $1 \le j \le p$, in order to estimate y_j, we need to derive the state estimate for (6). To do so, $u_l, 1 \le l \le m$, y_j, and $f_k(y), 1 \le k \le q$ are filtered by the following $n-$dimensional filters:

$$\dot{\lambda}_l = A_0\lambda_l + e_n u_l, \quad 1 \le l \le m \tag{8}$$

$$\dot{\eta}_j = A_0\eta_j + e_n y_j \tag{9}$$

$$\dot{\nu}_k = A_0\nu_k + e_n f_k(y), 1 \le k \le q \tag{10}$$

where $A_0 = \bar{A} - K(1\ 0\ \cdots\ 0)$ and $K = (K_1 \cdots K_n)^T$ is chosen such that A_0 is Hurwitz, and for any $1 \le i \le n$, $e_i = (e_{i,1}, \cdots, e_{i,n})^T \in R^n$ is defined by $e_{i,i} = 1$ and $e_{i,j} = 0$ for $j \ne i$.

After some matrix manipulations, it can be shown that

$$a(A_0)e_n = a - K$$

$$b_{jl}(A_0)e_n = b_{jl}, \quad 1 \le l \le m$$

$$d_{jk}(A_0)e_n = d_{jk}, \quad 1 \le l \le q \tag{11}$$

where $a(A_0)$, $b_{jl}(A_0), 1 \le l \le m$, and $d_{jk}(A_0), 1 \le l \le q$ are matrix polynomials with $a(s)$, $b_{jl}(s), 1 \le l \le m$, and $d_{jk}(s), 1 \le k \le q$ being defined earlier.

Now the estimate for x_j is formed as

$$\hat{x}_j = \sum_{l=1}^{m} b_{jl}(A_0)\lambda_l + \sum_{k=1}^{q} d_{jk}(A_0)\nu_k - a(A_0)\eta_j \tag{12}$$

Using (6) and (8)-(12), it can be verified that the estimation error $\varepsilon_j = (\varepsilon_{j,1}, \varepsilon_{j,2}, \cdots, \varepsilon_{j,n})^T = x_j - \hat{x}_j$ satisfies $\dot{\varepsilon}_j = A_0\varepsilon_j$. Denote $\xi_{ji} = A_0^i\eta_j, 0 \le i \le n-1$, $\xi_{jn} = -A_0^n\eta_j$, $\upsilon_{li} = A_0^i\lambda_l, 0 \le i \le n-1, 1 \le l \le m$, and $\varphi_{ki} = A_0^i\nu_k, 0 \le i \le n-1, 1 \le k \le q$, then (12) can be rewritten as

$$x_j = \xi_{jn} - \sum_{i=0}^{n-1} a_i\xi_{ji} + \sum_{l=1}^{m}\sum_{i=0}^{n-1} b_{jl,i}\upsilon_{li}$$

$$+ \sum_{k=1}^{q}\sum_{i=0}^{n-1} d_{jk,i}\varphi_{ki} + \varepsilon_j \tag{13}$$

It can be checked that all the ξ, υ, and φ signals and their derivatives are explicitly available:

$$\xi_{jn} = -A_0^n\eta_j, \quad \dot{\xi}_{jn} = A_0\xi_{jn} + ky_j,$$

$$\xi_{ji} = A_0^i\eta_j, \quad \dot{\xi}_{ji} = A_0\xi_{ji} + e_{n-i}y_j,$$

$$\upsilon_{li} = A_0^i\lambda_l, \quad \dot{\upsilon}_{li} = A_0\upsilon_{li} + e_{n-i}u_l,$$

$$\varphi_{ki} = A_0^i\nu_k, \quad \dot{\varphi}_{ki} = A_0\varphi_{ki} + e_{n-i}f_k(y), \tag{14}$$

where $0 \le i \le n-1, 1 \le l \le m, 1 \le k \le q$.

It should be pointed out the estimate given by (12) can not be applied directly since the parameters $a_i, 0 \le i \le n-1$, $b_{li}, 0 \le i \le n-1, 1 \le l \le m$, and $d_{ki}, 0 \le i \le n-1, 1 \le k \le q$ are unknown.

5. ADAPTIVE FAULT DETECTION

For each $1 \le j \le p$, under a no fault scenario, it follows from (6) and (13) that

$$\dot{y}_j = \xi_{jn,2} - (\xi_{j(2)} + e_1^T y_j)a + \sum_{l=1}^{m}(\upsilon_{l(2)} + e_1^T u_l)b_{jl}$$

$$+ \sum_{k=1}^{q}(\varphi_{k(2)} + e_1^T f_k(y))d_{jk} + \varepsilon_{j,2} \tag{15}$$

where the notations are defined as

$\xi_{jn}^T = (\xi_{jn,1}, \xi_{jn,2}, \cdots, \xi_{jn,n}), \xi_{(2)} = (\xi_{j(n-1),2}, \cdots, \xi_{j0,2})$

and

$\upsilon_{l(2)} = (\upsilon_{l(n-1),2}, \cdots, \upsilon_{l0,2}), \varphi_{k(2)} = (\varphi_{k(n-1),2}, \cdots, \varphi_{k0,2}).$

The output equation given by (15) is desirable because an estimate for each y_j is allowed to be designed separately. However, unlike in Chen and Saif (2007c), y_j is no more independent from other outputs because of the existence of nonlinear functions $f_k(y)), 1 \le k \le q$. As a result, sensor fault isolation can not be achieved using the idea in Chen and Saif (2007c). This is the reason why only fault detection is considered in this paper.

In model based fault diagnosis, to detect faults, residuals are generated and monitored. To this end, for each $1 \le j \le p$, an estimate for the output y_j will be constructed based on (15). By utilizing the adaptive technique, an estimate of the output y_j is given as

$$\dot{\hat{y}}_j = -c_{y_j}(\hat{y}_j - y_j) + \xi_{jn,2} - (\xi_{j(2)} + e_1^T y_j)\hat{a}_{y_j}$$

$$+ \sum_{l=1}^{m}(\upsilon_{l(2)} + e_1^T u_l)\hat{b}_{jl} + \sum_{k=1}^{q}(\varphi_{k(2)} + e_1^T f_k(y))\hat{d}_{jk} \tag{16}$$

where $\hat{a}_{y_j}$, $\hat{b}_{jl}, 1 \le l \le m$, and $\hat{d}_{jk}, 1 \le k \le q$ are the estimates of a, $b_{jl}, 1 \le l \le m$, and $d_{jk}, 1 \le k \le q$, $c_{y_j} > 1$ is a positive design constant.

The update laws for the unknown parameter vectors are given below

$$\dot{\hat{a}}_{y_j} = \gamma_{a_{y_j}}(\xi_{j(2)} + e_1^T y_j)^T(\hat{y}_j - y_j)$$

$$\dot{\hat{b}}_{jl} = -\gamma_{b_{jl}}(\upsilon_{l(2)} + e_1^T u_l)^T(\hat{y}_j - y_j), 1 \le l \le m$$
$$\dot{\hat{d}}_{jk} = -\gamma_{d_{jk}}(\varphi_{k(2)} + e_1^T f_k(y))^T(\hat{y}_j - y_j), 1 \le k \le q \quad (17)$$

where $\gamma_{a_{y_j}}$, $\gamma_{b_{jl}}, 1 \le l \le m$, and $\gamma_{d_{jk}}, 1 \le k \le q$ are positive design constants.

By letting $j = 1, 2, \cdots, p$, all the outputs, that is, $y_1, y_2, \cdots, y_p$ can be estimated adaptively based on (16) and (17). Define $r_j(t) = \hat{y}_j - y_j, j = 1, 2, \cdots, p$, the following result is obtained.

Theorem 1. Under Assumptions A1 and A2, if there is no fault in the control system and the controller designed maintains inputs and outputs that are all bounded, then $lim_{t\to\infty} r_j(t) = 0$ for all $1 \le j \le p$.

Proof. It follows from (15) and (16) that

$$\dot{r}_j(t) = -c_{y_j} r_j(t) - (\xi_{j(2)} + e_1^T y_j)(\hat{a}_{y_j} - a) + \sum_{l=1}^{m}(\upsilon_{l(2)} + e_1^T u_l)(\hat{b}_{jl} - b_{jl}) + \sum_{k=1}^{q}(\varphi_{k(2)} + e_1^T f_k(y))^T(\hat{d}_{jk} - d_{jk}) - \varepsilon_{j,2} \quad (18)$$

where ε_j satisfies $\dot{\varepsilon}_j = A_0 \varepsilon_j$ and A_0 is Hurwitz.

Choose a Lyapunov function as

$$V_j = \frac{1}{2} r_j^2 + \frac{1}{2\gamma_{a_{y_j}}}(\hat{a}_{y_j} - a)^T(\hat{a}_{y_j} - a) + \sum_{l=1}^{m} \frac{1}{2\gamma_{b_{jl}}}(\hat{b}_{jl} - b_{jl})^T(\hat{b}_{jl} - b_{jl}) + \sum_{k=1}^{m} \frac{1}{2\gamma_{d_{jk}}}(\hat{d}_{jk} - d_{jk})^T(\hat{d}_{jk} - d_{jk}) + \varepsilon_j^T P_0 \varepsilon_j \quad (19)$$

where P_0 is the positive definite solution of $P_0 A_0 + A_0^T P_0 = -I$.

Differentiate the above Lyapunov function with respect to t and use (18), one obtains

$$\dot{V}_j = -c_{y_j} r_j^2 + \frac{1}{\gamma_{a_{y_j}}}[\dot{\hat{a}}_{y_j}^T - \gamma_{a_{y_j}}(\xi_{j(2)} + e_1^T y_j) r_j](\hat{a}_{y_j} - a) + \sum_{l=1}^{m} \frac{1}{\gamma_{b_{jl}}}[\dot{\hat{b}}_{jl}^T + \gamma_{b_{jl}}(\upsilon_{l(2)} + e_1^T u_l) r_j](\hat{b}_{jl} - b_{jl}) + \sum_{k=1}^{m} \frac{1}{\gamma_{d_{jk}}}[\dot{\hat{d}}_{jk}^T + \gamma_{d_{jk}}(\varphi_{k(2)} + e_1^T f_k(y)) r_j](\hat{d}_{jk} - d_{jk}) - r_j \varepsilon_{j,2} - \varepsilon_j^T \varepsilon_j \quad (20)$$

By substituting (17) into the above equation, it is easy to get

$$\begin{aligned} \dot{V}_j &= -c_{y_j} r_j^2 - r_j \varepsilon_{j,2} - \varepsilon_j^T \varepsilon_j \\ &= -(c_{y_j} - 1) r_j^2 - (r_j + \frac{1}{2}\varepsilon_{j,2})^2 \\ &\quad -(\varepsilon_{j,1}^2 + \frac{3}{4}\varepsilon_{j,2}^2 + \varepsilon_{j,3}^2 + \cdots + \varepsilon_{j,n}^2) \\ &\le -(c_{y_j} - 1) r_j^2 \\ &\le 0 \ (since \ \ c_{y_j} > 1) \end{aligned} \quad (21)$$

Since $\dot{V}_j \le 0$, $V_j(t)$ is bounded. Hence $r_j(t)$, the estimates $\hat{a}_{y_j}$, $\hat{b}_{jl}, 1 \le l \le m$, and $\hat{d}_{jk}, 1 \le k \le q$ are all bounded. Because $y_j(t)$ and $u_l(t), 1 \le l \le m$ are bounded, it follows that all ξ, υ, and φ signals are bounded. Hence it follows from (18) that $\dot{r}_j(t)$ is bounded. From (21), it can be shown that $\int_0^\infty r_j^2 dt$ is bounded. This together with the boundness of $\dot{r}_j(t)$ proves that $lim_{t\to\infty} r_j(t) = 0$. This completes the proof. ¶

Remark 1. The approach taken to construct the output estimates adaptively is quite different from adaptive observer based techniques proposed in the literature for nonlinear systems. The main advantage here is that it is no longer necessary to require the original systems to be detectable, and there are no matched conditions on the unknown system parameters.

Theorem 1 serves as a foundation for adaptive fault detection. If there is no fault in presence, according to Theorem 1, $r_1(t), \cdots, r_p(t)$ will all tend to zero. Hence, a fault is declared if any of them becomes nonzero. To be specific, an adaptive fault detection scheme is proposed as follows.

(1) Solve equations (16) and (17) to obtain $\hat{y}_j$ for $j = 1, 2, \cdots, p$.
(2) For each $j = 1, 2, \cdots, p$, compute $r_j(t) = \hat{y}_j - y_j$ and define residuals as $|r_j(t)|$.
(3) Choose a threshold $\epsilon_{Thre,j}$ for each $|r_j(t)|$.
(4) For each $1 \le j \le p$, compare the residual $|r_j(t)|$ with the threshold $\epsilon_{Thre,j}$. If any residual goes beyond its corresponding threshold, faults are detected.

Remark 2. It follows from (18) that both sensor faults and actuator faults may cause all residuals to exceed the threshold. Hence, any one of the residuals $|r_j(t)|, j = 1, 2, \cdots, p$ may be sufficient to fulfill the task of fault detection. Because there are p residuals in total, one has redundant information for fault detection. This redundancy is desirable because it allows for a more reliable fault decision. For example, if all residuals exceed the threshold, one can trigger a fault alarm more confidently and thus be able to reduce false alarms. In addition, one can also define new residuals based on $|r_j(t)|, j = 1, 2, \cdots, p$. For example, one can use $r_{all} = \sum_{j=1}^{m} |r_j(t)|$ to indicate a fault.

Theoretically speaking, the threshold, that is, $\epsilon_{Thre,j}$ could be chosen arbitrarily small. However, in practical situations, because other unconsidered uncertainties may exist, too small $\epsilon_{Thre,j}$ may lead to too many false alarms. On the other hand, too large $\epsilon_{Thre,j}$ may increase the missed detections. Trade-off has to be made on the choice of a suitable threshold.

In the following, some insights will be given on threshold selection through investigating the relation between the design constant c_{y_j} and the threshold ϵ_{Thre}.

Denote $M_j(t) = -(\xi_{j(2)} + e_1^T y_j)(\hat{a}_{y_j} - a) + \sum_{l=1}^{m}(\upsilon_{l(2)} + e_1^T u_l)(\hat{b}_{jl} - b_{jl}) + \sum_{k=1}^{q}(\varphi_{k(2)} + e_1^T f_k(y))^T(\hat{d}_{jk} - d_{jk}) - \varepsilon_{j,2}$, then, using (18), one gets

$$r_j(t) = r_j(0)e^{-c_{y_j}t} + e^{-c_{y_j}t}\int_0^t e^{c_{y_j}\tau}M_j(\tau)d\tau \qquad (22)$$

Assume that $|M_j(t)| \leq M_{j0}$, it follows from (22) that

$$|r_j(t)| \leq (r_j(0) - \frac{M_{j0}}{c_{y_j}})e^{-c_{y_j}t} + \frac{M_{j0}}{c_{y_j}} \qquad (23)$$

In steady state, one will always have $|r_j(t)| \leq \frac{M_{j0}}{c_{y_j}}$. Based on this, it is easy to see that faults can not be detected if one chooses $\epsilon_{Thre,j} > \frac{M_{j0}}{c_{y_j}}$. With fixed M_{j0}, the upper bound of $|r_j(t)|$ will decrease as c_{y_j} increases, which implies the missed detections might increase as c_{y_j} increases. Therefore, c_{y_j} should be chosen as small as possible to reduce the missed detection rate.

6. AN EXAMPLE AND SIMULATION RESULTS

In this section, a single-link flexible robot manipulator model in Kanellakopoulos et al. (1991) is used to test the proposed adaptive fault detection scheme. Under certain assumptions, the model takes the following form

$$\begin{aligned} J_1\ddot{q}_1 + F_1\dot{q}_1 + K(q_1 - \frac{q_2}{N}) + mgdcosq_1 = 0 \\ J_2\ddot{q}_2 + F_2\dot{q}_2 - \frac{K}{N}(q_1 - \frac{q_2}{N}) = K_t i \\ LDi + Ri + K_b\dot{q}_2 = u \end{aligned} \qquad (24)$$

According to Kanellakopoulos et al. (1991), after suitable change of variables, (24) can be made into the following form

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta_1 x_1 \\ \dot{x}_2 &= x_3 + \theta_2 x_1 + \theta_3 cosx_1 \\ \dot{x}_3 &= x_4 + \theta_4 x_1 + \theta_5 cosx_1 \\ \dot{x}_4 &= x_5 + \theta_6 x_1 + \theta_7 cosx_1 \\ \dot{x}_5 &= \theta_8 cosx_1 + b_0 u \end{aligned} \qquad (25)$$

Due to lack of space, the physical meanings of all variables and notations in this section are omitted but can be found in Kanellakopoulos et al. (1991). Assume that $y_1 = x_1$ and $y_2 = x_2$, it is easy to see that (25) takes the form (1). Therefore, the adaptive fault detection scheme developed can be readily applied to the single-link flexible robot manipulator model.

Simulations are done based on (25). The design parameters are chosen as $K = (17.5\ 120\ 402.5\ 659\ 420)^T$, all the $c-$ constants and the $\gamma-$ constants are chosen equal to 2. Three types of faults are considered.

- Case A– The actuator is stuck at a constant value, that is, $u = 0$ after $t > 20s$.
- Case B– The sensor for x_1 has a scaling error, that is, $y_1(t) = 0.8x_1(t)$ after $t > 5s$, where $x_1(t)$ is the real output, and $y_1(t)$ is the measurement provided by the sensor.

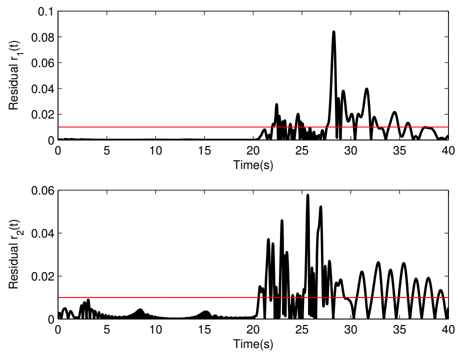

Fig. 1. Adaptive detection of actuator faults

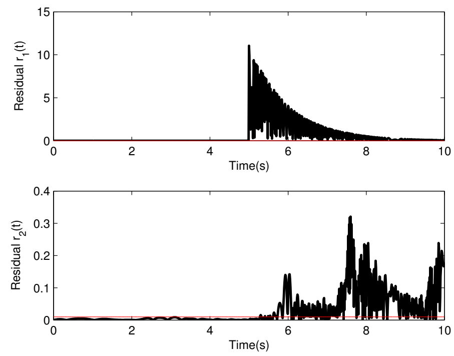

Fig. 2. Adaptive detection of scaling sensor faults

- Case C– The sensor for x_1 has an additive fault, that is, $y_1(t) = x_1(t) + 0.2sin(t - 10)$ after $t > 10s$, where $x_1(t)$ is the real output, and $y_1(t)$ is the measurement provided by the sensor.

The results for the above three cases are presented in Figure 1, Figure 2, and Figure 3, respectively. After the presence of an actuator fault at $t = 20s$, the second residual in Figure 1 exceeds the threshold at $t = 20.57s$, which means the fault is detected correctly within one second. After the presence of a scaling sensor fault at $t = 5s$, the first residual in Figure 2 exceeds the threshold at $t = 5.01s$, which means the fault is detected correctly very quickly, namely, within $0.02s$. For the additive sensor fault occurring at $t = 10s$, the first residual in Figure 2 exceeds the threshold at $t = 10.03s$, which means the fault is detected correctly within $0.04s$. Moreover, if one would like to have a more solid detection decision, one can wait until both residuals exceed their thresholds. In this way, the fault is detected within $2s$, $1s$, and $4s$ according to Figure 1, Figure 2, and Figure 3, respectively.

7. CONCLUSIONS

Adaptive fault detection problem was studied and solved for a class of MIMO nonlinear systems with unknown parameters. The output estimator design rather than state observer design was used in developing an adaptive fault detection scheme, which can be used to both detectable and undetectable systems. Through decomposing a MIMO system with p outputs into p MISO systems, the difficult fault detection problem for a MIMO system was formulated as a group of simpler fault detection problems

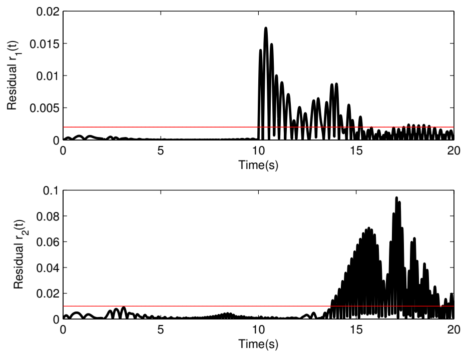

Fig. 3. Adaptive detection of additive sensor faults

for a group of separate MISO systems. This technique leads to more solid fault detection decisions. The proposed adaptive fault detection scheme was tested on a single-link flexible robot manipulator model, and computer simulation results have shown that its satisfactory performance.

Although the paper addressed and successfully solved fault detection problem for the considered class of unknown MIMO systems, fault isolation problem has not been solved yet. Thus, one future research topic is to solve the fault isolation problem for unknown MIMO nonlinear systems.

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