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Control of Multivariable Systems Using Modified Local Optimal Controller

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Abstract: In this paper, a modified local optimal control approach is proposed for multivariable systems. As for unknown plant dynamics, system identification must be performed to obtain a model based on which the local optimal controller is designed and implemented. The proposed method guarantee closed loop stability characteristic when dealing with non-minimum phase plant which is a considerable advantage over the original local optimal controller. In addition to computational efficiency and structure simplicity, experimental results on a lab-based test rig confirm the effectiveness and robustness of the proposed local optimal controller over a conventional genetically tuned PID. Copyright © 2008 IFAC.

Keywords: Multivariable local optimal controller, and system identification.

1. INTRODUCTION

Due to the increasing complexity of process control systems, multivariable process control has been of considerable attention over the past decades and numerous theoretical and practical works have been proposed in this area of research (Skogestad, and Postlethwaite, 2005). Among the existing methods of multivariable control, the Local Optimal Control (LOC) is a new method which was first introduced by Lyantsev, *et al.* (2004). However, existing LOC approach is incapable of dealing with non-minimum phase systems.
In this paper, a modification has been made for controlling non-minimum phase systems using LOC approach. The controller has been designed and implemented on a laboratory based multivariable process control test rig. The results of the proposed controller are compared with a conventional PID controller tuned by Genetic Algorithm (GA). Since the multivariable plant is unknown, system identification should be incorporated to provide the LOC with the system's model. Therefore, different open loop Multi-Input Multi-Output (MIMO) system identification techniques are used to obtain the required model (Ljung, 1999; Simani, 2005). This paper is organised as follows: Section 2 is a brief description of the system rig to be controlled. Section 3 introduces and compares two different techniques of open loop MIMO system identification employed. In section 4 the details of the modified local optimal controller design are proposed and compared with GA-based PID controller. Concluding remarks are made in section 5.

2. THE TEST RIG PROCESS DESCRIPTION

The system rig is Bytronic Process Control Unit (PCU) which is based around a fluid flow process, where flow and temperature can be controlled. This reflects a typical process control situation such as in the food and drink manufacturing and petrochemical industry (Bytronic ltd, 1998).
In this system a fluid is pumped in a closed path from a sump through a cooling fan to a process tank where the fluid is heated and then is drained back to the sump. The scheme of PCU is shown in figure 1.

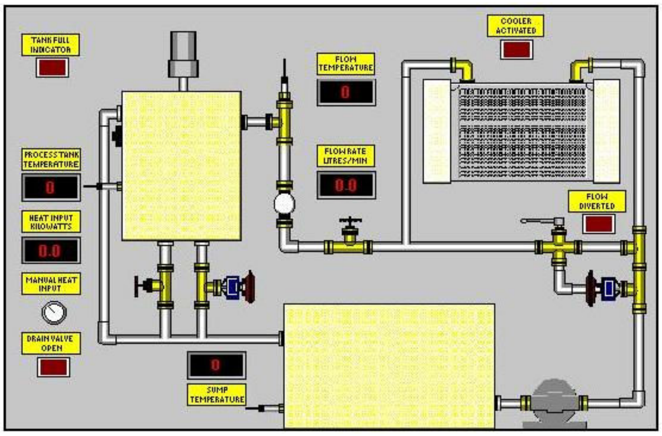

Fig. 1. Bytronic process control unit.

As shown in fig. 1, the PCU consists of a sump (liquid reservoir), a pump, flow meter, cooler, and a process tank containing an electrical heating element, temperature sensor, stirrer, and a high-level switch for safety. The system is connected to a power supply unit that provides the input power to all the system elements. Also the process inputs and outputs are connected to a computer control module that works as an interface between the PCU and PC-based controller. The system was modified by replacing the existing I/O interface module with a NI PCI-DIO-96 digital I/O to enable working in Matlab environment. The overall block diagram of the I/O interface system is illustrated in fig. 2.

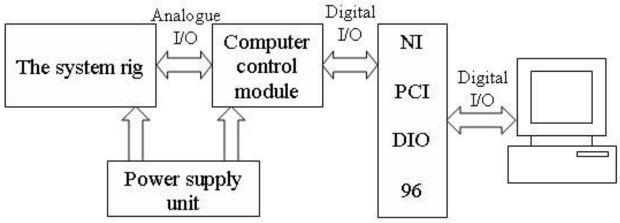

Fig. 2. Overall block diagram of the I/O interface system

The system is considered as a two-input, two output system. The inputs are the input DC voltage to the pump, and heater power. The outputs are the fluid flow rate, and the fluid temperature.
The input power to the heater element is computer-controlled using Pulse Width Modulation (PWM) technique. In fact the corresponding control signal manipulates the mark/ space ratio of PWM signal which in turn controls the average current applied to the heating element.

3. OPEN LOOP SYSTEM IDENTIFICATION

From fluid dynamics, the fluid flow rate depends on the pump speed, meaning that the input power to the heater has no effect on the fluid flow rate. However, the fluid temperature varies with both pump speed and the heating power. In addition, the cooling fan has an additional effect on temperature. Therefore, the process is complicated in terms of the non-symmetrical effect of inputs on the output signals. As such, there are two different approaches available for the purpose of system identification. In the first option, namely approach#1, the plant is considered as two sub-systems according to number of outputs, one of which is a Single-Input Single-Output (SISO) system and the other is a two input single output system. Then, each sub-system is identified separately and both are combined into one system equivalent to the original system (Simani, 2005). In approach#2, the plant is identified as a MIMO system (two inputs two outputs). Both approaches have been considered and compared in this paper.

3.1 Approach# 1 to System Identification

In this section, each subsystem in approach#1 is introduced and identified then these subsystem models are merged into an equivalent multivariable model.

Approach#1, Subsystem#1.
For this subsystem the input is the DC voltage to the pump and the output is the fluid flow rate.
Prior to the system identification several initial open loop tests must be performed to determine the characteristics of the system and design the excitation signal (Ljung, 1999), based on which the system's time constant is 700 ms and its cut-off frequency is 6 rad/sec. As such, the sampling time is chosen as 125 ms and the frequency band for the excitation signal is chosen accordingly.
The input to the pump can vary between 0 and 12 V. The Input-Output (I/O) curve for this subsystem shows that the system can be considered as linear when working between 3 and 9 V.
According to the previous tests discussed, the excitation signal must vary between 3-9 V in amplitude and its power spectrum must be flat for frequencies from 0-6 rad/sec. Chirp and multi-sine signals can be considered as the excitation signals satisfying mentioned requirements (Ljung, 1999).

Identification using Chirp input (experiment1).
Chirp signal is a sinusoidal signal with a continuously changing frequency over interval $\Omega : \omega_1 \le \omega \le \omega_2$ within time period $0 \le t \le M$. It is represented as in (1) (Ljung, 1999).

$$u(t) = A\cos(\omega_1 t + (\omega_2 - \omega_1)t^2/(2M)) \quad (1)$$

where A is the amplitude of the signal.
In the case studied a Chirp input voltage with linear swept-frequency from 0 to 6 rad/sec over a time period of 100 seconds is applied as the excitation signal. Using the input-output data, the system is modeled using Output Error (OE) method (Ljung, 1999) and a second order model (m_1) is obtained as in (2). Estimated parameters are in table 1.

$$y(i) = -a_1 y(i-1) - a_2 y(i-2) + b_1 u(i-1) + b_2 u(i-2) \quad (2)$$

Identification using multi-sine input (experiment2)
In this case the excitation signal considered is a sum of sinusoids as in the following equation.

$$u(t) = \sum_{k=1}^{d} a_k \cos(\omega_k t + \phi_k) \quad (3)$$

where ϕ_k is chosen by Schroeder phase choice (Ljung, 1999).
The excitation signal is applied for 600 s to the system under consideration. Similar to Chirp session, OE method is considered with the I/O data from 300-400 s and a second order model (m_2) is obtained as in (2). The model parameters are given in table 1.

Table 1 model parameters for each experiment

model	a_1	a_2	b_1	b_2
m_1	-0.9078 (±0.04621)	0.1557 (±0.03499)	0.0136 (±0.001813)	0.0336 (±0.003953)
m_2	-0.8323 (±0.04477)	0.1378 (±0.03306)	0.0167 (±0.002262)	0.0472 (±0.004665)

Subsystem#1 Model Validation
The two sets of output data generated by each experiment mentioned above are used to validate the two models obtained. The Mean Square Error (MSE) and models fit for each set of output data are listed in table 2.

Table 2. MSE and fitness for each model with each output data

	Experiment1		Experiment 2	
	MSE	Fitness	MSE	Fitness
m_1	0.00054	91.89%		79.63%
m_2		72.92%	0.00017	90.13%

As in table 2, the two subsystems are subjected to a cross validation, i.e., to validate the model generated using I/O data of certain experiment with the data generated from the other experiment. Fig. 3 and 4 show the error signals for cross validation between the measured output and the simulated output of each model. Results confirm that m_1 can be regarded as the final model in approach#1.

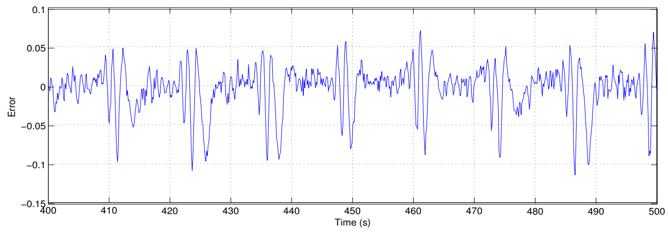

Fig. 3. Cross validation error for model m_1

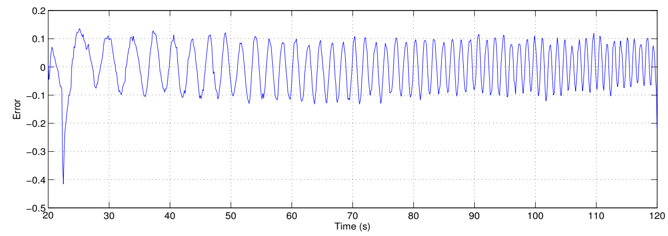

Fig. 4. Cross validation error for model m_2

Approach#1, Subsystem#2

The second subsystem will be formed by DC voltage to the pump (u_1) and the power to the heater (u_2)as inputs and the fluid temperature as the output. It must be noted that while such inputs can directly contribute to increase the temperature, decreasing temperature will be associated with the temperature difference between the fluid and the air surrounding, and conductivity of the cavity. Therefore, to identify this subsystem two models must be taken in account; one for increasing temperature, and the other for decreasing temperature. In this paper the first model will be considered. Open loop tests result in time constant about 300 s which is higher than subsystem#1 (700 ms). The sampling time will be the same as subsystem#1, i.e., 125 ms.

Subsystem#2 identification

The excitation signals are chosen to be multi step signals as follows.

$$u_1 = \begin{cases} 3.765 & 0 \le t < 400 \\ 6.118 & 400 \le t < 800 \\ 8.455 & 800 \le t < 1200 \end{cases} \tag{4-a}$$

$$u_2 = \begin{cases} 0 & 0 \le t < 200 \\ 29.412 & 200 \le t < 600 \\ 58.825 & 600 \le t < 1000 \\ 88.235 & 1000 \le t < 1200 \end{cases} \tag{4-b}$$

The output of subsystem#2 subjected to (4-a,b) is shown in fig. 5.

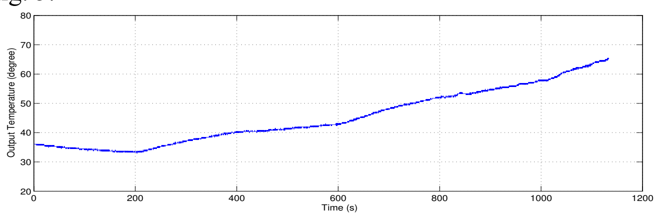

Fig. 5. Output fluid temperature of the system

According to this experiment the OE model of this subsystem is obtained as follows.

$$y(i) = -\sum_{n=1}^{4} a_n y(i-n) + \sum_{m=1}^{4} b_{1m} u_1(i-m) + \sum_{k=1}^{4} b_{2k} u_2(i-k) \tag{5}$$

where the MSE of that model is 0.016 and:

a_1=-1.558, a_2=-0.4417, a_3=1.558, a_4=- 0.5584
b_{11}=-0.0346, b_{12}=0.0345, b_{13}= 0.0346, b_{14}=-0.0345
b_{21}=0.0046, b_{22}=-0.0114, b_{23}= 0.0091, b_{24}=-0.0023

Subsystem#2 Model Validation

Two sets of output data are used to validate the model, one using the same output data used to produce the model (experiment3), and the other using fresh data from another new experiment (experiment4). For this new experiment; the first excitation input u_1 is as in (4-a), while the second excitation input u_2 is shown in (6).

$$u_2 = \begin{cases} 0 & 0 \le t < 200 \\ 7.843 & 200 \le t < 600 \\ 23.53 & 600 \le t < 1000 \\ 39.215 & 1000 \le t < 1200 \end{cases} \tag{6}$$

The resulting error signal between the measured output and the model simulated output associated with the above mentioned experiments are illustrated fig. 6 and 7. The model validation gives 95.52% fitness for experiment3, and 96.38% for fresh data of experiment4, confirming the validation of experiment4 (see fig.7).

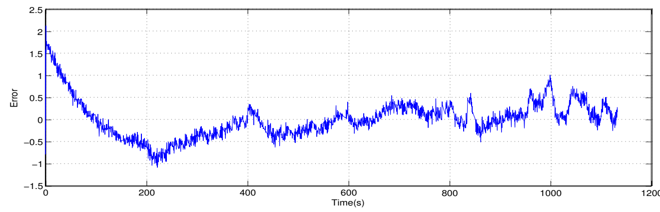

Fig. 6. The model error signal with output data from experiment3

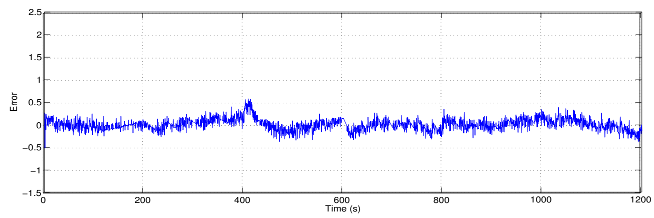

Fig. 7. The model error signal with output data from experiment4

3.2 Approach# 2 Identification of Multivariable model

In this section, the plant is regarded as a MIMO system and results for the identification are investigated. In this case, the excitation signals are chosen as (4-a) and (6) and two models are obtained using system identification toolbox of Matlab , namely a multivariable ARX model (M_1) of 16 free parameters, and a state space model (M_2) of 4 states and 44 free parameters (see (Ljung, 2007) for model structure**).**

Multivariable Model Validation

To validate the multivariable models obtained directly in section 3.2 *(M_1, M_2)* and the multivariable model obtained by merging the subsystem models obtained in section3.1 in one model equivalent to the multivariable model *(M_3)*, the output data used in section 3.2 to produce models M_1, M_2 is employed to validate the MIMO models M_1, M_2,M_3. Fig. 8 and 9 show the output responses as measured from the experiment and the simulated output responses of each model (with mean removed from each).

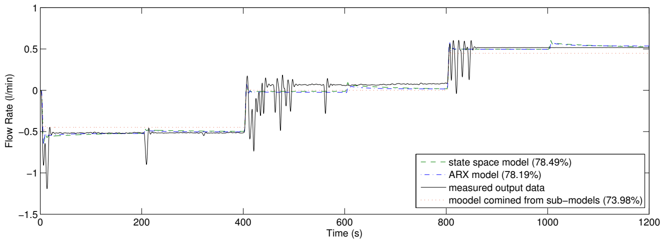

Fig. 8. Flow rate (output 1) validation.

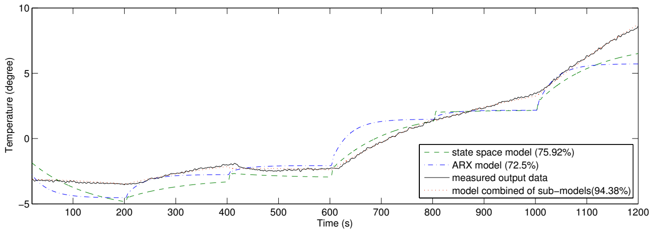

Fig. 9. Fluid temperature (output 2) validation

Although the output data used are fresh data for M_3 and not fresh for M_1 and M_2, figures show that the model M_3 is the best model obtained in terms of output tracking. In addition, the fitting table 3 confirms the effectiveness of M_3 model.

Table 3. Fits for different models

model	Fits	
	Flow rate (Output1)	Fluid temperature (Output2)
M_1	78.19%	72.5%
M_2	78.49%	75.92%
M_3	73.98%	94.38%

4. SYSTEM CONTROL

The main purpose of this section is to design a local optimal controller for the PCU (Lyantsev, et al., 2004) and to compare it with a genetically tuned PID controller. For simplicity, the pump subsystem is considered first.

4.1 Pump control

Fig. 10 shows the closed loop system step response.

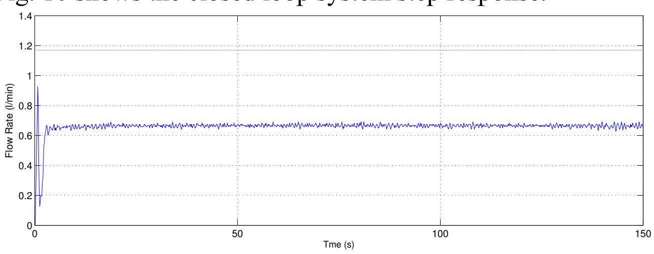

Fig. 10. The reference input and the closed loop response of the pump

From this figure it is clear that the system needs to be controlled using PI controller.

Genetically tuned PI controller

GA is used for tuning the controller parameters (K_P, K_I) (Fleming and Purshouse, 2002; Kwok and Wang, 1994). The multi-objective function is used to minimize the MSE of the tracking error and eliminate the overshoot. Fig. 11 shows the response of the pump controlled by PI controller when the reference input is changed from *0.8 l/min* to *1.2 l/min*. Two curves illustrated are obtained from simulated model, and the real PCU system.

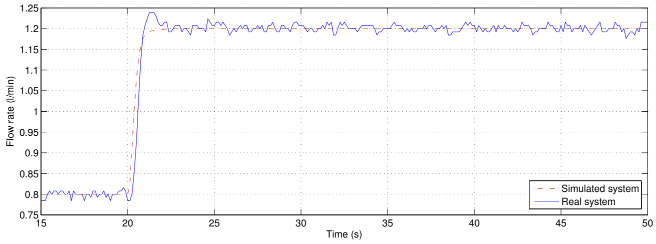

Fig. 11. The pump response with PI controller (simulation, PCU)

The difference observed between the simulated model and the real system is because modeling error and uncertainty are involved in the controller parameters tuning made by GA. As such, the controller does not show a robust performance. Some fine tuning to reduce the K_I parameter leads to the following change in PCU response as in fig. 12.

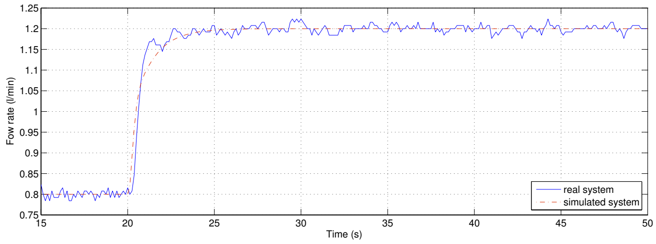

Fig. 12. The output response after fine tuning of K_I

The Local Optimal Controller (LOC) Design

The local optimal controller is designed and implemented as proposed in (Lyantsev, et al., 2004). For the pump model (SISO case) two states x_1=y(i-1), x_2=y(i-2) are considered and the following equation is driven from the model in (2).

$$y(i+1)-y(i)=h[\alpha_1\delta y(i)+\alpha_2\delta y(i-1)+\beta_1\delta u(i)+\beta_2\delta u(i-1)] \quad (7)$$

where h is a weighting coefficient indicating the level of uncertainty involved in the plant dynamics (Lyantsev, et al., 2004).

From (7), *u(i)* can be obtained as a summation of the system input and output with different time delays and different coefficients. Fig. 13 shows the block diagram of the system with local optimal controller as in (Lyantsev, et al., 2004). The existing LOC approach as in fig. 13 is not well-suited for non-minimum phase systems as it leads to closed loop instability.

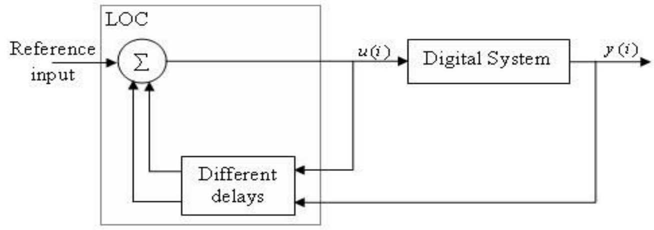

Fig. 13. Block diagram of the local optimal controller.

As such, modification is made to this method by using decremented difference $\delta u(i)$ and integrating it by a discrete time integrator to obtain *u(i)*. Fig. 14 illustrates the modification made in this paper.

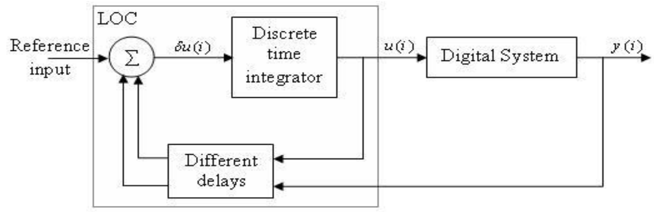

Figure 14. Modified block diagram of the local optimal controller.

Fig. 15 shows the response of the model with the proposed modified LOC using Simulink for different values of the controller parameter, namely *h (h=2, 3, 4, and 5)*. Obviously, increasing *h* result in a better tracking performance of the model under control and less overshoot can be observed but longer rise time obtained.

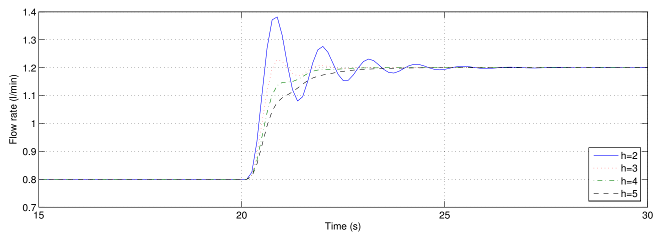

Fig. 15. Simulated output response for different h

For *h=5* the controller is designed for PCU system and figure 16 shows the output response for both real and simulated system. From this figure it is clear that the real and simulated outputs are approximately identical.

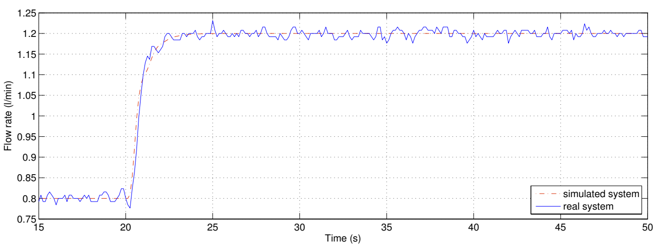

Fig. 16 The output response for both real and simulated system at h=5

Comparison of PI and the modified LOC

A frequency domain comparison between GA-based PI and the modified LOC is made as shown in fig. 17. The gain margin and phase margin for each controller is shown in table 4.

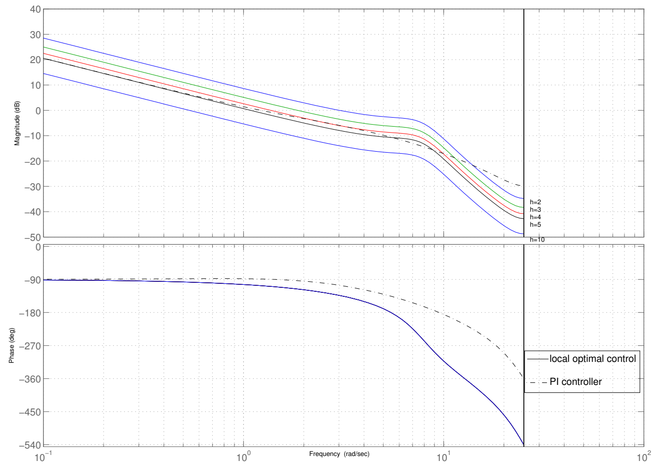

Fig. 17. Bode plot of the pump with PI and local optimal controllers.

Table 4. Phase and gain margin of each controller

Controller type		Gain margin	Phase margin
Local optimal control	h=2	1.3731	46.6198
	h=3	2.0596	63.9724
	h=4	2.7461	71.0365
	h=5	3.4326	75.0118
	h=10	6.8653	82.6261
GA based PI controller		6.6417	91.2797

4.2 Multivariable system control

In this section the LOC and the genetically tuned PID will be designed, implemented, and compared in terms of performance on PCU system.

Genetically tuned PID Controller

To design PID controller for the multivariable system under consideration, two techniques were considered.

1) To design a compensator (decoupler) so that the multivariable system is became as two SISO systems then a diagonal matrix of PID controllers is designed (Skogestad, and Postlethwaite, 2005; Miklosovic and Gao, 2005). However, the decoupler increases the order of the system so this technique will not be used here.

2) To design a full matrix of digital PID controllers without a decoupler as expressed in Z-domain in (8).

$$C(s)=\begin{bmatrix} K_{P11}+\dfrac{K_{I11}}{1-Z^{-1}}+K_{D11}(1-Z^{-1}) & K_{P12}+\dfrac{K_{I12}}{1-Z^{-1}}+K_{D12}(1-Z^{-1}) \\ K_{P21}+\dfrac{K_{I21}}{1-Z^{-1}}+K_{D21}(1-Z^{-1}) & K_{P22}+\dfrac{K_{I22}}{1-Z^{-1}}+K_{D22}(1-Z^{-1}) \end{bmatrix} \quad (8)$$

A GA program is used to tune the twelve controller parameters represented in (8) using multi-objective function to minimize MSE between the reference input and the system's output and to have no over shoot (Fleming and Purshouse , 2002; Kwok and Wang, 1994).

The modified LOC

The local optimal controller is designed according to (Lyantsev, et al., 2004) for the models obtained for each output in (2) and (5). Therefore, the (9), (10) can be obtained for the local optimal controller of this multivariable system.

$$y_1(i+1)-y_1(i)=h_1[\alpha_{11}\delta y_1(i)+\alpha_{21}\delta y_1(i-1)+\beta_{11}\delta u_1(i)+\beta_{21}\delta u_1(i-1)] \quad (9)$$

$$y_2(i+1)-y_2(i)=h_2[\sum_{n=0}^{3}\alpha_{2n}\delta y_2(i-n)+\sum_{m=0}^{3}\beta_{2m}\delta u_1(i-m)+\sum_{k=0}^{3}\gamma_{2k}\delta u_2(i-k)] \quad (10)$$

By solving these two equations for $\delta u_1(i)$ and $\delta u_2(i)$ the LOC can be constructed as in fig. 14.

Multivariable control results

Fig. 18 and 19 show the two output responses for the real system using genetically tuned PID controller and local optimal controller respectively, where the reference input for the flow rate is changed from *0.8* to *1.2 l/min* at *t=500 s* and the reference input for the fluid temperature is changed from *room temperature* to *60* degree at *t=20 s*.

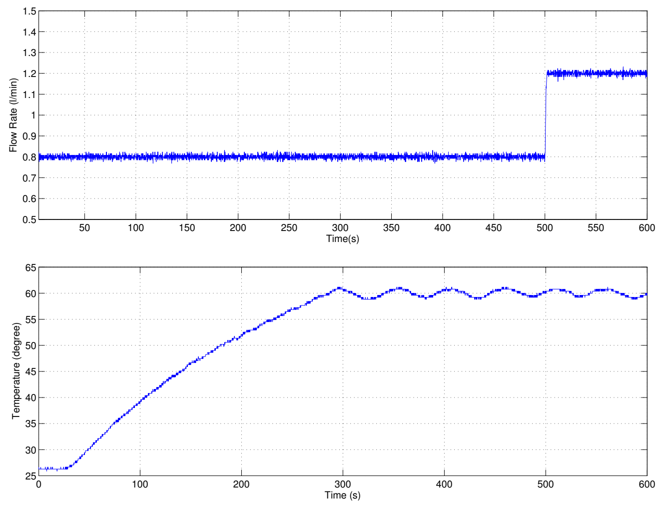

Fig. 18. Multivariable outputs for the PID controlled system (12 parameters tuned genetically)

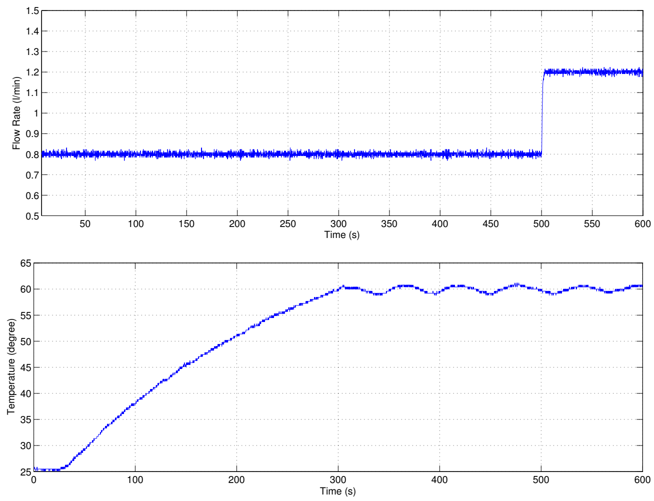

Fig. 19. Multivariable outputs for the local optimal controlled system (h_1=5, h_2=10)

It is clear from these two figures that approximately the same results are obtained. However, from design simplicity point of view, the local optimal controller is simpler and it has only two integer scalar parameters *(h_1, h_2)* to be tuned (Lyantsev, et al., 2004). Because of its simplicity LOC can easily be designed or adjusted in automatic mode.

The oscillations for the output temperature in both types of controllers *(60±1)* are due to the sensor sensitivity.

5. CONCLUSION

A modified LOC associated with system identification has been proposed. The best system identification results are obtained by approach#1 where two subsystems are formed and merged into a MIMO model. The proposed modified LOC performs significantly more efficient than genetically tuned PID in terms of robust performance. In addition, compared to a genetically tuned PID, the modified LOC scheme proposed is considerably easier in design due to less numerical computations and parameters to be tuned. Furthermore, when non-minimum phase systems are concerned, the proposed modified LOC seems to be the most appropriate substitute for existing LOC approach. The method has been verified and confirmed by a considerable amount of systems simulation and implementations on practical PCU systems.

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