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Fault Detection System Design for Networked Control System with Stochastically Varying Transmission Delays

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Abstract: This paper studies the observer based fault detection problem in networked control systems. It is assumed that the transmission delays caused by the communication network are varying and their probability distribution is unknown but knowledge about its stochastic characteristics is known *a priori*. First the dynamic modelling of a proposed networked control system under different sampling frequencies using the packet-oriented data transmission method is derived. Based on this developed system model, an approach for fault detection system is presented. Main attention is paid to the residual evaluation task in order to reduce false alarm rate caused by the uncertainties due to the effect of the communication network. A study based on simulation platform to verify the results is given. *Copyright* ©2008 IFAC

Keywords: Fault detection, networked control system, statistical evaluation, stochastic varying delays, packet-oriented data transmission, false alarm rate.

1. INTRODUCTION

NCS are in general composed of a large number of interconnected devices (system nodes) that exchange data through communication networks. As all engineering systems are subject to malfunction due to components fault, their sensitivity to failures with the higher system complexity is increased consequently. This demands for more attention to the reliability and safety of networked control system. The study of Fault detection (FD) in NCS is recently a new research topic, which gained more attention in the past years. Different problems aiming an optimal design of FD in NCS has been studied, for instance, by Persis [2003], Zhang et al. [2004], Ye et al. [2004], and Ding et al. [2007].

The objective of this paper is to study the problems related to the design of observer based networked fault detection system (NFDS) with stochastically varying transmission delays over the communication network. Only *a priori* knowledge of stochastic characteristics of the delays are known. Their probability distribution is assumed to be unknown. Therefore, due to the randomness and poor information about the stochastic properties of delays, design of residual generator (to minimize their effects on the residual dynamic) will be a difficult task, i.e. using a discrete time Markovian jump technique as studied in Zhong et al. [2005]. To overcome this problem, the handling of uncertainties caused by these delays will be taken into account in the residual evaluation.

Additional important motivation of this study is, the observation that the FD performance in NFDS is considerably related to the variance of the data transmission delays. In this respect, to ensure a low false alarm rate (FAR), not only the mean value of the delays but also their variance have to be taken into account by the residual evaluation and threshold determination. Thus the major focus is paid to design an evaluation strategy for the residual signal. Additionally, attention is paid to the asynchronization working mode of the operated systems. To synchronaize the information exchange between them, a packet-oriented data transfer strategy is applied.

The paper will be organized as follows. The problem formulation is given in section 2. System description and modelling of the NCS is stated in section 3. Then, in section 4 the design of a model based NFDS is given. Section 5 includes the results of simulation study.

2. PROBLEM FORMULATION

The NCS structure considered in this study shown in Fig.1 includes:

- a local station (controlled process),
- a central supervision station (FD System),
- a communication network (Ethernet).

The controlled process consists of a plant, equipped with a number of sensors and actuators as well as a local controller implemented on a microprocessor with limited computing power and a network interface. The central supervision station in which the FD system is implemented, is located remotely from the controlled process. Due to the embedded structure of the controller, the local station will have a limited computing power than the supervision station. For the purpose of data transmission between the controlled process and the supervision station, Ethernet is adopted in this study. In Ethernet the transmission delay is non-deterministic and depends strongly on the network load. For our purpose of detecting possible faults in the plant, in the sensors and in the actuators, the following problems will be studied

- modelling of the NCS structure under consideration as illustrated in Fig.1 as an LTI with stochastic uncertainty and with $\mu(k), \upsilon(k)$ as input and output signals, respectively.
- design a model based NFDS. The main focus has to be paid on the residual evaluation and threshold determination.

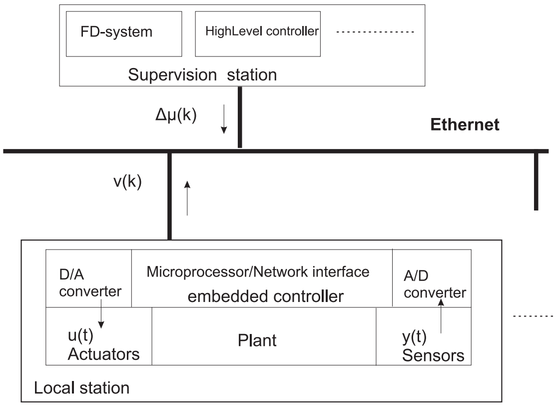

Fig. 1. NCS structure under consideration

3. NCS MODEL

Modelling of the information structure between the central station and the local station will be derived by taking into account the impact of the variable delay (as a main effect of the communication network) as well as the different working frequencies of both stations on the system dynamic.

3.1 Technical features of proposed NCS structure

Suppose that the plant with the embedded sensors, actuators and the local controller is described by

$$\dot{x}(t) = Ax(t) + Bu(t) + E_x d(t) + F_x f(t) \tag{1}$$
$$y(t) = Cx(t) + F_y f(t) \tag{2}$$

where $x \in R^n, u \in R^p, y \in R^m, d \in R^q$ and $f \in R^{k_f}$ denote the state, the input, the output, the unknown vector and the fault vector, respectively. Matrices $A \in R^{n\times n}$, $B \in R^{n\times p}$,$C \in R^{m\times n}$, $E_x \in R^{n\times q}$, $F_x \in R^{n\times k_f}$ and $F_y \in R^{m\times k_f}$ are known and of appropriate dimensions. The model of the A/D converter is described by

$$y(k) = y(kT_s) \tag{3}$$

The D/A converter can be modelled as

$$u(t) = u(k), \quad kT_s \le t < (k+1)T_s \tag{4}$$

where T_s denotes the sampling time of the local system (controlled process). Considering that the controlled process can be a part of a distributed system and the supervision station may be connected to additional operating units or subsystems by the Ethernet. It is assumed that:

- the communication between the controlled process and the supervision station is periodic and coordinated by the supervision station,
- the period time, denoted by T is considerably larger than the sampling times of the A/D- and D/A-converters embedded in the controlled process.

The data transmission strategy adopted in this study is described as follows:

- at time instant $t = kT$, the supervision station sends a reference signal $\Delta\mu(k)$ to the controlled process via Ethernet. The controlled process builds $\mu(k) = \mu(k-1) + \Delta\mu(k)$ and saves it,
- at time instant $t = kT$, the controlled process sends a packet of sensor data collected during the time interval $((k-1)T, kT]$.

Due to the transmission delay over Ethernet denoted by τ, $\Delta\mu(k)$ will arrive in the control process at $t = kT + \tau$, which gives

$$u(kT + \tau) = \mu(k-1) + \Delta\mu(k), \Delta\mu(k) = \Delta\mu(kT)$$

In this study, it is assumed that:

- τ is a random variable which can be written into
$$\tau = E(\tau) + \tau_\Delta = \bar{\tau} + \tau_\Delta \tag{5}$$
where $E(\tau) = \bar{\tau}$ is the mean value of τ and is known, τ_Δ is a random variable with poor statistical information (probability distribution unknown) but its mean value and *upper bound* of variance are known and constant,
$$E(\tau_\Delta) = 0, var(\tau_\Delta) = \sigma_\tau^2 \tag{6}$$
- $\tau < T$.

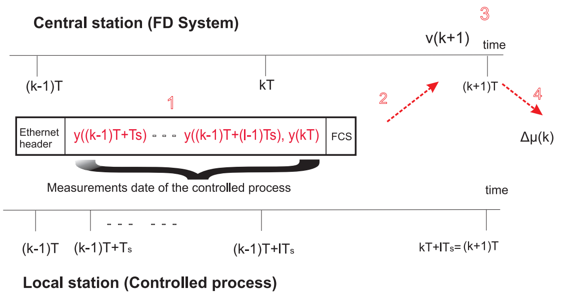

Fig. 2. Packet-oriented data transfer between a local station and central supervision station

Assume that the data packet sent by the controlled process to the supervision station at time instant $t = kT$ includes sensor data $y((k-1)T+T_s), \cdots, y((k-1)T+(l-1)T_s), y((k-1)T+lT_s) = y(kT)$, where l is a (constant) integer and denotes the length of the data buffer where $T = lT_s$. As a result, the supervision station will receive the data packet at time instant $t = kT + \tau$ with τ denoting the transmission delay. On the assumption that $\tau < T$, at time instant $t = (k+1)T$, the data vector

$$\upsilon(k+1) = col\,\{\, y((k-1)T + T_s) \;\ldots\; y(kT)\,\} \tag{7}$$

is available at the supervision station, as described in Fig. 2. This means that the local station will collect the output signals (measurements) during the working period T of the supervision station and send them through Ethernet packet at the start of each T. The idea behind is one side to synchronize the information exchange between both systems and on the other side to reduce the data traffic in Ethernet.

3.2 NCS model

Discretizing plant model (1) with sampling time T_s and with the assumption that the mean delay of the communication network $\bar{\tau}$ approximately equal to αT_s, $1 \le \alpha \le l$, i.e. $\bar{\tau} = \alpha T_s$. It yields, after a straightforward computation the following NCS model

$$\begin{aligned} x(k+1) &= A_{dc}x(k) + \bar{B}_{d12}\bar{\mu}(k-1) + \bar{B}_{d2}\Delta\bar{\mu}(k) \\ &\quad + \Delta\bar{B}\Delta\bar{\mu}(k) + \bar{E}_{xd}d^c(k) + \bar{F}_{xd}f^c(k) \qquad (8) \\ \upsilon(k+1) &= \bar{C}_d x(k-1) + \bar{D}_\mu\bar{\mu}(k-2) + \bar{E}_y^c\bar{d}(k) \\ &\quad + \left(\bar{D}_d + \Delta\bar{D}\right)\Delta\bar{\mu}(k-1) + \bar{F}_y\bar{f}^c(k) \qquad (9) \end{aligned}$$

where

$$\begin{aligned} \bar{\mu}(k-1) &= col\,\{\,\mu((k-1)T)\ \ldots\ \mu((k-1)T+(l-1)T_s))\,\} \\ \Delta\bar{\mu}(k) &= col\,\{\,\Delta\mu_1(kT)\ \ldots\ \Delta\mu_l(kT)\,\} \\ \bar{d}^c(k) &= col\,\{\,d^s(kT)\ \ldots\ d^s(kT+(l-1)T_s)\,\} \\ \bar{f}^c(k) &= col\,\{\,f^s(kT)\ \ldots\ f^s(kT+(l-1)T_s)\,\} \\ \upsilon(k+1) &= col\,\{\,y((k-1)T+T_s)\ \ldots\ y(kT)\,\} \\ \bar{\mu}(k-2) &= col\,\{\,\mu((k-2)T)\ \ldots\ \mu((k-2)T+(l-1)T_s)\,\} \\ A_{dc} &= A_{ds}^l,\ \bar{B}_{d12} = \bar{B}_{d1} + \bar{B}_{d2} \\ \bar{B}_{d12} &= \left[A_{ds}^{l-1}B_{d12}\quad A_{ds}^{l-2}B_{d12}\ldots B_{d12}\right] \\ \bar{B}_{d1} &= \left[A_{ds}^{l-1}B_{d1}\ldots A_{ds}^{l-\alpha}B_{d1}\ 0_{n\times((l-\alpha-1)\times p)}\right] \\ \bar{B}_{d2} &= \left[0_{n\times((l-\alpha)\times p)}\ A_{ds}^{l-\alpha-1}B_{d2}\ldots B_{d2}\right] \\ \Delta\bar{B} &\approx \left[0_{n\times((l-\alpha)\times p)}\ A_{ds}^{l-\alpha-1}B\ldots B\right]e^{A(T-\bar{\tau})}\tau_\Delta \\ \bar{E}_{xd} &= \left[A_{ds}^{l-1}E_{xdd}\quad A_{ds}^{l-2}E_{xdd}\ldots E_{xdd}\right] \\ \bar{F}_{xd} &= \left[A_{ds}^{l-1}F_{xdd}\quad A_{ds}^{l-2}F_{xdd}\ldots F_{xdd}\right] \end{aligned}$$

$$B_{d12} = B_{d1} + B_{d2},\ B_{d1} = \int_0^{\bar{\tau}} e^{A(T-t)}B\,dt$$

$$B_{d2} = \int_{\bar{\tau}}^{T} e^{A(T-t)}B\,dt,\ E_{xdd} = \int_0^{T_s} e^{At}E_xE_x^Te^{A^Tt}dt$$

$$\bar{C}_d = col\,\{\,(CA_{ds})\ (CA_{ds}^2)\ \ldots\ (CA_{ds}^l)\,\},\ \bar{D}_d = \bar{D}_{\mu 2}$$

$$F_{xdd} = \int_0^{T_s} e^{At}F_xF_x^Te^{A^Tt}dt,\ \bar{D}_\mu = \left[\bar{D}_{\mu 1} + \bar{D}_{\mu 2}\right]$$

$$\bar{D}_{\mu_1} = \begin{bmatrix} CB_{d1} & 0 & \ldots & 0 \\ CA_{ds}B_{d1} & CB_{d1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA_{ds}^{l-1}B_{d1} & CA_{ds}^{l-2}B_{d1} & \ldots & CB_{d1} \end{bmatrix}$$

$$\bar{F}_y = \begin{bmatrix} C\bar{F}_{xdd} & 0 & \ldots & 0 \\ CA_{ds}F_{xdd} & C\bar{F}_{xdd} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA_{ds}^{l-1}F_{xdd} & CA_{ds}^{l-2}F_{xdd} & \ldots & C\bar{F}_{xdd} \end{bmatrix}$$

$$\bar{F}_{xdd} = F_{xdd} + F_y$$

$$\Delta\bar{D} = -\begin{bmatrix} Ce^{A(T_s-\bar{\tau})}B\tau_\Delta & \ldots & 0 \\ 0 & \ldots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \ldots & Ce^{(lT_s-\bar{\tau})}B\tau_\Delta \end{bmatrix}$$

with $(\bar{B}_{d12}, \bar{B}_{d1}, \bar{B}_{d2}, \Delta\bar{B}) \in R^{n\times(l\times p)}$, $\bar{E}_{xd} \in R^{n\times(l\times q)}$, $\bar{F}_{xd} \in R^{n\times(l\times k_f)}$, $\bar{D}_{\mu_1} \in R^{m\times(l\times p)}$, $\Delta\bar{D} \in R^{m\times(l\times p)}$ and $\bar{F}_y \in R^{m\times(l\times k_f)}$. The matrices $\bar{D}_{\mu 2} \in R^{m\times(l\times q)}$ and $\bar{E}_y \in R^{m\times(l\times k_f)}$ are equal to $\bar{D}_{\mu 1}$ by replacing B_{d1} with B_{d12} and E_{xdd}, respectively. In summary, (8),(9) describe the dynamics of the overall system (central station and local station). The sampling time of the system is T. The packet-oriented data transfer between the local station and central supervision station is considered in the modelling. As a result, the NCS model is represented by a linear time uncertain system. The uncertainties $\Delta\bar{B}, \Delta\bar{D}$ are caused from the network-induced delay jitter τ_Δ. The parameters $\bar{B}_{d12}, \bar{B}_{d2}, \bar{D}_{mu}$ and $\bar{D}_d$ are influenced by the mean delay value $\bar{\tau}$.

4. MODEL BASED NETWORKED FD SYSTEM DESIGN

An NFDS consists of a residual generator and residual evaluator placed in the central supervision station. First, a statistical properties of the residual dynamic will be discussed. After that a residual generator will be designed. A statistical based evaluation of the residual signal is provided.

4.1 Residual generation

Under the following assumptions

A1. $\left(\bar{C}_d, A_{dc}\right)$ is detectable,

A2. $rank\begin{bmatrix} A_{dc} - e^{j\theta}I & \bar{E}_{xd} \\ \bar{C}_d & \bar{E}_y \end{bmatrix} = n + (l\times m)$ for all $\theta \in [0, 2\pi)$,

and based on the model in (8)-(9), the following observer based standard residual generator is given :

$$\begin{aligned} \hat{x}(k+1) &= A_{dc}\hat{x}(k) + \left(\bar{B}_{d12}\quad \bar{B}_{d2}\right)\begin{pmatrix} \bar{\mu}(k-1) \\ \Delta\bar{\mu}(k) \end{pmatrix} \\ &\quad + L\left(\zeta(k+1) - \hat{\zeta}(k+1)\right) \qquad (10) \\ \hat{\upsilon}(k+1) &= \bar{C}_d\hat{x}(k-1) + \bar{D}_\mu\bar{\mu}(k-2) + \bar{D}_d\Delta\bar{\mu}(k-1) \qquad (11) \\ \zeta(k) &= \upsilon(k+1),\ \eta(k) = \zeta(k+1) - \hat{\zeta}(k+1) \\ r(k+1) &= V\eta(k) \qquad (12) \end{aligned}$$

where L, V are the observer gain and postfilter, $\eta(k)$ is the output estimation error and $r(k)$ is a residual signal. It is interesting to notice that the control input vector does not to be transferred to the central station in order to construct the residual generator. It can be reconstructed using the process measurements (outputs) assuming that the local control law is known in the central station. This fact reveals that we can transmit only the process outputs for the FD in the central station. This leads to reduce the load of the network by means of less data transfer.

Statistical properties on residual dynamic: Due to parameter uncertainties, the residual generator depends on the dynamics of the plant under observation:

$$x_r = \begin{pmatrix} x(k) \\ e(k) \end{pmatrix} \qquad (13)$$

where the estimation error $e(k)$is defined as $e(k) = x(k) - \hat{x}(k)$. The overall system dynamic, which includes the plant and the residual generator can be expressed as

$$x_r(k+1) = A_r x_r(k) + B_r\widetilde{\mu}(k) + \bar{E}_{rx}\bar{d}^c(k) + \bar{F}_{rx}\bar{f}^c(k) \quad (14)$$

$$r(k+1) = C_r x_r(k) + D_r\widetilde{\mu}(k) + E_{ry}\bar{d}^c(k) + F_{ry}\bar{f}^c(k) \quad (15)$$

where

$$\widetilde{\mu}(k) = \begin{pmatrix} \bar{\mu}(k-1) \\ \Delta\bar{\mu}(k) \end{pmatrix}, A_r = A_{r,0} = \begin{pmatrix} A_{dc} & 0 \\ 0 & A_L \end{pmatrix},$$

$$B_r = B_{r,0} + B_{r,\Delta}\tau_\Delta, B_{r,0} = \begin{pmatrix} \bar{B}_{d12} & \bar{B}_{d2} \\ 0 & 0 \end{pmatrix},$$

$$B_{r,\Delta} = \begin{pmatrix} 0 & \bar{B} \\ 0 & \bar{B} - L\bar{D} \end{pmatrix}, D_r = D_{r,0} + D_{r,\Delta}\tau_\Delta, D_{r,0} = 0,$$

$$C_r = C_{r,0} = \begin{pmatrix} 0 & V\bar{C}_d \end{pmatrix}, D_r = D_{r,\Delta} = \begin{pmatrix} 0 & V\bar{D} \end{pmatrix},$$

$$E_{rx} = E_{rx,0} = \begin{pmatrix} (\bar{E}_{xd} - L\bar{E}_y)^T & \bar{E}_y^T \end{pmatrix}^T, E_{ry} = E_{ry,0} = \bar{E}_y,$$

$$F_{rx} = F_{rx,0} = \begin{pmatrix} (\bar{F}_{xd} - L\bar{F}_y)^T & \bar{F}_y^T \end{pmatrix}^T, F_{ry} = F_{ry,0} = \bar{F}_y$$

where $\bar{B}$ and $\bar{D}$ are equal to $\Delta\bar{B}$ and $\Delta\bar{B}$ excluding the uncertainty part τ_Δ. It is assumed that the plant is mean square stable. Since the observer gain L has no influence on the system in (14)-(15), the overall system dynamic (plant and residual generator) is assumed as mean square stable. Furthermore, it is assumed that the delay jitter τ_Δ is independent on $x(k)$,$\Delta\bar{\mu}(k)$,$d(k)$,$e(k)$ with the properties in (6). The mean of $r(k)$ can be expressed by

$$x_r(k+1) = A_{r,0}x_r(k) + B_{r,0}\widetilde{\mu}(k-1) + \bar{E}_{ry,0}\bar{d}^c(k) + \bar{F}_{rx,0}\bar{f}^c(k) \quad (16)$$

$$\bar{r}(k+1) = C_{r,0}x_r(k) + E_{ry,0}\bar{d}^c(k) + \bar{F}_{ry,0}\bar{f}^c(k) \quad (17)$$

which is equivalent to

$$\bar{e}(k+1) = A_L\bar{e}(k) + E_{xd,L}\bar{d}^c(k) + F_{xd,L}\bar{f}^c(k) \quad (18)$$

$$\bar{r}(k+1) = V\left(\bar{C}_d\bar{e}(k) + \bar{E}_y\bar{d}^c(k) + \bar{F}_y\bar{f}^c(k)\right) \quad (19)$$

where $A_L = (A_{dc} - L\bar{C}_d)$, $E_{xd,L} = \left(\bar{E}_{xd} - L\bar{E}_y\right)$, $F_{xd,L} = \left(\bar{F}_{xd} - L\bar{F}_y\right)$ and $r(k) \in R^{k_r}$ is a residual signal. The post filter V and the observer feedback matrix L are the design parameters for the residual signal. The main objective of the design of the residual generator is to improve the sensitivity of the FD system to faults while keeping robustness against disturbances. Thus, the selection of the design parameters L, V can be formulated as an optimization problem such as

$$\sup_{L,V} J = \sup_{L,V} \frac{\|G_{rf}\|_-}{\|G_{rd}\|_\infty} \quad (20)$$

where

$$G_{rd}(z) = V\bar{C}_d\left(zI - A_{dc} + L\bar{C}_d\right)^{-1}E_{xd,L} + V\bar{E}_y \quad (21)$$

$$G_{rf}(z) = V\bar{C}_d\left(zI - A_{dc} + L\bar{C}_d\right)^{-1}F_{xd,L} + V\bar{F}_y \quad (22)$$

The optimal solution to the optimization problem in (20) can be carried out using the (so-called) unified solution (by taking into account the assumption $A1, A2$), which is discussed in Ding et al. [2000]. Note that the model parameter uncertainties $\Delta\bar{B}$ and $\Delta\bar{D}$ are not taken into account in the relations (21) and (22). These uncertainties can also increase the FAR and therefore reduce the effectiveness of the FD system. This means, it has to be ensured, that the false alarm rate of the FD system has to be minimized as much as from the effect of these uncertainties which will be investigated in the residual evaluation.

4.2 Statistical based residual evaluation

Residual evaluation is an important step of model based FD approach, Frank et al. [1997]. This stage include a calculation of the residual evaluation function and a determination of a detection threshold. The decision for successful fault detection is finally made based on the comparison between the results obtained from the residual evaluation function and the determined threshold. This can be realized based on the following logic:

$$J_e = \|r(k)\|_2 \begin{cases} \leq J_{th} \rightarrow & \text{fault-free} \\ > J_{th} \rightarrow & \text{fault} \end{cases} \quad (23)$$

where J_e is a function of the residual signal which in most cases the L_2-norm is used to measure its size, J_{th} is the threshold which is be to determined based on the residual dynamic in the fault free cases. There are two popular strategies residual evaluation, the statistical testing method is one of them, which is established and another one is the (so-called) norm-based residual evaluation Gertler [1998], Emani et al. [1997], Frank et al. [1997]. Due to stochastic behaviour of the uncertainties in the NCS model, applying robust control theory methods for deterministic uncertain systems will lead to conservative results by increasing the FAR.

Since the NCS model in (8) and (9) is described as stochastic process with stochastic uncertainties, it is reasonable to apply a residual evaluation strategy based on statistical testing, Ding et al. [2006]. This technique will be studied next with aim of possible application in networked control systems. The following tasks will be considered

- study of the stochastic stability and bound on the statistic features of the residual generator.
- define a suitable statistic feature for the residual evaluator.
- the detection threshold can be set to the maximum bounds of the statistic feature of the residual signal.

The mean of $r(k)$ can be expressed ($\bar{f}^c = 0$) as

$$\bar{e}(k+1) = A_L\bar{e}(k) + E_{xd,L}\bar{d}^c(k) \quad (24)$$

$$\bar{r}(k+1) = V\left(\bar{C}_d\bar{e}(k) + \bar{E}_y\bar{d}^c(k)\right) \quad (25)$$

First, the estimation of bound of mean of $r(k+1)$ can be calculated using the following theorem

Theorem 1. Given system (24)-(25), the constants α_1,α_2 and suppose that $\bar{e}(0) = 0$ Then,$\forall k$

$$\bar{r}^T(k+1)\bar{r}(k+1) = \alpha_1\delta_{d,2} + \alpha_2\delta_{d,\infty} \quad (26)$$

if the following three LMI's hold for some $P > 0$

$$\begin{bmatrix} P & PA_L & PE_{xd,L} \\ A_L^T P & P & 0 \\ E_{xd,L}^T P & 0 & I \end{bmatrix} > 0 \quad (27)$$

$$\begin{bmatrix} P & \bar{C}_d^T V^T \\ V\bar{C}_d & \alpha_1 \end{bmatrix} > 0 \quad (28)$$

where

$$||\bar{d}^c(k)||_2 \leq \delta_d, \forall k\sqrt{(\bar{d}^c(k))^T\bar{d}^c(k)} \leq \delta_{d,\infty}$$

Note that $\alpha_2^{1/2}$ is the maximum singular value of the matrix $V\bar{E}_y$. The proof of Theorem 1 is similar to the one mentioned in Ding [2008] and omitted due to the limited space. Next, the variance of $r(k+1)$ will be considered. It will be denoted by

$$\sigma_r = E\left((r(k+1) - \bar{r}(k+1))^T (r(k+1) - \bar{r}(k+1))\right) \tag{29}$$

where $\bar{r}(k+1) = E(r(k+1))$. Assume that the unknown disturbance $\bar{d}^c(k)$ and the reference control input $\Delta\bar{\mu}(k)$ are L_2-bounded. The estimation of the bound on the variance of $r(k+1)$ can be obtained according to the following theorem.

Theorem 2. Given system (14)-(15) and constants $\gamma_1 > 0$,$\gamma_2 > 0$. Then the following relation

$$\begin{aligned}\sigma_r &= E\left((r(k+1) - \bar{r}(k+1))^T (r(k+1) - \bar{r}(k+1))\right)\\ &< \gamma_1 \sum_{j=0}^{k} \left(\bar{d}^{cT}(j)\bar{d}^c(j) + \Delta\bar{\mu}^T(j)\Delta\bar{\mu}(j)\right)\\ &+ \gamma_2 \left(d^{cT}(k)\bar{d}^c(k) + \Delta\bar{\mu}^T(k)\Delta\bar{\mu}(k)\right)\end{aligned} \tag{30}$$

is hold if there exists $P > 0$ so that

$$M_1 < \begin{bmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{31}$$

$$M_C \leq \gamma_1 P, \tag{32}$$

$$M_D \leq \gamma_2 \tag{33}$$

where

$$M_1 = \begin{bmatrix} A_{r,0}^T \\ B_{r,0}^T \\ E_{r,0}^T \end{bmatrix} P \left[\, A_{r,0} \;\; B_{r,0} \;\; E_{r,0} \,\right] + \sigma_{\tau_\Delta}^2 B_{r,\Delta}^T P B_{r,\Delta} \tag{34}$$

$$M_C = C_{r,0}^T C_{r,0}, M_D = \sigma_{\tau_\Delta}^2 D_{r,\Delta}^T D_{r,\Delta} \tag{35}$$

The Proof is mentioned in Ding [2008]. Based on the results of statistical features of the residual signal, a residual evaluation strategy is developed. The construction of the evaluation residual function will be described at next.

Residual evaluation function: To design a residual evaluation function, the size of the residual signal is calculated at each time instant, in order to compare it with the threshold. The following evaluation function is adopted

$$J_e = \sqrt{\left(\frac{1}{N}\sum_{i=1}^{N} r(k-i)\right)^T \left(\frac{1}{N}\sum_{i=1}^{N} r(k-i)\right)} \tag{36}$$

Due to stochastic character of the uncertainties, a finite evaluation window N is used, Ding [2008].

Selection of threshold: Based on the results on the bounds of the statistic feature of the residual signal in the previous subsection, the threshold can be determined as

$$\begin{aligned}J_{th}(k) &= \sqrt{\sup_{\bar{d}^c(k),\bar{f}^c(k)=0} \bar{r}(k+1)^T\bar{r}(k+1)}\\ &+ \sqrt{\alpha(N) \sup_{\bar{d}^c(k),\Delta\mu(k)} \sigma_r}\end{aligned} \tag{37}$$

where $0 < \alpha(N) \leq 1$. Note the value of $\alpha(N)$ is constant and depends on the length of the evaluation window N, Ding et al. [2006]. The decision logic for the FD system can be defined as

$$J_e > J_{th}(k) \Rightarrow fault \tag{38}$$

$$J_e \leq J_{th}(k) \Rightarrow fault - free \tag{39}$$

As it is shown in (37), the threshold selection for NFDS is determined based on the maximum bound on the feature of residual signal as well as reference control signal $\Delta\mu(k)$ which is online available. An algorithm for the calculation of the detection threshold is presented as follows

Algorithm: Given system (8)-(9), residual generator (12) and the residual evaluation function (36) and assume that $\bar{d}^c(k)$ is an L_2-norm bounded signal as $||d(k)||_2 \leq \delta_{d,2}$. The following tasks have to be considered

- determine α_1,α_2 by iteratively solving the optimisation problem subjects to the LMIs (27)-(28) for some $P > 0$ as described in theorem (1).
- iteratively solve the optimisation problem γ_1,γ_2 subject to the LMIs (31)-(33) for some $P > 0$, as given in theorem (2).
- the threshold is given by

$$J_{th}(k) = \alpha_1\delta_{d,2} + \alpha_2\delta_{d,\infty} + \sqrt{\alpha(N)\beta} \tag{40}$$

where

$$\begin{aligned}\beta &= \gamma_1\left(\delta_{d,2} + \Delta\bar{\mu}^T(j)\Delta\bar{\mu}(j)\right)\\ &+ \gamma_2\left(\delta_{d,\infty} + \Delta\bar{\mu}^T(k)\Delta\bar{\mu}(k)\right)\end{aligned}$$

Note that β depends on the σ_τ.

5. A SIMULATION EXAMPLE

In order to illustrate the effectiveness and applicability of the results obtained in this study, a simulation platform is established as illustrated in Fig.3. The linux PC acts as router and emulates the Ethernet properties by using the emulation software NIST Net [2005]. It connects the local station PC to the central supervision station PC and vice versa. The required delay, jitter are tunable and can be adjusted. The algorithms for the programming of the controlled process (local station PC) and the networked model-based fault detection (central station PC) are implemented with the help of MATLAB/Simulink. As local process, an inverted pendulum is implemented which is described as by the a linear time invariant model as mentioned in (1)-(2), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2.4707 & -0.7576 & 0.0007 \\ 0 & 0 & 0 & 1 \\ 0 & 4.7569 & 20.3458 & -00185 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.2471 \\ 0 \\ -0.4757 \end{bmatrix},$$

$$C = I_{4\times4}, E_x = \left[\, B \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right], F_y = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It is assumed that the local station and the supervision station have the sampling time $T_s = 0.03$sec and $T = 0.09$sec, respectively. Furthermore, the mean delay value is 0.02sec and its variance equals to 0.05sec. The response of the evaluated residuals generated by NFDS is presented in Fig. 4. The actuator fault is assumed as step signal occurs at 15sec. The Fig. 5 shows the response of the residual signal by a sensor (angle) fault assumed as step signal

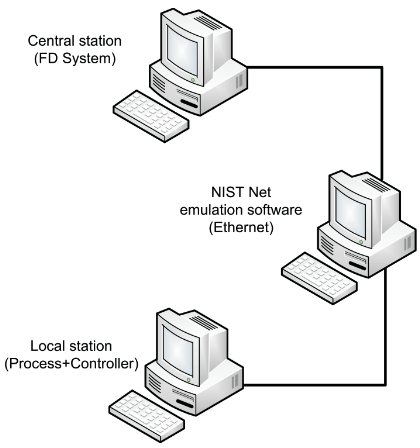

Fig. 3. Sketch of the simulation platform

occurs at 17 sec. It can be seen that the fault has been correctly detected since the residual exceeds the threshold $J_{th}(k)$. The threshold is also adaptive to the influence of the online information of the reference control input received from the central supervision station.

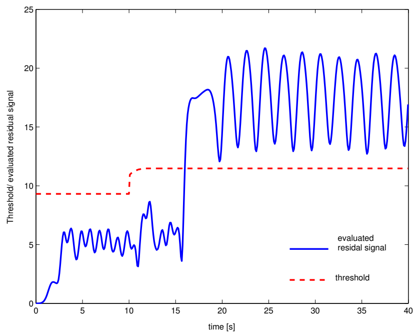

Fig. 4. Evaluated residual in case of actuator fault

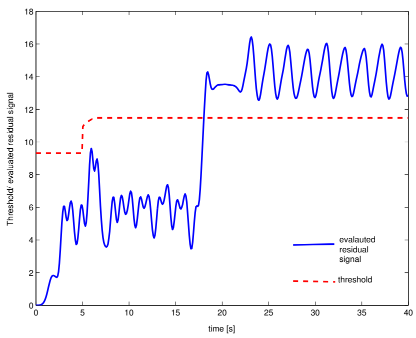

Fig. 5. Evaluated residual in case of sensor (angle) fault

6. CONCLUSION

The paper considers an approach for FD in networked control system with stochastically time varying transmission delays. A suitable model that reflect the dynamic behaviour on the NCS structure under consideration is derived. A packet-oriented data transmission is applied. The network's effects (uncertainties) which have stochastic behaviour are taken into account by the design of evaluation strategy for the NFDS. The problem of handling uncertainties caused by the communication network for the purpose of fault detection is also solved by the proper design of the detection threshold. A simulation platform was established to test and analyze the applicability of the obtained theoretical results.

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