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Design of Sensor Fault Diagnosis Method for Nonlinear Systems described by Linear Polynomial Matrices Formulation: Application to a Winding Machine

D. Theilliol*. J-C. Ponsart*, M. Rodrigues, S. Aberkane*, J. Yamé***

**Centre de Recherche en Automatique de Nancy - CNRS UMR 7039 -Nancy Université*
BP 239 - 54506 Vandoeuvre Cedex - France.
Phone: +33 383 684 466 - Fax: +33 383 684 462
email: didier.theilliol@cran.uhp-nancy.fr
*** Laboratoire d'Automatique et de Génie des Procédés – CNRS UMR 5007 – Université Claude Bernard Lyon 1*
CPE Lyon – Bd du 11 Novembre 1918 – 69622 Villeurbanne Cedex France
Phone: +33 472 431 892 - Fax: +33 472 431 682

Abstract: In this paper, a sensor model-based fault diagnosis method for a particular class of nonlinear systems is developed. A polynomial matrices representation is considered for modeling the dynamic behavior of a class of nonlinear systems. According to nonlinear representation via a polytopic transformation, the nonlinear faulty system can be considered as a nonlinear system with the presence of additive unknown inputs. Under fault isolation conditions, the main contribution of the paper relies on the use of an accurate observer that performs fault detection and isolation over the whole operating range of the nonlinear system. The effectiveness and performance of the proposed method are illustrated via real tests on a winding machine subject to sensor faults.

1. INTRODUCTION

Process monitoring is necessary to ensure effectiveness of process control and consequently a safe and a profitable plant operation. Sensor or actuator failure, equipment fouling, feedstock variations, product changes and seasonal influences may affect controller performance and as many as 60% of industrial controllers problems (T.J. Harris et al., 1999). Fault Detection and Isolation (FDI) refers to the task of inferring the occurrence of faults in a process and finding the root causes of the faults with various strategies according to the knowledge on the system: quantitative models (Venkatasubramanian et al., 2003a), qualitative models (Venkatasubramanian et al., 2003b), historical data (Venkatasubramanian et al., 2003c). Among quantitative models, fault diagnosis based on analytical models is developed for exact and uncertain linear mathematical description of the system, several books are dedicated to these topics such as (Gertler, 1998), and (Chen and Patton, 1999). FDI for nonlinear systems remains a challenge due to the problem of discriminating between disturbances and faults through a wide range of operating conditions. Different techniques based on an exact knowledge of the nonlinear system allow to generate residuals insensitive to fault by specific decoupling methods (Alcorta-Garcia and Frank, 1997), (Kinnaert, 1999) or geometric approach (De Persis and Isidori, 2001), (Hammouri et al., 2001).

The aim of this paper is to develop a sensor fault diagnosis method for nonlinear system with polynomial matrices state space representation. In order to achieve this objective, it should be emphasized that the approach presented in this paper relies on a recent method presented in (Rodrigues, 2006). Thus, this paper addresses an original contribution that could allow to detect and isolate sensor fault in nonlinear systems based on a polynomial to polytopic transformation. According to the associated faulty polytopic state space representation, Polytopic Unknown Input Observers is designed to generate residuals decoupled to sensor faults.

Based on appropriate observers, the developed technique enables to supervise nonlinear systems under polynomial matrices state space representation through an accurate bank of residuals within a Generalized Observer Scheme (GOS). The effectiveness and performances of the technique are illustrated on a winding machine example.

The paper is organized as follows. In section 2, we state the problem under consideration. Section 3 is devoted to the design of the sensor fault diagnosis module. Section 4 gives some experimental results to illustrate the effectiveness and performance. Conclusion and further work are discussed in the last section.

2. PROBLEM STATEMENT

Consider the following discrete nonlinear system:

$$\begin{cases} x_{k+1} = A(\lambda_k)\, x_k + B(\lambda_k)\, u_k \\ \;\; y_k = C x_k \end{cases} \qquad (1)$$

where the system matrix A and the control matrix B are assumed to be linear polynomial matrices depending on a bounded positive time varying parameter noted λ_k $(0 < \lambda_{min} < \lambda_k < \lambda_{max})$ and verifying:

$$G(\lambda) = G_0\lambda^0 + G_1\lambda^1 + G_2\lambda^2 + \cdots + G_\alpha\lambda^\alpha \qquad (2)$$

where G stands for A or B, α defines the polynomial degree and $\forall\, i \in [0, 1, \cdots, \alpha]$, $A_i \in \Re^{n\times n}$ and $B_i \in \Re^{n\times p}$ are constant matrices. Matrix $C \in \Re^{m\times n}$ defines the output matrix, $x \in \Re^n$ is the state vector, $u \in \Re^p$ is the control input vector and $y \in \Re^m$ is the output vector

Due to abnormal operation or material aging, sensor faults can occur in the system. A sensor fault can be represented by additive and/or multiplicative faults as follows:

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$$w_j^f = \beta_k w_j + w_0 \qquad (3)$$

where w_j and w_j^f represent the j^{th} normal and faulty measurements (i.e., $w_k = y_k$), w_0 denotes a constant offset and $0 \le \beta_k \le 1$ denotes a gain degradation of the j^{th} sensor (constant or variable).

Therefore, when a sensor fault occurs, the discrete state space representation defined in (1) becomes as:

$$\begin{cases} x_{k+1} = A(\lambda_k)\, x_k + B(\lambda_k)\, u_k \\ y_k = Cx_k + Ff_k \end{cases} \qquad (4)$$

where F represents the sensor fault distribution matrix and f is the faulty vector.

The presence of such faults may lead to performance deterioration, instability of the system or the loss of the process. The next section is dedicated to the development of an efficient model-based fault diagnosis method in order to provide an efficient monitoring tool in the operator's decision.

3. MODEL-BASED FAULT DIAGNOSIS DESIGN

3.1 From polynomial to polytopic faulty representation

As recently proposed by (Hetel *et al.*, 2007), each polynomial matrice can be defined on a convex polytope with $\alpha+1$ vertices Δ_j^G calculated as follows:

$$\begin{cases} \Delta_1^G = G_\alpha \lambda_{min}^\alpha + \cdots + G_2 \lambda_{min}^2 + G_1 \lambda_{min}^1 + G_0 \lambda_{max}^0 \\ \Delta_2^G = G_\alpha \lambda_{min}^\alpha + \cdots + G_2 \lambda_{min}^2 + G_1 \lambda_{max}^1 + G_0 \lambda_{max}^0 \\ \Delta_3^G = G_\alpha \lambda_{min}^\alpha + \cdots + G_2 \lambda_{max}^2 + G_1 \lambda_{max}^1 + G_0 \lambda_{max}^0 \\ \vdots = \vdots \\ \Delta_{\alpha+1}^G = G_\alpha \lambda_{max}^\alpha + \cdots + G_2 \lambda_{max}^2 + G_1 \lambda_{max}^1 + G_0 \lambda_{max}^0 \end{cases} \qquad (5)$$

The convex polytope formulation is achieved by the computation of parameter $\rho_j(\lambda)$ ($\forall\ j \in [0, 1, \cdots, \alpha+1]$) established following the recursive algorithm:

$$\begin{cases} \rho_1(\lambda) = 1 - \dfrac{\lambda - \lambda_{min}}{\lambda_{max} - \lambda_{min}} \\ \rho_{\alpha+1}(\lambda) = \dfrac{\lambda^\alpha - \lambda_{min}^\alpha}{\lambda_{max}^\alpha - \lambda_{min}^\alpha} \\ \rho_\tau(\lambda) = \dfrac{\lambda^{\tau-1} - \lambda_{min}^{\tau-1}}{\lambda_{max}^{\tau-1} - \lambda_{min}^{\tau-1}} - \sum_{j=\tau+1}^{\alpha+1} \rho_j(\lambda) \ with\ \tau = 2 \cdots \alpha \end{cases} \qquad (6)$$

Therefore parameter $\rho_j(\lambda)$ lie in a specific convex set:

$$\Omega = \Big\{ \rho(\lambda) \in \Re^{\alpha+1}, \rho = [\rho_1\ \rho_2\ \cdots \rho_{\alpha+1}]^T \quad \forall j\ \rho_j \ge 0\ and\ \sum_{j=1}^{\alpha+1} \rho_j = 1 \Big\}$$

Whereupon, $\forall\ j \in [0, 1, \cdots, \alpha+1]$, Δ_j^G defines a convex polytope such that:

$$G(\lambda) = \sum_{j=1}^{\alpha+1} \rho_j(\lambda) \Delta_j^G \qquad (7)$$

Based on the previous equation, the faulty discrete state space representation (4) can be expressed as a polytopic system:

$$\begin{cases} x_{k+1} = \sum_{j=1}^{\alpha+1} \rho_j(\lambda_k)\big(A_j x_k + B_j u_k\big) \\ y_k = Cx_k + Ff_k \end{cases} \qquad (8)$$

with $\forall\ j \in [0, 1, \cdots, \alpha+1]\ \rho_j \ge 0$ $\quad \sum_{j=1}^{\alpha+1} \rho_j(\lambda) = 1$ with $0 < \lambda_{min} < \lambda_k < \lambda_{max}$.

3.2 Residual generator synthesis: unknown input observer

Before designing the residual generator, a preliminary work consists in rewriting system (8) using Park *et al.* approach (Park *et al.*, 1994). These authors have developed a technique such that a system affected by a sensor fault can be written as a system represented by an actuator fault. Assume a new pseudo-fault input $\bar{f}_k$ such as:

$$f_{k+1} = \gamma f_k + \bar{f}_k \qquad (9)$$

where $\gamma \in \Re^{q \times q}$ defined by $\gamma = diag(\gamma_1, \ldots, \gamma_q)$ is always satisfied with $0 < q \le m$.

From (8) and (9), a new faulty polytopic system representation including this auxiliary state can be introduced:

$$\begin{cases} \bar{x}_{k+1} = \sum_{j=1}^{\alpha+1} \rho_j(\lambda_k)\big(\bar{A}_j \bar{x}_k + \bar{B}_j u_k + \bar{F}\bar{f}_k\big) \\ y_k = \bar{C}\bar{x}_k \end{cases} \qquad (10)$$

with $\bar{x}_k = \begin{bmatrix} x_k \\ f_k \end{bmatrix}$, $\bar{A}_j = \begin{bmatrix} A_j & 0 \\ \underline{0} & \gamma \end{bmatrix}$, $\bar{B}_j = \begin{bmatrix} B_j \\ \underline{0} \end{bmatrix}$, $\bar{F} = \begin{bmatrix} \underline{0} \\ I \end{bmatrix}$, $\bar{C} = \begin{bmatrix} C & F \end{bmatrix}$ ($\underline{0}$ means the zero matrix and *I* the identity matrix of appropriate dimensions).

In order to provide efficient fault detection and isolation, the synthesis of a residual decoupled to sensor fault is dealt with a Polytopic Unknown Input Observer as proposed by (Rodrigues, 2006) in a multi-model framework. It should be noted that Polytopic Unknown Input Observer was recently proposed in (Millerioux and Daafouz, 2004) for communication purposes but not for fault diagnosis.

Under the assumptions that the necessary conditions for the existence of an Unknown Input Decoupled Observer, defined by (Hou and Muller, 1994) in linear case, are fulfilled

cdt i) the number of measurements is greater than the number of unknown inputs i.e. $q \le m$ (always fulfilled with sensor faults);

cdt ii) unknown input matrix is a full column rank i.e. equal to q.

a Polytopic Unknown Input Observer associated to (10) is defined such that:

$$\begin{cases} z_{k+1} = \sum_{j=1}^{\alpha+1} \rho_j(\lambda_k)\left(S_j z_k + T\overline{B}_j u_k + K_j y_k\right) \\ \hat{\overline{x}}_{k+1} = z_{k+1} + H^* y_{k+1} \end{cases} \quad (11)$$

or also with notation $(\bullet)(\rho_k) = \sum_{j=1}^{\alpha+1} \rho_j(\lambda_k)(\bullet_j)$ as:

$$\begin{cases} z_{k+1} = S(\rho_k) z_k + T\overline{B}(\rho_k) u_k + K(\rho_k) y_k \\ \hat{\overline{x}}_{k+1} = z_{k+1} + H^* y_{k+1} \end{cases} \quad (12)$$

where $\hat{\overline{x}}$ denotes the estimated state and z the observer state vector.

The error estimation $e_k = \overline{x}_k - \hat{\overline{x}}_k$ between (10) and (12) is equivalent to:

$$\begin{aligned} e_{k+1} &= \overline{x}_{k+1} - \left(z_{k+1} - H^* y_{k+1}\right) \\ &= \overline{x}_{k+1} - z_{k+1} + H^*\left(\overline{C}\overline{x}_{k+1}\right) \\ &= \left(I - H^*\overline{C}\right)\overline{x}_{k+1} - S(\rho_k) z_k \\ &\quad - T\overline{B}(\rho_k) u_k - K(\rho_k) y_k \end{aligned} \quad (13)$$

By taking into account the gain decomposition $K(\rho_k)$ such as $K(\rho_k) = K^1(\rho_k) + \Pi(\rho_k)$, (12) leads to:

$$\begin{aligned} e_{k+1} &= \left(I - H^*\overline{C}\right)\left(\overline{A}(\rho_k)\overline{x}_k + \overline{B}(\rho_k) u_k + \overline{F} f_k\right) \\ &\quad - K^1(\rho_k)\overline{C}\overline{x}_k - \Pi(\rho_k) y_k - S(\rho_k)\left(\overline{x}_k - e_k - H^* y_k\right) \\ &- T\overline{B}(\rho_k)\overline{u}_k \end{aligned} \quad (14)$$

Consequently, the estimation error and the residual is equivalent to:

$$\begin{cases} e_{k+1} = S(\rho_k) e_k - \left[S(\rho_k) - \left(I - H^*\overline{C}\right)\overline{A}(\rho_k) - K^1(\rho_k)\overline{C}\right]\overline{x}_k \\ \quad - \left[T - \left(I - H^*\overline{C}\right)\right]\overline{B}(\rho_k) u_k - \left[\Pi(\rho_k) - S(\rho_k) H^*\right] y_k \\ \quad + \left(I - H^*\overline{C}\right)\overline{F} f_k \\ r_k = \overline{C} e_k \end{cases} \quad (15)$$

Thus, $S(\rho_k)$, $K(\rho_k) = K^1(\rho_k) + \Pi(\rho_k)$, H^* and T matrices of the Polytopic Unknown Input Observer (12) are designed to be insensitive only to $\overline{f}_k$ such as:

$$\begin{cases} S(\rho_k) = T\overline{A}(\rho_k) - K^1(\rho_k)\overline{C} \\ T = \left(I - H^*\overline{C}\right) \\ \Pi(\rho_k) - S(\rho_k) H^* = 0 \\ T\overline{F} = 0 \end{cases} \quad (16)$$

The synthesis of the Polytopic Unknown Input Observer is realized through the resolution of equations (16) under the condition that $S(\rho_k)$ is stable. The necessary and sufficient conditions for the existence of a Polytopic Unknown Input Observer are directly extended from the linear case presented in (Chen and Patton, 1999):

i) $rank(\overline{C}\overline{F}) = rank(\overline{F}) = q \leq m$;

ii) $j \in [1, \cdots, \alpha+1]$, $(T\overline{A}_j, \overline{C})$ are detectable pairs.

If condition *i)* is fulfilled, then :

$$H^* = \overline{F}\left(\overline{C}\overline{F}\right)^+ \quad (17)$$

Condition *ii)* ensures that a gain $K^1(\rho_k)$ can be synthesized in order to obtain a Hurwitz matrix $S(\rho_k) = T\overline{A}(\rho_k) - K^1(\rho_k)\overline{C}$ in order to generate an estimation error and consequently a residual vector which tends asymptotically to zero in fault-free case otherwise in faulty case.

If the previous conditions hold true, equation (15) becomes:

$$\begin{cases} e_{k+1} = S(\rho_k) e_k \\ r_k = \overline{C} e_k \end{cases} \quad (18)$$

The definition of Polytopic Unknown Input Observer requires the design of a specific gain $K(\rho_k) = K^1(\rho_k) + \Pi(\rho_k)$ to generate a residual which is decoupled from disturbances. $\Pi(\rho_k)$ is determined as a solution of (16) whereas the gain $K^1(\rho_k)$ should be synthesized in order to obtain a Hurwitz matrix $S(\rho_k)$ equivalent to:

$$S(\rho_k) = T\overline{A}(\rho_k) - K^1(\rho_k)\overline{C} \quad (19)$$

In order to achieve this objective for convex sets, a classical pole assignment by LMI (Oliviera et al., 1999) (Chilali and Gahinet, 1996) is considered in this paper. Pole assignment by LMI ensures the polytopic observer stability and its poles will be constrained in a specified and appropriate region of the complex plane (Rodrigues *et al.*, 2005).

3.3 Generalized polytopic unknown input observer scheme

While a single residual is sufficient to detect a fault, a set of residuals is required for fault isolation. Several methods have been proposed in the literature to generate structured residuals and to perform the fault diagnosis (Isermann and Ballé, 1996). The basic idea of the proposed approach is to reconstruct the state of the system from the subsets of measurements. The objective is to build a bank of observers so that each is driven by all inputs and all outputs except the j^{th} measurement variable. Signal y_j is not used in the j^{th} observer due to the fact that y_j is assumed to be corrupted by the fault and therefore does not carry the relevant information. This fault diagnosis scheme is similar to the well known Generalized Observer Structure (GOS) proposed by (Frank, 1990). According to the proposed approach, the bank of unknown input observers generates an incidence matrix as follows where each column is called the coherence vector associated to each fault signature:

Table 1. Incidence matrix

Fault	F_0	F_1	F_2	•••	F_m
$\lVert r \rVert_{y_1}$	0	0	1	1	1
$\lVert r \rVert_{y_2}$	0	1	0	1	1
•••	0	1	1	0	1
$\lVert r \rVert_{y_m}$	0	1	1	1	0

Therefore, without (F_0) sensor faults, the bank of decoupled observers generates some zero-mean residuals. Otherwise, the Polytopic Unknown Input Observers, insensitive to a sensor fault (F_j), is easily isolated based on the GOS structure. Decision making is then carried-out according to an elementary logic (Leonhardt and Ayoubi, 1997) which can be described as follows: a fault indicator is equal to one if the residual vector generated by the bank is equal to a column of the incidence matrix and is equal to zero otherwise. The element which is associated with the indicator being equal to one is then declared to be faulty.

In the next section, an example is provided in fault-free case and sensor faulty case to illustrate the performance and limitations of the developed method.

4. THE WINDING MACHINE

4.1 System description

The winding process is composed of a plastic web and three reels, respectively called the unwinding, pacer and rewinding reels but the radius are unmeasurable. Each reel is coupled with a DC-motor via gear reduction. The angular speed of each reel (S_1, S_2, S_3) and both tensions between the reels (T_1, T_3) are measured by tachometers and tension meters. Each motor is driven by a local controller composed of one or two PI controllers. The first control loop adjusts the motor current (I_1, I_2, I_3), and its integration time constant is about 40 ms, while the second loop controls the angular speed with an integration time constant equal to approximately 200 ms. The set-points of those controllers $\left(I_1^*/S_1^*, I_2^*/S_2^*, I_3^*/S_3^*\right)$ are computed by a programmable logic controller (PLC) in order to control both tensions and the linear velocity of the strip (300 m length, 5 cm broad and 0.2 mm thickness). Under specific experimental investigation which lasts 40 minutes, the radius of the unwinding reel varies from 230 to 50 mm. A real-time development environment (Simulink Real-Time Workshop + dSPACE) based on a PC computer is used instead of the PLC to improve new control law for instance. System inputs and outputs are given in the interval $[0 \quad 100\%]$ corresponding to $[-10\text{V} \quad +10\text{V}]$.

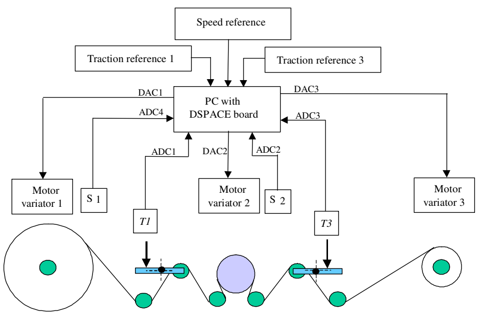

Fig.1. Architecture of the winding process control.

4.2 Nonlinear state space representation

As proposed by (Ponsart and Theilliol, 2004), the dynamic behaviour of the winding process can be represented with a dependence on the unwinding reel radius R :

$$\begin{cases} x_{k+1} = A(R_k)\, x_k + B(R_k)\, u_k \\ y_k = x_k \end{cases} \tag{20}$$

where $x_k = x(k\,T_e)$, with sampling period $T_e = 0.1\text{s}$, and $y = x = [T_1 \quad S_2 \quad T_3]^T$, $u = [U_1 \quad U_2 \quad U_3]^T$. Each coefficient of matrices A and B is expressed in the following polynomial form $(\forall\, i, j = 1, \ldots, 3)$:

$$\{a_{i,j}, b_{i,j}\}(R_k) = \lambda_{i,j}^0 + \lambda_{i,j}^1 R_k + \lambda_{i,j}^2 R_k^2 + \lambda_{i,j}^3 R_k^3 + \lambda_{i,j}^4 R_k^4 + \lambda_{i,j}^5 R_k^5 + \lambda_{i,j}^6 R_k^6 \tag{21}$$

where $\lambda_{i,j}^{\sigma}$ $(\sigma = 0, \ldots, 6)$ are constant values of polynomial form.

For technical reason, the radius is estimated via the following expression:

$$R_k = R_{k-1} + \frac{h}{2\pi} S_{1,k} \tag{22}$$

where h is the strip thickness.

Sensors faults can occurred only in $y = x = [T_1 \quad S_2 \quad T_3]^T$. S_1 which defines the angular speed of reel 1, is assumed to be fault free. According to §3.1., the discrete state space representation (20) of the winding machine can be expressed as a polytopic system:

$$\begin{cases} x_{k+1} = \displaystyle\sum_{j=1}^{7} \rho_j(R_k)\left(A_j x_k + B_j u_k\right) \\ y_k = x_k + F f_k \end{cases} \tag{23}$$

with $\forall\, j \in [0, 1, \cdots, 7]\ \rho_j \geq 0$ $\quad \displaystyle\sum_{j=1}^{7} \rho_j(R) = 1$ and $0 < R_{min}(= 70mm) < R_k < R_{max}(= 210mm)$.

4.3 Control loop

Based on the polynomial model, an input-output linearizing control law ((Fossard and Normand-Cyrot, 1995), (Isidori, 1995)) has been used to control this unstable process in open loop. This method is straightforward to apply to winding machine and ensures a suitable control of traction and speed. The controller design in the classical input-output linearizing form is composed of two main parts:

- a linearizing state feedback which linearizes polynomial model and decouples MIMO system into several SISO sub-systems,
- a stabilized state feedback.

Therefore, each decoupled sub-system is equivalent to an exact delay such as:

$$y_{i,k} = v_{i,k-1} \tag{24}$$

where $i \in [1, \ldots, 3]$ represents the number of the output and of the new input v.

To ensure closed loop stability, a proportional output feedback is applied to each decoupled sub-system. Then, the discrete input-output transfer is expressed as:

$$\frac{y_i(z)}{y_{i,ref}(z)} = \frac{(1-K_i)\, z}{z - K_i} \quad \forall\, i = 1 \cdots 3 \tag{25}$$

where the gain stabilisation dynamic is adjusted by K_i and the reference input is $y_{i,ref}$.

For illustration purposes, different scenarios have been conducted under simulated environments and are presented in the next paragraph.

4.4 Results and comments

For the winding machine, the conditions considered in §3.2 are fulfilled, and a generalized polytopic unknown input observer scheme is tested in the fault-free and the faulty cases. Various tests are presented in this section for different values of radius R in order to illustrate the effectiveness and performance of the FDI method. First, the fault free case is considered for the real process. The output responses to reference variations are illustrated in Fig 2 where step responses are considered for a range of 150s with 10% of their corresponding operating values. The dynamic behavior of the outputs demonstrates that the disturbances rejection is synthesized correctly. Indeed, it can be verified that the static errors are cancelled. Moreover, the outputs are decoupled.

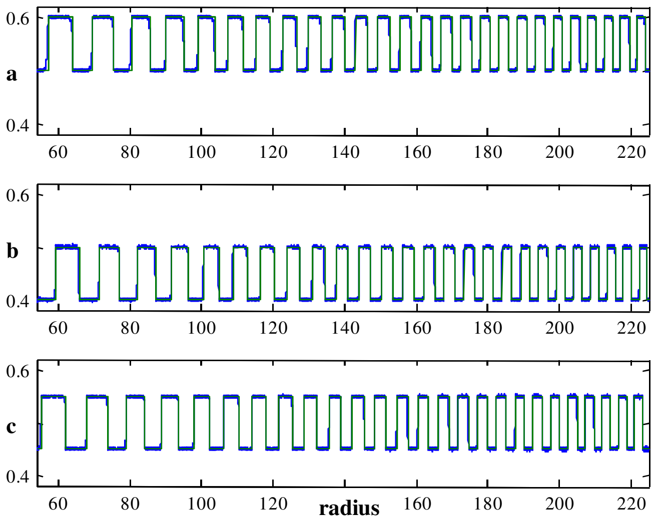

Fig.2. Fault free case: Responses (**a**-T_1, **b**-S_2, **c**- T_3) to reference change versus radius R_k

These results are similar whatever the value of the radius. It can be noted that the residual norm vector issued from the bank of Polytopic Unknown Input Observer insensitive to a specific sensor fault are close to zero.

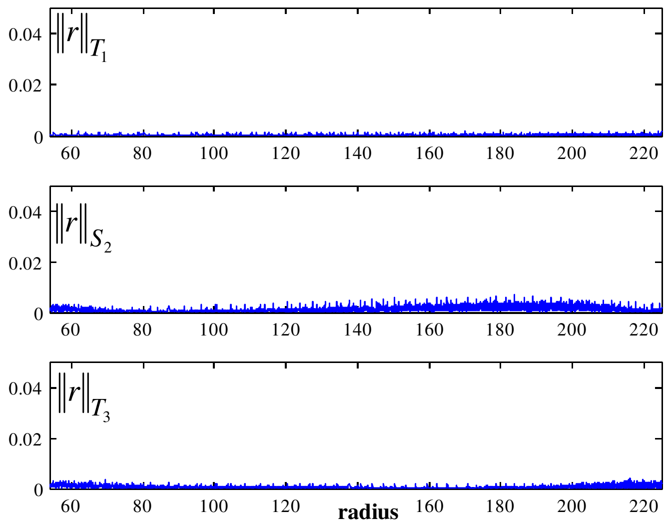

Fig.3. Residual vector norms in Fault free case

A sensor fault on the tension T_1 is supposed to occur and disappear at different times. As defined in (2), a constant gain on the tension T_1 is created and added with $\beta = 0$ and $\beta_0 = 0.05$. This bias can be observed in Fig 4. The control law tries to cancel the static error created by the corrupted output. Consequently, the real output is different from the reference input as illustrated in Fig 4.

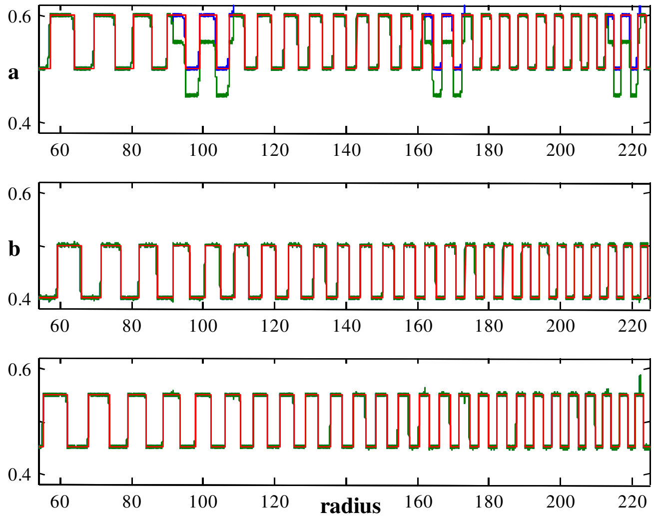

Fig.4.a. Sensor faulty case: Responses (**a**-measured T_1 and realT_1, **b**-S_2, **c**- T_3) to reference change versus radius R_k .

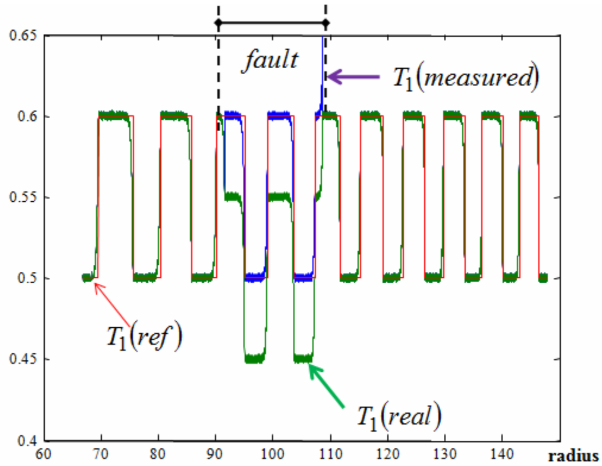

Fig.4.b. Sensor faulty case: Response (**a**-T_1) to reference change versus radius R_k (Zoom around $R \approx 100mm$)

According to the incidence matrix defined in the previous section, only the Polytopic Unknown Input Observer synthesized in order to be insensitive to fault on the tension provides a residual vector equal to zero means as presented in Fig 5.

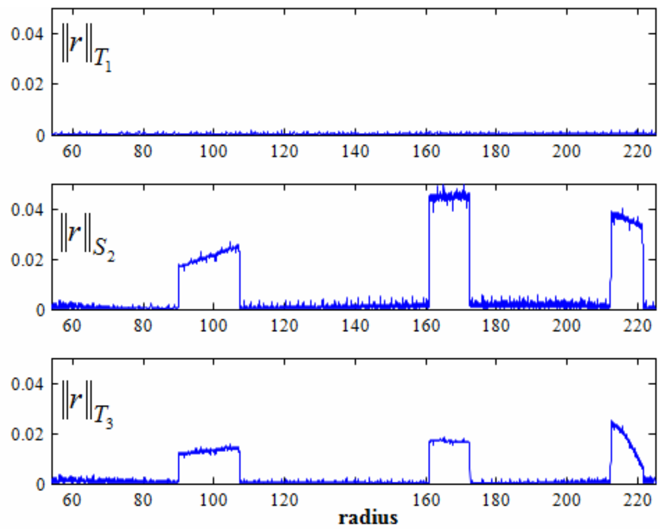

Fig.5. Residual vector norms in Faulty case

The results show that the bank of *g*eneralized polytopic unknown input observer is very effective in detecting and isolating the fault for the whole operating conditions. The residual norm vector should be evaluated through a classical statistical threshold test in order to generate alarms for the operating system.

The generalized polytopic unknown input observer scheme is able to indicate which sensor is faulty and represents an efficient tool in the operator's decision winding process.

5. CONCLUSION

In this paper, a sensor model-based fault diagnosis method for a particular class of nonlinear systems has been developed. Using a polynomial to polytopic transformation, Polytopic Unknown Input Observers that provide decoupled residuals have been synthesized and designed through an appropriate bank in order to detect and isolate sensor faults over the whole operating conditions. Moreover, the experimental results dedicated to web transport process clearly show that sensor faults are detected, isolated thanks to the accurate fault diagnosis module. Based on fault diagnosis module, human operator can access information on the health of the process in order to recognize an abnormal behavior and to keep it safe.

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