Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Lipschitz Numbers: A Medium for Delay Estimation

Iman Makaremi*, Alireza Fatehi*, Babak Nadjar Araabi**

**Advanced Process Automation & Control (APAC) Research Group, Faculty of Electrical Eng., K.N. Toosi University of Technology, Tehran, Iran, (e-mail: i.makaremi@ieee.org, fatehi@kntu.ac.ir)*
***Control & Intelligent Processing Center of Excellence, School of Electrical & Computer Eng. University of Tehran, Tehran, Iran, (e-mail: araabi@ut.ac.ir)*

Abstract: The paper deals with problem of estimating input channel delay in nonlinear system with a model-free approach. The proposed method is based on Lipschitz theory. It is an extension to the Lipschitz method which was proposed for determining the order of a model. Our algorithm consists of two parts which in the first one estimation is made on the proper number of dynamics on the input and in the second part the pure delay of the input is obtained. The method is applied for estimation of the delay of two different models and the estimation was as accurate as possible.

1. INTRODUCTION

Nonlinear system identification is a kind of the optimization problems that in many cases suffers from curse of dimension. This problem gets much more complicated if the input channels have delays. There are some methods for estimating input channel delay in literature. Many of them such as step response test (Astrom *et al*, 1995) use simple ideas, but it can be used when it is possible to apply a step input to the plant. Cross correlation (Cryer, 1986) analysis and mutual information (Trappenberg *et al*, 2006) are also used. These methods can indicate where the relevant dynamics of the input lie and by means of that it is possible to estimate the first relevant dynamic, the input channel delay. However these methods are dependent on richness of the data.

Some other approaches identify the input channel delay together with other parameters of the model during the identification. (Ren *et al*, 2005) proposes a recursive algorithm for identification of systems with unknown time delay based on a modified least square method. (Drakunov *et al*, 2006) uses variable structure observers for the same purpose. The proposed algorithms which are based on use of system model as sliding surfaces make it possible to estimate the delay and parameters of the model simultaneously. There are several other proposed methods based on observers for linear systems with delays e.g. (Wang *et al*, 1999), (Darouach, 2001), and (Yao *et al*, 1997). But, all of these methods are applicable for linear systems. None of them deal with a model considering its nonlinear behavior. Also, the current methods which estimate the input channel delay with other model parameters during the identification task are burdensome.

What we concern about is estimating input channel delay in nonlinear systems with a model-free approach without using a manipulated variable. By estimating the input channel delay before identifying other parameters of the model, the size of search space shrinks and the result is more accurate. Hence the identification task gets easier. Although some methods like cross correlation and mutual information has this properties, but we show in the section 4 that they are not always successful.

In this paper we propose a method based on Lipschitz numbers to indicate the input channel delay of nonlinear systems. This method is on based on (He *et al*, 1993) proposed method which is based on Lipschitz numbers and h determines the order of a model. We use the same idea; by adding a second part to their algorithm, we introduced a new method to estimate the input channel delays.

In the next section this method is described briefly. Then in section 3, we explain the delay estimation algorithm and after that, in section 4, the results of implying it on two different models are compared with those of cross correlation and mutual information analysis. Conclusion comes at the end.

2. LIPSCHITZ METHOD

There are some noticeable methods that can determine the order of a system without developing a model. (He *et al*, 1993) presented a very effective method for determining the order of a nonlinear system with its input and output data. This method is based on the Lipschitz theorem which states that every continuous mapping has bounded gradient which can be estimated by the maximum of the gradients at the known points. In the Lipschitz method, some numbers known as 'Lipschitz numbers' are calculated which represent the smoothness of the mapping. These numbers are based on the following quotient:

$$L_{ij}^{n}=\frac{\left|y(i)-y(j)\right|}{\sqrt{\left(u_1(i)-u_1(j)\right)^2+\cdots\left(u_n(i)-u_n(j)\right)^2}} \qquad (1)$$

where y is the output of the system and $u_l=u(t-l)$ are the probable dynamics both on input and output. From L_{ij}^{n}, the Lipschitz number can be calculated as:

$$L^{n}=\left(\prod_{k=1}^{p}\sqrt{n}L^{n}(k)\right)^{\frac{1}{p}} \qquad (2)$$

978-1-1234-7890-2/08/$20.00 © 2008 IFAC

10.3182/20080706-5-KR-1001.2502

where $L^n(k)$ is the $k-$th largest quotient among all L_{ij}^n. The amount of p is about 1 or 2 percent of the amount of data used for the calculations. As long as the denominator of L_{ij}^n does not include the relevant dynamics, L^n has a large value and when all pertinent dynamics are taken in, it diminishes. Afterward, adding irrelevant dynamics does not cause a great decrease in L^n. Therefore the number of relevant dynamics is achieved (Fig. 1).

A known input channel delay is a presumption of using the Lipschitz method (He *et al*, 1993). However, this parameter is unknown in many system identification problems. Therefore, in a case that a system has delays on its inputs, it is impossible to use this method directly.

3. DELAY ESTIMATION METHOD

In this part, we propose our method for estimating the input channel delay. As stated previously, this method is based on the Lipschitz theorem. Specifically it is based on the presumption that the input dynamics are sequential and they have an effect on output after a certain delay. Also it should be noticed that despite the Lipschitz method proposition which uses delayed input(s) and output, just delayed inputs are considered here for calculating Lipschitz numbers. We eschew using delayed output because this assumption will not affect the accuracy of our computations while we are going to determine the input channel delay of the system while it is possible to do it in its conventional way.

In the first step, the Lipschitz method is performed using the input data of the plant. u_l's are only inputs to the plant. With mean of that, the number of dynamics before the break point D_0 is obtained. If the system has input channel delay, then D_0 is not the strict number of dynamics of the model. Therefore, we should eliminate the improper dynamics in order to find the adequate number of them. As far as our concern is to estimate the input channel delay in external dynamic model, the non-relevant dynamics are the first ones in the sequence obtained in the first step. For this purpose, having a D_0-member set of delayed inputs, the calculation is done in a reverse way. It is actually sweeping the inputs from beginning to the end. It means inputs are removed one by one from the beginning and then the Lipschitz numbers are calculated again. For instance for $d-$th input in the subsequent set, the Lipschitz number is calculated with $\{u_d, u_{d+1}, \cdots, u_{D0-1}, u_{D0}\}$ set as below:

$$LB_{ij}^d = \frac{|y(i)-y(j)|}{\sqrt{\left(u_d(i)-u_d(j)\right)^2+\cdots\left(u_{D_0}(i)-u_{D_0}(j)\right)^2}} \tag{3}$$

and

$$LB^d = \left(\prod_{k=1}^{p}\sqrt{(D_0-d+1)}\,LB^d(k)\right)^{\frac{1}{p}}. \tag{4}$$

Based on the Lipschitz idea, if a delayed input is not germane to the model, then removing it will not affect the quantity of the Lipschitz number greatly. But if one of the relevant dynamics of the model is removed, this amount increases to a great extent. As a result, by indicating the first sudden increase in LB^d s, simply the delay of the system can be revealed.

4. ILLUSTRATIVE EXAMPLES

To study the capability of this method, we exploit it to estimate the input channel delay in two different examples.

4.1- Model 1

The first model to study is represented as below:

$$\dot{y}(t) = -y(t)-y^3(t)+\frac{1}{9}x^2(t-4.2)-\sin\left(x(t-4.9)\right) \tag{7}$$

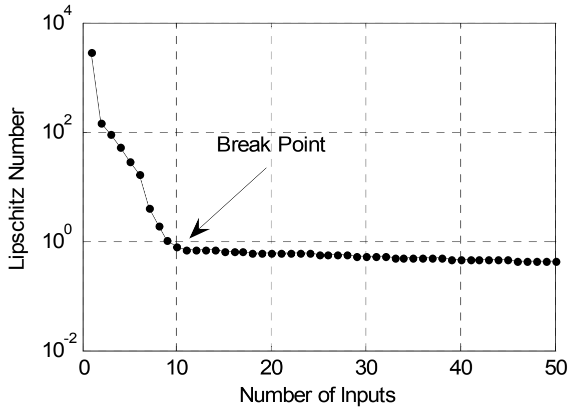

Fig. 1. The place where Lipschitz numbers do not decrease extremely shows that all relevant dynamics are included.

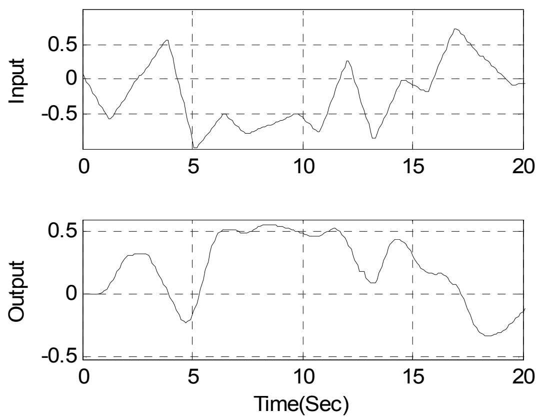

Fig. 2. Input and output of the first model. The input is discrepant slopes and random time interval for each of them.

The model has two dynamics on its input at delays 5.2 and 5.9 (It has one sample inherent delay due to implementation using sample time equal to one). The model is fed with discrepant slopes and random time interval for each of them and the input and output are sampled with the rate of one sample per second (See Fig.2). Fig.3 shows the Lipschitz numbers for the first 30 samples of the input. It shows a horizon of $D_0 = 20$ dynamics. This time is approximately the settling time of the plant dynamics. Choosing this number of dynamics as the appropriate amount for performing the second part, we obtained what is shown in Fig.4. As it was mentioned formerly, the place that a sudden jump occurs shows the first germane dynamic of the model is removed. The first jump took place at the sixth delay. It means that the last removed sample had an effect on the output. Hence, it can be inferred that the input channel delay is five seconds due to the sampling time.

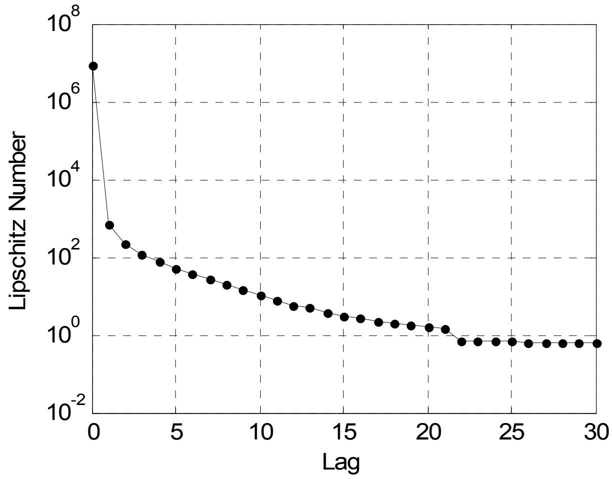

Fig. 3. Indices for the first 30 delays of the input of Model 1.

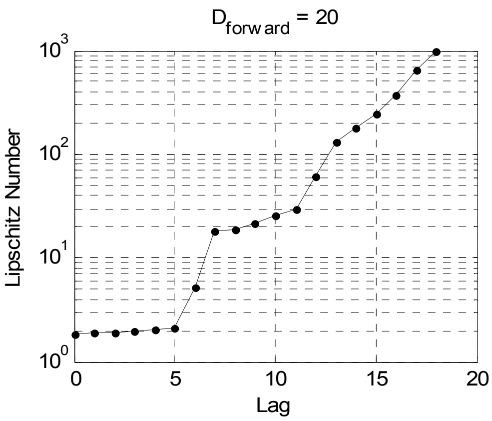

Fig. 4. The result of performing the delay estimation method on Model 1. As it's clear, the first jump occurs while the fifth delay of the input is removed.

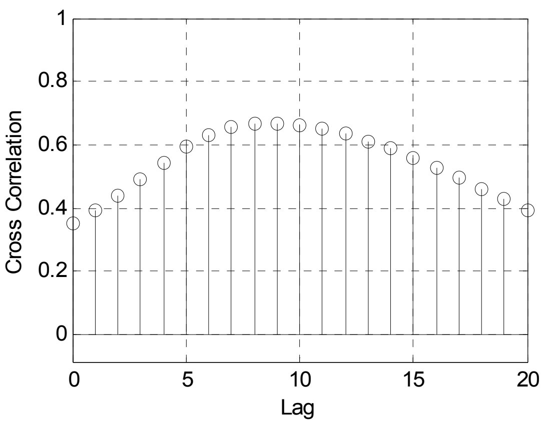

Fig. 5. The result of the Cross Correlation Analysis on Model 1. This analysis does not show the pertinent delays as well.

To compare this method with two conventional methods for estimating the appropriate dynamics of the system, we also check the outcome of cross correlation and mutual information analysis. Figs 5 and 6 respectively show their results. They show that cross correlation analysis is not successful to give an acceptable answer. Also mutual information analysis does not obtain the exact dynamics.

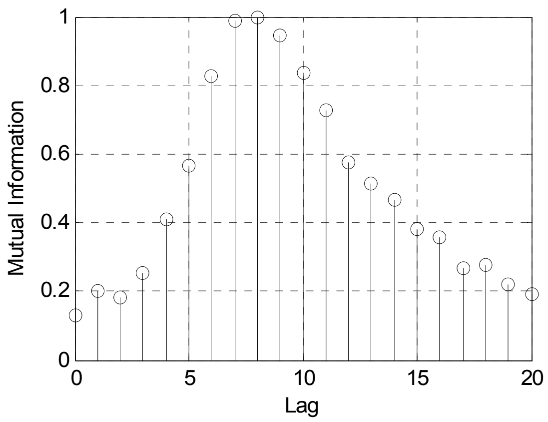

Fig. 6. The result of Mutual Information test on Model 1. It does not obtain the exact dynamics.

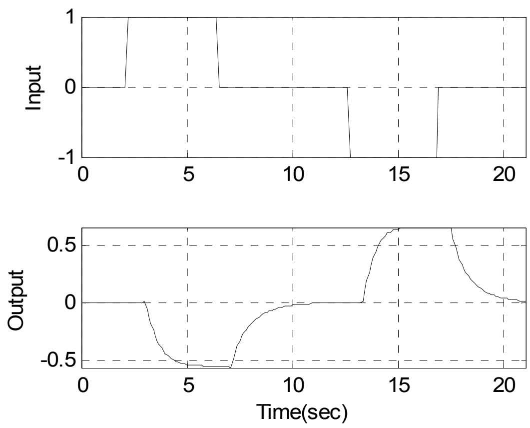

Fig. 7. The input and output of the first model. The input is two pulses.

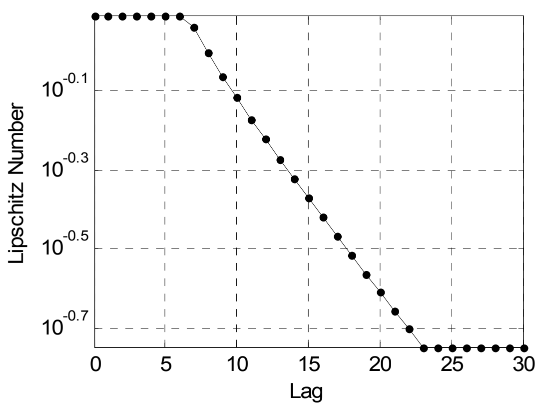

Fig. 8. Indices for the first 30 delays of the input of Model 1. The pulse input.

To study the effect of the input shape on the outcome of this method, a pulse-like input is applied to the same model (See Fig 7) and the these methods are exploited. The results are shown in Figs 8 through 11. The proposed method shows that the first relevant input is at five. But Figs 10 and 11 illustrate that cross correlation and mutual information do not reveal any information about them. It can be inferred that the results obtained by cross correlation and mutual information get unreliable when the distribution of input and output data are not rich enough. But the proposed method is not that dependent on this issue as it benefits from gradient of the model. However, it is recommended to use signals with more variation in order to be sure that the consequence is surely trustworthy.

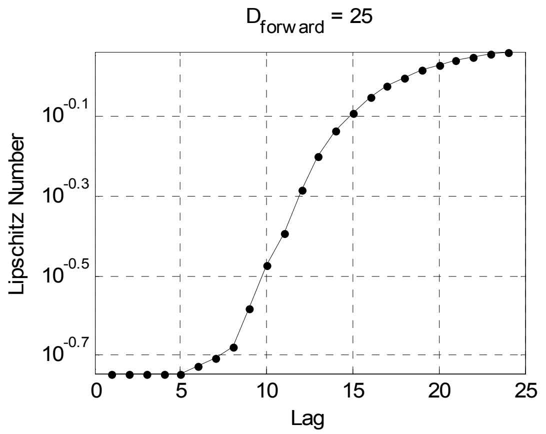

Fig. 9. The result of performing the delay estimation method on Model 1 with the pulse input. The first jump occurs while the fifth delay of the input is removed.

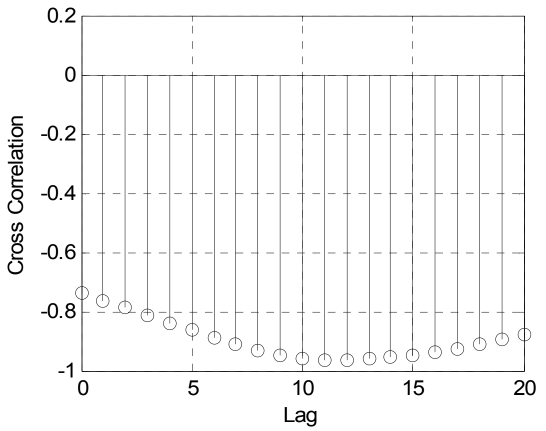

Fig. 10. The result of the Cross Correlation Analysis on Model 1with the pulse input. This analysis does not show the pertinent delays at all.

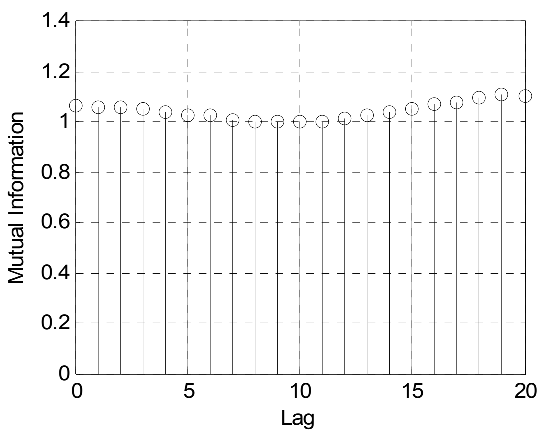

Fig. 11. The result of Mutual Information test on Model 1 with the pulse input. Nothing can be inferred from it.

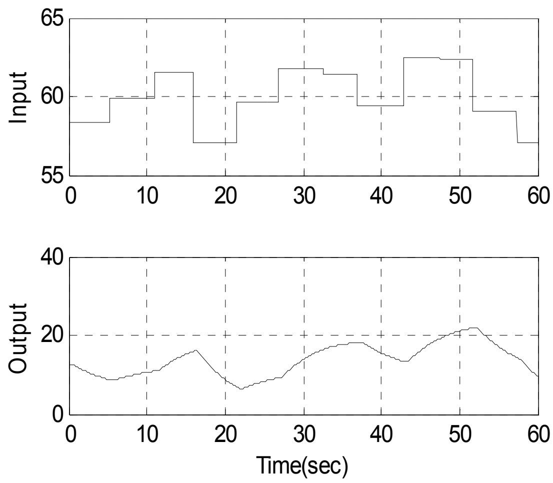

Fig. 12. Input and output shape of the Model 2 when the outlet valve is 40% open.

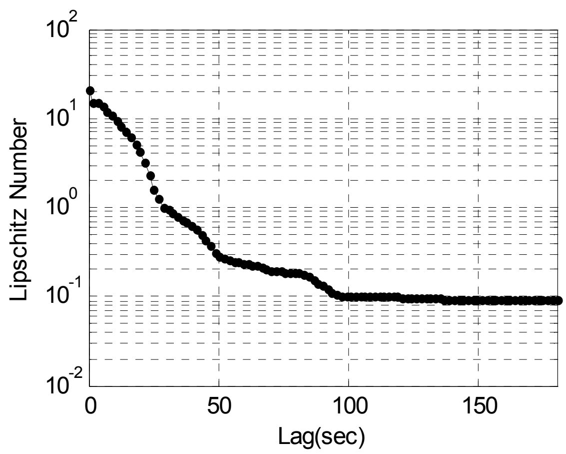

Fig. 13. The Lipschitz indices for the input channel delays in Model 5 while the outlet valve is at 40%.

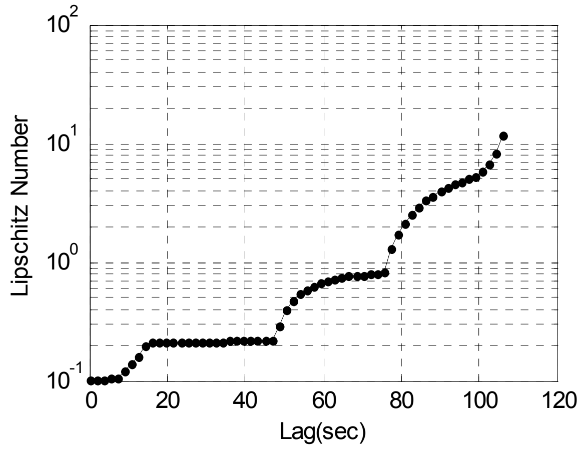

Fig. 14. The result of performing the delay estimation method on Model 2 while the outlet valve is at 40%. The first jump is occurred when the dynamic related to the 7.2 second lagged input is plucked out.

4.2- Model 2

The model of a pilot level control process is used in this part. It is an academic set-up in Process Lab at K.N. Toosi Univ. of

Tech[1]. This process has been simulated in two parallel projects, (Jalili, 2005) and (Maghoul, 2005). In this paper, we use the proposed model in (Maghoul, 2005).

This system has a congenial delay about five seconds. But the specific character of this model is that for different outlet valve position, the time delay of the system varies. We used the proposed method on this model for two common percentages of the outlet valve, which are 40 and 50 degrees. The input channel delay is about 7 and 5.2 seconds when the output valve position is respectively at 40 and 50 due to the change of speed of water flow.

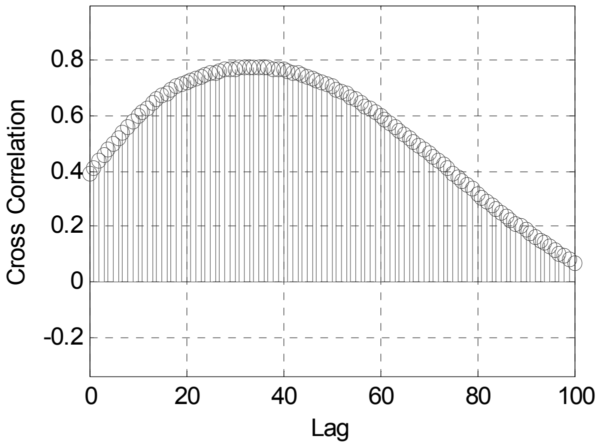

Fig. 15. The result of Cross Correlation Analysis on Model 2 while the outlet valve is at 40%. It wrongly shows that that the relevant dynamics of the system is located around 30 seconds.

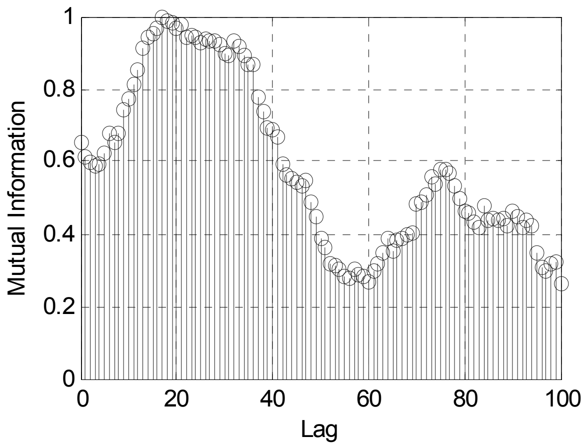

Fig. 16. The result of Mutual Information test on Model 2 while the outlet valve is at 40%. It wrongly shows that that the relevant dynamics of the system is located around 20 seconds.

We input the model of the plant with a stair-like input when the outlet valve position was 40, shown in Fig. 12. The output of the plant is also illustrated in the same figure. Then we used our proposed method and the conventional analysis to compute the delay of the system and the range that relevant dynamics lie. The sampling time is 0.2 second. Fig. 13 shows the Lipschitz numbers and Fig. 14 demonstrates the outcome of the proposed method. The horizontal axis represents the lagged input in seconds. As it is shown, the first jump is occurred when the dynamic related to the 7.2 second lagged input is plucked out. But cross correlation and mutual information analysis did thoroughly wrong (Figs 15-16). Cross correlation shows that the relevant dynamics of the system is located around 30 to 40 seconds and from mutual information analysis it's obtained that they are about 20 second.

We checked out the other working condition of the tank. Fig 17 shows the input and output of the plant when the outlet valve is fixed at 50. Figs 18-19 illustrate the consequents of our delay estimation method. With means of that, the proposed method estimated the input channel delay at 5.4 second which is close to the actual delay, 5.2 second. In comparison with what two other analyses bring to us (Figs 20-21) it is revealed that the new method estimations are much more accurate. Cross correlation and mutual information depict the relevant dynamics around 30 second and 20 second respectively.

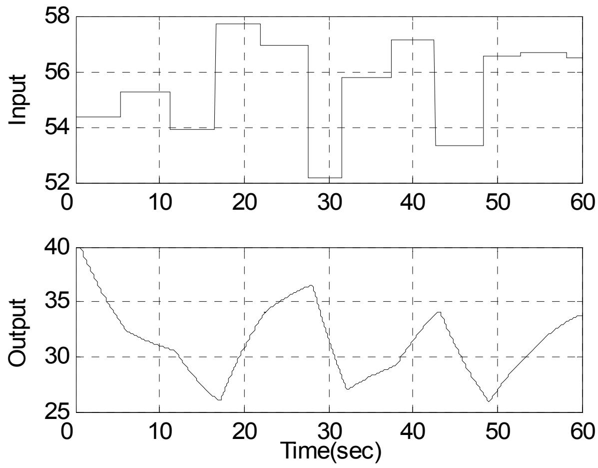

Fig. 17. Input and output shape of the Model 2 when the outlet valve is 50% open.

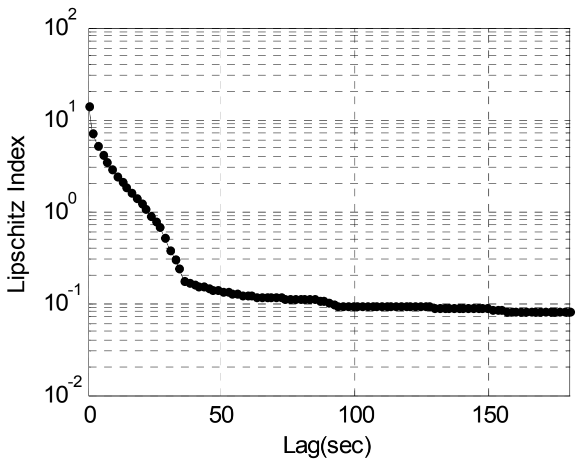

Fig. 18. The Lipschitz indices for the input channel delays in Model 2 while the outlet valve is at 50%.

5. CONCLUSION

In this paper, we proposed a novel approach based on Lipschitz numbers to estimate the input channel delay. This method benefits from the concept that every continuous map has bounded gradient. In a two-step algorithm, the input

[1] Homepage of the process lab at KN Toosi Univ. of Tech., http://saba.kntu.ac.ir/eecd/pcl

channel delay can be estimated without developing a model for the system. Knowing the delay, one is extricated from an exhaustive search to estimate delay with other parameters during the identification task. In a comparison with two conventional methods which are useful to approximate the input channel delay and dynamics, our method surpasses them with its very accurate estimations.

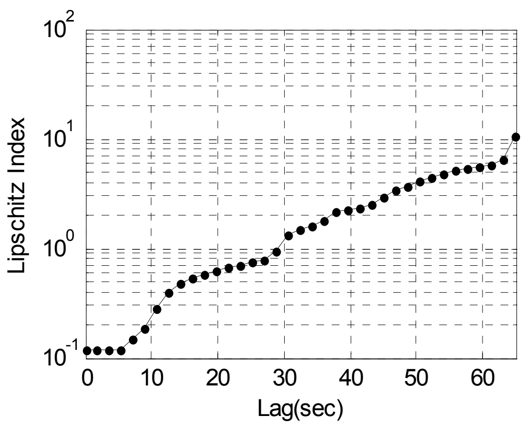

Fig. 19. The result of performing the delay estimation method on Model 2 while the outlet valve is at 50%. The first jump is occurred when the dynamic related to the 5.4 second lagged input is plucked out.

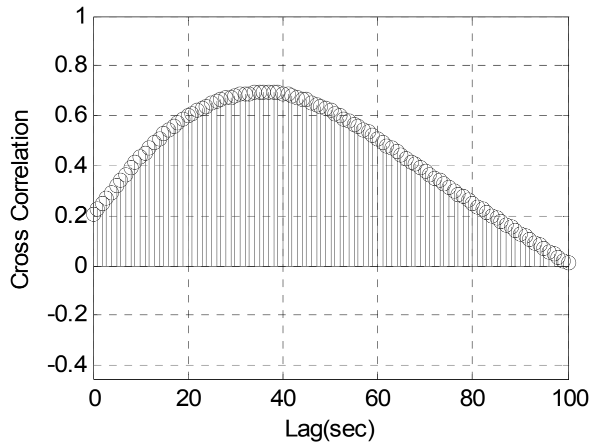

Fig. 20. The result of Cross Correlation Analysis on Model 2 while the outlet valve is at 50%. It wrongly shows that that the relevant dynamics of the system is located around 30 to 40 seconds.

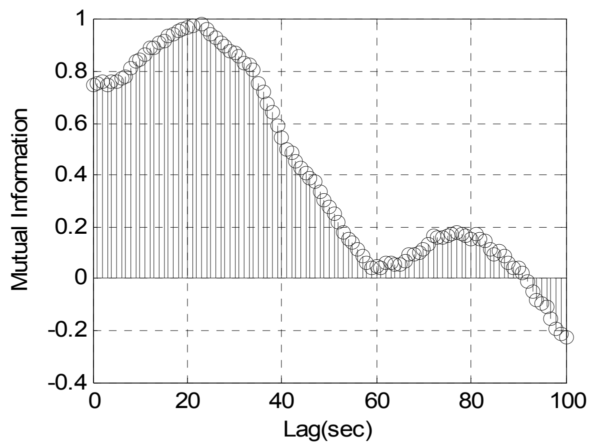

Fig. 21. The result of Mutual Information test on Model 2 while the outlet valve is at 50%. It wrongly shows that that the relevant dynamics of the system is located around 20 seconds.

REFERENCES

Astrom, K.J., and Hagglund. H.(1995), *PID controllers: Theory, design, and tuning*, 2nd ed. Research Triangle Park, NC: Instrument Society of America.

Cryer, J.(1986), *Time series analysis, Cross Correlation*, Wadsworth Pub. Co.

Drakunov, S. V., Perruquetti, W., Richard, J.-P., Belkoura, L. (2006), *Delay identification in time-delay systems using variable structure observers*, Annual Reviews in Control., **Vol. 30**, 143-158.

Darouach, M. (2001), *Linear functional observers for systems with delays in state variables*, IEEE Trans. Automat. Control, **Vol. 46**, issue 3, 491–496.

He, X., and Asada, H. (1993), *A new method for identifying orders of input-output models for nonlinear dynamic systems*, Proc. ACC, 2520–2524.

Jalali. J.(2005), *Designing and applying an adaptive controller for pilot level control process*, B.Sc. Project, Control Department, Faculty of Electrical Engineering, K.N. Toosi Univ. of Tech., Tehran, Iran.

Maghoul P.(2005), *Physical identification of pilot level control process and flow control process and design and apply a MIMO controller*, M.Sc. Thesis, Control Department, Faculty of Electrical Engineering, K.N. Toosi Univ. of Tech., Tehran, Iran.

Ren, X. M., Rad, A. B., Chan, P. T., Lo, W. L.(2005), *Online identification of continuous-time systems with unknown time delay*, IEEE Trans. Automat. Control., **Vol. 50**, issue 9, 1418-1422.

Trappenberg, T., Ouyang, J., and Back, A. (2006), *Input variable selection: mutual information and linear mixing measures*, IEEE Trans. Knowledge & Data Eng., **Vol. 18**, issue 1, 37-46.

Wang, Z., Huang, B., and Unbehausen, H. (1999), *Robust H1 observer design of linear state delayed systems with parametric uncertainty: The discrete-time case*, Automatica., **Vol. 35**, issue 6, 1161–1167.

Yao, Y. X., Zhang, Y. M , and Kovacevic, R. (1997), *Functional observer and state feedback for input time-delay systems*, Int. J. of Control, **Vol. 66**, issue 4, 603–617.