

Discrete-time Output Feedback Sliding Mode Control of a Large Pressurized Heavy Water Reactor

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Abstract: In this paper a novel method is presented to design a sliding mode spatial control for a large Pressurized Heavy Water Reactor (PHWR) using a new formulation of Multirate Output Feedback (MROF). In the new formulation of MROF, the outputs of the system should be some of the states of the system or system should be in special form. The non-linear model of PHWR including xenon and iodine dynamics is characterized by 70 state variables and 14 inputs and outputs each. Linear nodal model is obtained by linearizing the non-linear dynamic equations of the reactor about the full power operating point. The 14 outputs of the PHWR are the power levels in 14 zones, also these are the 14 states of the system. The PHWR model is perfectly suitable for the application of this new formulation in which the states of the system can be computed using reduced system matrix inversion. The PHWR is an ill-conditioned system and the computation of the states of the system using existing MROF require the larger matrix inversions which sometimes may not possible. The proposed approach avoids this difficulty and produces the similar result as it is produced by the existing technique. The proposed control method does not require state information of the system for feedback purposes and hence may be easier to implement. From simulation of the non-linear model of the reactor in representative transients, the proposed control scheme is found to be superior to other methods.

Keywords: Nonlinear system control; Output feedback control; Robust control; Tracking.

1. INTRODUCTION

Fission reactions in a nuclear reactor give rise to several fission fragments and their decay products. Among these substances, ^{135}Xe is considered to be the most important product in operation and control of thermal reactors due to its exceptionally large thermal neutron absorption cross section, see Duderstadt et al. (1976). This isotope is formed to a small extent as a direct product of fission, but a major proportion in a reactor originates from the radioactive decay of ^{135}I with a half life of 6.7 h. The radioactive decay rate of ^{135}Xe is less than that of ^{135}I . An immediate effect of an increase in neutron flux in the reactor is an increase in the rate of consumption of ^{135}Xe , which in turn results into an increase in the neutron flux further. The result is, continued decrease in the xenon concentration and a steady increase in the neutron flux until the delayed production of xenon, by the decay of increasing concentration of iodine brings about an increase in the amount of xenon. This process may continue for about 10 h and then the reverse starts taking place. Because of increased concentration of xenon,

the neutron flux starts decreasing. In this manner, the neutron flux may undergo a slow variation caused due to xenon. However, this does not represent a serious concern in control and operation of a nuclear reactor, as such oscillations in neutron flux are easily controlled by a suitable control rod program.

A serious situation may arise in a large nuclear reactor in which the different regions of the core may undergo variations in neutron flux in opposite phase. If the oscillations in the power distribution are left uncontrolled, the power density and the rate of change of power at some locations in the reactor core may exceed their respective thermal limits, resulting into increased chances of fuel failure. So, in large thermal nuclear reactors, it becomes necessary to employ automatic power distribution control systems, besides the system for control of global power. The objective is to maintain the core power distribution close to a desired shape.

The neutron flux signal and the other states are required for the feedback purposes for many of the conventional spatial control systems designed based on state feedback

principle, refer to Tiwari (1999). In a reactor, in particular, the measurement of xenon and iodine concentrations would be required. This represents a major drawback, as measurement of xenon and iodine concentrations are rather difficult. An alternative approach to overcome this problem is to make use of an observer based design. But, this increases the implementation cost and reduces the reliability of the control system. Hence, it is desirable to go for an output feedback design. The static output feedback is one of the most investigated problems in the control theory and applications, see Syrmos et al. (1997). However, a major limitation of this approach is that the stability of the closed loop system is not guaranteed. The dynamic output feedback controller involves more dynamics and is complex to design. Recently, a new approach known as multirate output feedback, see e.g., Chammas et al. (1979), Hagiwara et al. (1988), Werner (1998), and Werner et al. (1995), has drawn much attention of many researchers. In contrast to the observer based design, where the states converge asymptotically to the actual states, exact computation of states in just one sampling period is feasible if multirate output feedback is employed.

In this paper a new formulation of multirate output feedback, proposed by Bandyopadhyay et al. (2007) is used to compute the states from the outputs and past inputs. The new formulation avoids the higher dimension ill-conditioned system matrix inversions and results identical to those produced by the existing methods are obtained. In Bandyopadhyay et al. (2007) computed states by new approach are used for sliding mode control. In recent years, sliding mode control has attracted the attention of many researchers, see Emelyanov (1967), Furuta et al. (2000), Gao et al. (1993), and Utkin (1977). Research in discrete time sliding mode control has directly been conducive with advancement of digital computers, see e.g., Bartoszewicz (1996), Furuta (1990), and Gao et al. (1995). The sliding mode control considered in Bandyopadhyay et al. (2007) is based on the work of Gao et al. (1995), which has main drawback of chattering. PHWRs employ computer control and therefore discrete-time sliding mode control is particularly suited for implementation. Hence, in this paper the sliding mode control proposed by Bartolini et al. (1995) is considered to obtain spatial control for a large PHWR. The method eliminates chattering unlike the method used in Bandyopadhyay et al. (2007). Also in the formulation by Bandyopadhyay et al. (2007) uncertainty is not considered. This paper presents the new formulation of MROF with uncertainty and application to a PHWR control problem.

The rest of the paper is organized in the following sequence. Section 2 presents the state space representation of nodal core model of the PHWR and Section 3 presents the control approach. The results and discussion are presented in Section 4, followed by the conclusion and the references.

2. PHWR MODEL AND ITS STATE SPACE REPRESENTATION

2.1 Model of PHWR

To design a spatial control system, simplified dynamic equations are obtained usually from neutron diffusion

equations. The simplified model of large PHWR is obtained on the basis of the nodal approach. Fourteen fictitiously divided zones are considered as fourteen small cores, each of which is coupled to its neighbouring zones through neutron diffusion.

The following set of coupled differential equations represents nodal core model of the PHWR :

$$\frac{dP_i}{dt} = -\frac{\alpha_{ii}}{l_i}P_i + \sum_{j=1}^z \frac{\alpha_{ij}}{l_i}P_j + \left(\frac{-K_i(H_i - H_{i0}) - \frac{\bar{\sigma}_{X_i}X_i}{\Sigma_{ai}} - \beta}{l_i} \right) P_i + \lambda C_i, \quad (1)$$

$$\frac{dC_i}{dt} = -\lambda C_i + \frac{\beta}{l_i}P_i, \quad (2)$$

$$\frac{dI_i}{dt} = \gamma_I \Sigma_{f_i} P_i - \lambda_I I_i, \quad (3)$$

$$\frac{dX_i}{dt} = \gamma_X \Sigma_{f_i} P_i + \lambda_I I_i - (\lambda_X + \bar{\sigma}_{X_i} P_i) X_i, \quad (4)$$

$$\frac{dH_i}{dt} = -m_i q_i, \quad (5)$$

$$(i = 1, 2, \dots, z),$$

where P_i indicates the power level, C_i denotes effective one group delayed neutron precursor concentration, I_i denotes iodine concentration, X_i denotes xenon concentration and H_i is the water level of ZCC in the i^{th} zone of the reactor. The description and the values of the several reactor parameters and the steady state values of zonal powers are as given in Tiwari et al. (2000).

2.2 State Space Representation of Nodal Core Model

The set of equations (1)–(5) can be linearized around the steady state operating point and by defining the state, control and output vectors respectively as

$$x = \begin{bmatrix} \frac{\delta P_1}{P_{10}} & \dots & \frac{\delta P_z}{P_{z0}} & \frac{\delta C_1}{C_{10}} & \dots & \frac{\delta C_z}{C_{z0}} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \frac{\delta I_1}{I_{10}} & \dots & \frac{\delta I_z}{I_{z0}} & \frac{\delta X_1}{X_{10}} & \dots & \frac{\delta X_z}{X_{z0}} & \delta H_1 & \dots & \delta H_z \end{bmatrix}^T, \quad (7)$$

$$u = [\delta q_1 \dots \delta q_z]^T, \quad (8)$$

$$y = \begin{bmatrix} \frac{\delta P_1}{P_{10}} & \frac{\delta P_2}{P_{20}} & \dots & \frac{\delta P_z}{P_{z0}} \end{bmatrix}^T, \quad (9)$$

where δ denotes an incremental change, a linear model for the reactor is obtained in standard state space representation as

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Mx. \end{aligned} \quad (9)$$

3. CONTROL APPROACH

Consider the system given by (9). When the effect of perturbations and external disturbances is considered, it can be written as

$$\begin{aligned} \dot{x} &= Ax + Bu + D_c f, \\ y &= Mx = [E_p \ 0]x. \end{aligned} \quad (10)$$

Note that the output equation is in a special form such that the first few states are available directly as outputs. If the system is not in the above form, it is always possible to transform a given system with full row rank output matrix into the form of (10). Let the system be sampled at every τ s and a discrete representation of the system is given as

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k) + D_\tau f(k), \quad (11)$$

$$y(k) = [E_p : 0]x(k) = Mx(k), \quad (12)$$

where, $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$ and $\Phi_\tau, \Gamma_\tau, M$ and D_τ are of suitable dimensions. The disturbance part in (11) is considered to be bounded and satisfies the matching condition given in Gao et al. (1995). Let us assume that the system in (10) is sampled at every $\Delta = \frac{\tau}{N}$ instant, where the constant N is chosen to be greater than or equal to the observability index (ν) of (Φ, M) . Then the following system is obtained

$$x(k+1) = \Phi x(k) + \Gamma u(k) + Df(k), \quad (13)$$

$$y(k) = [E_p : 0]x(k) = Mx(k). \quad (14)$$

The system and input matrices for the τ system, given by (11) and the Δ system, given by (13) have the relation

$$\Phi_\tau = \Phi^N, \Gamma_\tau = \left(\sum_{i=0}^{N-1} \Phi^i \right) \Gamma, D_\tau = \left(\sum_{i=0}^{N-1} \Phi^i \right) D.$$

Now $x(k)$ is computed in the following way, where $f(k)$ and $u(k)$ are held constant during the interval τ

$$\begin{aligned} y(k) &= Mx(k), \\ y(k+\Delta) &= Mx(k+\Delta), \\ &= M\Phi x(k) + M\Gamma u(k) + MDf(k), \\ y(k+2\Delta) &= Mx(k+2\Delta), \\ &= M\Phi^2 x(k) + M\Phi\Gamma u(k) + M\Gamma u(k+\Delta) \\ &\quad + M\Phi Df(k) + MDf(k+\Delta), \\ &\vdots \\ y((k+1)-\Delta) &= M\Phi^{N-1}x(k) + M \sum_{i=0}^{N-2} \Phi^i \Gamma u(k) \\ &\quad + M \sum_{i=0}^{N-2} \Phi^i Df(k). \end{aligned}$$

Then a multirate output feedback representation, see Werner (1998) is given

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k) + D_\tau f(k), \quad (15)$$

$$\mathbf{y}_{k+1} = M_0 x(k) + D_0 u(k) + M_d f(k), \quad (16)$$

where,

$$\mathbf{y}_k = \begin{bmatrix} y(k) \\ y(k+\Delta) \\ y(k+2\Delta) \\ \vdots \\ y((k+1)-\Delta) \end{bmatrix}, \quad (17)$$

$$M_0 = \begin{bmatrix} M \\ M\Phi \\ M\Phi^2 \\ \vdots \\ M\Phi^{N-1} \end{bmatrix}, D_0 = \begin{bmatrix} 0 \\ M\Gamma \\ M\Phi\Gamma + M\Gamma \\ \vdots \\ M \sum_{i=0}^{N-2} \Phi^i \Gamma \end{bmatrix},$$

$$M_d = \begin{bmatrix} 0 \\ MD \\ M\Phi D + MD \\ \vdots \\ M \sum_{i=0}^{N-2} \Phi^i D \end{bmatrix}. \quad (18)$$

From (16), the following can be easily derived.

$$\begin{bmatrix} y_1(k+\Delta) \\ y_2(k+\Delta) \\ \vdots \\ y_p(k+\Delta) \\ \vdots \\ y_1((k+1)-\Delta) \\ y_2((k+1)-\Delta) \\ \vdots \\ y_p((k+1)-\Delta) \end{bmatrix} = \begin{bmatrix} M\Phi \\ M\Phi^2 \\ \vdots \\ M\Phi^{N-1} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$+ \begin{bmatrix} M\Gamma \\ M\Phi\Gamma + M\Gamma \\ \vdots \\ M \sum_{i=0}^{N-2} \Phi^i \Gamma \end{bmatrix} u(k) + \begin{bmatrix} MD \\ M\Phi D + MD \\ \vdots \\ M \sum_{i=0}^{N-2} \Phi^i D \end{bmatrix} f(k).$$

This can be represented in a simplified notation as

$$\bar{\mathbf{y}}_{k+1} = \bar{M}_{01} x_1(k) + \bar{M}_{02} x_2(k) + \bar{D}_0 u(k) + \bar{M}_d f(k), \quad (19)$$

where, $\bar{M}_{01} \in \mathbb{R}^{(N-1)p \times p}, \bar{M}_{02} \in \mathbb{R}^{(N-1)p \times (n-p)}$. Utilizing the special form of M , we have from (19)

$$\bar{\mathbf{y}}_{k+1} = \bar{M}_{01} y(k) + \bar{M}_{02} x_2(k) + \bar{D}_0 u(k) + \bar{M}_d f(k), \quad (20)$$

from which the value of $x_2(k)$ can be obtained in terms of $\bar{\mathbf{y}}_{k+1}$ and $u(k)$ as

$$x_2(k) = (\bar{M}_{02}^T M_{02})^{-1} \bar{M}_{02}^T (\bar{\mathbf{y}}_{k+1} - \bar{M}_{01} y(k) - \bar{D}_0 u(k) - \bar{M}_d f(k)). \quad (21)$$

Now from (11) and as $M = [E_p : 0]$ we have

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = [\Phi_{\tau 1} \quad \Phi_{\tau 2}] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \Gamma_\tau u(k) + D_\tau f(k),$$

$$x(k+1) = \Phi_{\tau 1} y(k) + \Phi_{\tau 2} x_2(k) + \Gamma_\tau u(k) + D_\tau f(k). \quad (22)$$

Where, $\Phi_{\tau 1}$ and $\Phi_{\tau 2}$ are the sub-matrices of Φ_τ of dimensions $n \times p$ and $n \times (n-p)$ respectively. Substituting the value of $x_2(k)$ from (21), it yields

$$\begin{aligned}
 x(k+1) = & (\Phi_{\tau_1} - \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{M}_{01})y(k) \\
 & + \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{y}_{k+1} \\
 & + (\Gamma_{\tau} - \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{D}_0)u(k) \\
 & + (D_{\tau} - \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{M}_d)f(k). \quad (23)
 \end{aligned}$$

This can also be written as

$$x_{k+1} = L_y \mathbf{y}_{k+1} + L_u u(k) + L_d f(k), \quad (24)$$

where,

$$\begin{aligned}
 L_y = & \left[(\Phi_{\tau_1} - \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{M}_{01}) \right. \\
 & \left. \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \right], \quad (25)
 \end{aligned}$$

$$\begin{aligned}
 L_u = & (\Gamma_{\tau} - \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{D}_0), \\
 L_d = & (D_{\tau} - \Phi_{\tau_2}(\overline{M}_{02}^T \overline{M}_{02})^{-1} \overline{M}_{02}^T \overline{M}_d). \quad (26)
 \end{aligned}$$

From (27), the state $x(k)$ can be computed as

$$x_k = L_y \mathbf{y}_k + L_u u(k-1) + L_d f(k-1). \quad (27)$$

Comparing the expressions for L_y , L_u , and L_d with those given elsewhere, e.g., Janardhanan et al. (2007), it is clearly evident that, if the system is in a special form, a lower dimensional matrix inversion is needed to obtain the state vector from output samples measurement. This state can be used for obtaining a sliding mode control based on output samples.

Let $s(k) = c^T x(k)$ be a stable sliding surface for the system (11)-(12). A state feedback sliding mode control can be obtained from the reaching law proposed by Bartolini et al. (1995), i.e.,

$$s(k+1) = 0. \quad (28)$$

Since the state equation of the system consists of uncertain term the above reaching law can't be directly used to obtain the control law. The bounds on the disturbance term can be considered as $d_l \leq \tilde{d}(k) = c^T D_{\tau} f(k) \leq d_u$ also the mean and spread of $\tilde{d}(k)$ can be defined as $d_0 = \frac{d_l + d_u}{2}$, $d_1 = \frac{d_u - d_l}{2}$. The control law can be obtained by ensuring at all instants of time the maximum deviation of the trajectory from the sliding surface is the spread of the disturbance d_1 . Hence, the reaching law can be modified as $s(k+1) = \tilde{d}(k) - d_0$, using this reaching law controller can be obtained as

$$u(k) = -[c^T \Gamma_{\tau}]^{-1} \{c^T \Phi_{\tau} x(k) + d_0\}. \quad (29)$$

From (27), the above state feedback control can be converted into output feedback control, but, (27) also consists of uncertain term which may lead to unknown control. Based on the additional uncertain term, reaching law can be further modified to ensure that the trajectory deviation will be maximum of $d_1 + e_1$ from the sliding surface as $s(k+1) = \tilde{d}(k) + \tilde{e}(k-1) - d_0 - e_0$. Hence, the output feedback control is given by

$$\begin{aligned}
 u(k) = & -(c^T \Gamma_{\tau})^{-1} \{c^T \Phi_{\tau} L_y \mathbf{y}_k + c^T \Phi_{\tau} L_u u(k-1) \\
 & + d_0 + e_0\}, \quad (30)
 \end{aligned}$$

where, $e_0 = \frac{e_l + e_u}{2}$, $e_1 = \frac{e_u - e_l}{2}$ and e_l , e_u are upper and lower bounds on uncertainty term $\tilde{e}(k) = c^T \Phi_{\tau} L_d f(k)$.

Initially, if more control effort is needed and if it is exceeding limits, we can limit the control by assuring the sliding mode motion as given in Bartolini et al. (1995).

4. NON-LINEAR SIMULATION RESULTS

Now the method presented in the Section 3 is applied to obtain a discrete-time output feedback sliding mode control for the large PHWR described in Section 2. For the discretization of the PHWR model the sampling period τ is chosen as 1 s, which is not very small for the implementation of the proposed control scheme. If the sampling period is less than 1 s, the gains of the controller will be more and for the value greater than 1 s, the reduction in the gain values is considerably less. For the implementation of multirate output feedback based algorithm presented in the Section 3, the selection of output sub-intervals is necessary for which it was already stated in Section 3, the number of output subintervals, N , must be greater than or equal to observability index, ν . The nodal model of PHWR has the observability index $\nu = 5$, thus, N is chosen as 5. Then, we have, $\Delta = 0.2$ s. Though more output subintervals can be considered, it is observed that there was no considerable improvement in the response of the closed loop system. We also obtained the model corresponding to reactor operation at 10% about the full power and the corresponding difference in the perturbed system and input matrices with the nominal system and input matrices was used to generate D_c in (10), f is considered as uncertainty. It is assumed that the variation of f is $\pm 15\%$ of system states and input signals.

To show the effectiveness of the control law, different transients are considered. First, consider the power maneuvering transient. Initially, the reactor is under steady state and it is assumed to be operating at 1800 MW with the desired zonal power distribution, refer Tiwari et al. (2000). Iodine, xenon and precursor concentrations are in equilibrium with the respective zonal power levels. Now, the demand is reduced uniformly at the rate of 10 MW/s to 1620 MW, in 18 s and held constant thereafter. During the transient, the variation of global power, zonal power levels and zonal xenon concentrations takes place as shown in Figs. 1, 2 and 3 respectively. Global power and signal to control valve of zone 1 vary during the first 150 s as depicted in Fig. 4 before settling to their respective steady state values of 1620 MW and 0 V respectively. The global power is 1611 MW (0.5498 % less than the demand) approximately at 76 s, and 1640 MW (1.2954 % greater than the demand) at 1578 s. Then, it settles within $\pm 1\%$ of new demand in approximately 3000 s. During the entire course of the transient, the global power is maintained close to demand with maximum error of 1.2954 %. The zonal power levels attains the steady state values within $\pm 1\%$ in about 480 s and xenon concentrations stabilize to their respective new steady state values in about 15 hrs. The variation of control input for zone 1 is within $\pm 1V$. The behavior of the system during the power maneuvering transient is thus, observed to be better than that obtained with periodic output feedback control by Tiwari et al. (2000) and comparable to that reported with fast output sampling by Sharma et al. (2003).

Next, we consider the transient featuring the control of power tilts. It is assumed that the initial values of zonal

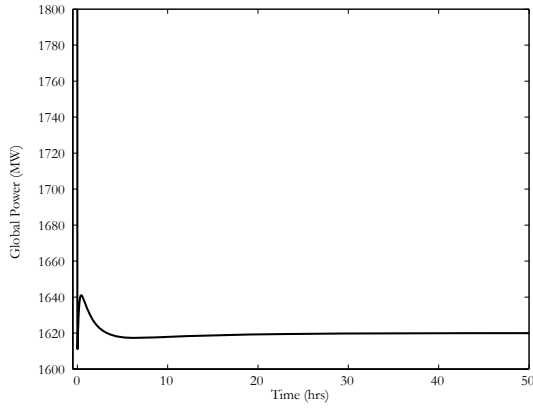


Fig. 1. Global power variation during power maneuvering from 1800 MW to 1620 MW.

power levels are as shown in Table 1, which is different from the desired distribution given in Tiwari et al. (2000). Although the global power is 1800 MW but the power distribution given in Table 1 is characterized by the presence of the axial tilt of about 2 %, side to side tilt of about 2 % and no top to bottom tilt, whereas the desired power distribution is characterized by 2 % top to bottom tilt and no axial and side to side tilts. The initial xenon, iodine and precursor concentrations are assumed to be at their respective steady state values corresponding to the zonal power levels given in Table 3. The deviations of zone 1, zone 2 and zone 8 power levels in time from their respective steady state values occurs as shown in Fig. 5. The axial, side to side and top to bottom tilts are corrected as shown in Fig. 6. During the transient however, slight deviation of global power from its desired level of 1800 MW occurs. This is depicted in Fig. 7. The simulation results obtained are found to be satisfactory. With periodic output feedback, the maximum deviation of the global power from its steady state value reported in Tiwari et al. (2000) is about 1.055 % and that with fast output sampling technique by Sharma et al. (2003), is about 1 % whereas with proposed control technique suggested here, it is 0.445 %. Obviously, the proposed technique is found to be more effective.

Table 1. Zonal power levels assumed for power tilt

Zone No.	Power (as fraction of total power)	Zone No.	Power (as fraction of total power)
1	0.062644	8	0.0852
2	0.069583	9	0.077033
3	0.06835	10	0.0685
4	0.05475	11	0.05475
5	0.072283	12	0.072283
6	0.0852	13	0.0752
7	0.077033	14	0.077033

5. CONCLUSION

A method for design of discrete-time sliding mode control in the presence of matched uncertainty has been arrived at in this paper based on an entirely new approach.

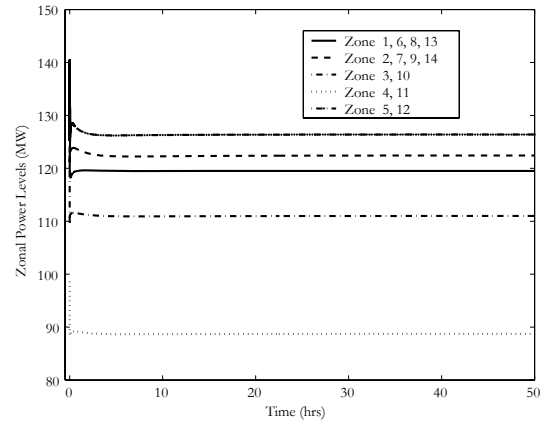


Fig. 2. Variation of zonal power levels during power maneuvering from 1800 MW to 1620 MW.

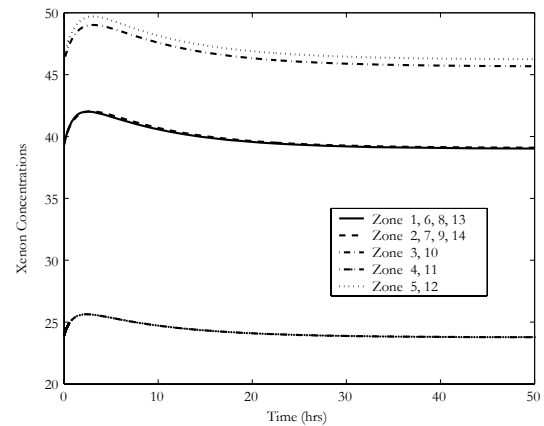


Fig. 3. Variation of xenon concentrations during power maneuvering from 1800 MW to 1620 MW.

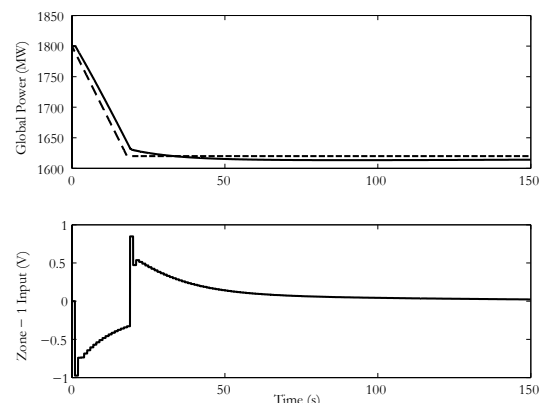


Fig. 4. Variation of global power and control input to zone 1 during power maneuvering (initial few seconds) from 1800 MW to 1620 MW.

Computations require lower order matrix manipulations as the output equation of the system is transformed into a suitable form before proceeding with the design. In addition, the sliding mode control is so designed to overcome the chattering problem.

The method has then been applied to the spatial control problem of a large PHWR. Its effectiveness has been demonstrated by simulations. The controller thus obtained

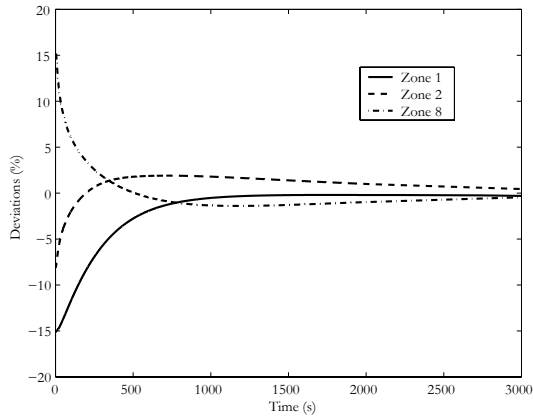


Fig. 5. Percentage deviations of zone 1, 2 and 8 power levels from their respective equilibrium values, during power tilt control.

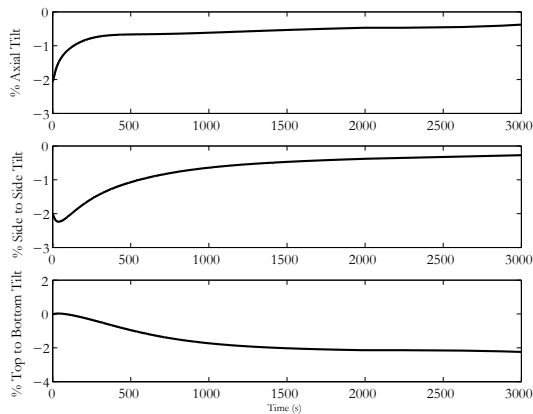


Fig. 6. Variation of axial, side to side and top to bottom tilts during power tilt control.

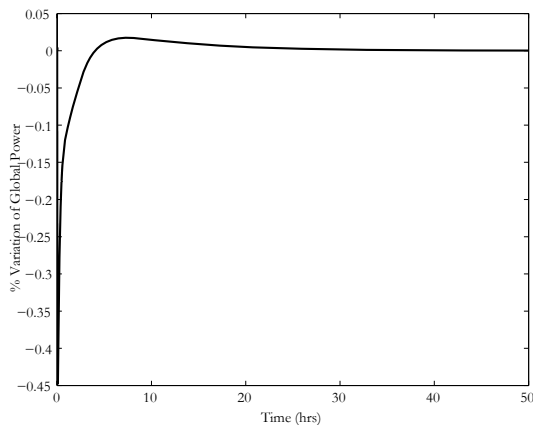


Fig. 7. Percentage variation of global power during power tilt control.

has been found to be superior to controllers based on fast output sampling and periodic output feedback techniques.

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