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Image-based camera-robot target-tracking satisfying multicriteria constraints

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Abstract: This paper presents an image-based camera control, mounted on a nonholonomic mobile robot platform, traking a mobile target as reference, via task function approach. The system stability is guaranteed by the Lyapunov theory. Due to parametric uncertainties (target depth), actuator (acceleration and velocity) and visual constraints, the gain is generated via LMIs (Linear Matrix Inequalities), in order to maximize the stability region associated with the closed loop. A convex optimization package was used to obtain the feedback gain, and simulations are presented to visualize the system behavior.

1. INTRODUCTION

Visual servoing aims at controlling robotic systems by the information provided by one or more cameras. There are some issues with this approach such as parametric uncertainties and actuator saturation. Therefore, robustness becomes essential in order to guarantee asymptotic stability. As showed in Malis and Rives [2003], the depth uncertainties of the target may reduce the stability domain of a system. Some recent works, such as García-Aracil et al. [2005], proposed a weighted feature task function, in order to allow changes in the visibility of image features during the control task. Despite this, ensuring visibility during the motion is an important issue. Coupling path planning in image space and image-based control, as proposed by Mezouar and Chaumette [2002], introduces constraints, such as object in the trajectory, assuring the convergence for the initial configuration. Improvements in image-based visual servoing, using image moments, were proposed by Tahri and Chaumette [2005].

In this paper we propose a generalization of the technique described by Gao [2006] and Gao et al. [2006]. Gao's technique allows the tracking moving targets with the camera perpendicular to the target. The technique described in this paper, allows tracking moving targets with both the camera and the robot perpendicular to the moving target. As a consequence, the dimension of the task function is doubled in comparison to the previous works, increasing the complexity of the design, but allowing better behavior of the angle between the camera and the robot platform, as both of them must be aligned. This should be useful in tasks such as inspection and surveillance.

This paper is organized as follows: In section 2 the mathematical model of the camera and the robot, divided into robot model and kinematic screw of the camera, is described. Section 3 presents the problem formulation, divided into task function definition and problem statement. In section 4 the control synthesis is developed, separated into preliminaries, theoretical issues and optimization. A quadratic Lyapunov function was used in order to guarantee the system asymptotic stability, and a modified sector condition was used in order to consider the saturation nonlinearity, allowing to be obtained LMI conditions [Boyd et al., 1994] for design purposes. Thus, the control gains can be obtained by a convex optimization algorithm . The simulation results are shown in the section 5, and the overall paper is discussed in section 6.

2. MATHEMATICAL MODEL

2.1 Camera Model

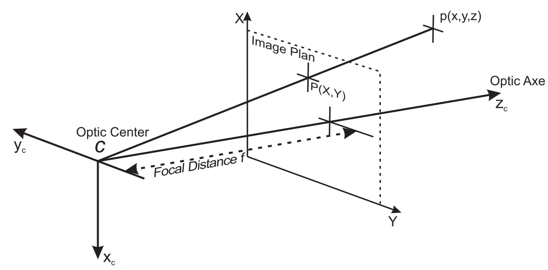

Fig. 1. Pinhole camera model

The camera model considered here is the pinhole [Gao, 2006], as is shown by Fig. 1, where the image plan is 1 m from the optical center. The coordinate system $R_C\,[x_c\; y_c\; z_c]$ represents the camera's coordinate system, and its origin is the optical center C of the camera, where the axis Cz_c corresponds to the optical axis, and the axis Cx_c points vertically down. One point p is projected in the image plan as P, as a perspective projection, and its coordinates are $x_P = [x\; y\; z]$ with respect to R_C, and its coordinates in the image plan are done by the homogenic metric coordinates:

$$\begin{bmatrix} X & Y & 1 \end{bmatrix}' \text{with } X = \frac{x}{z},\ Y = \frac{y}{z} \quad (1)$$

2.2 Robot Model

Figure 2 shows the schematic diagram of the robot and the camera.

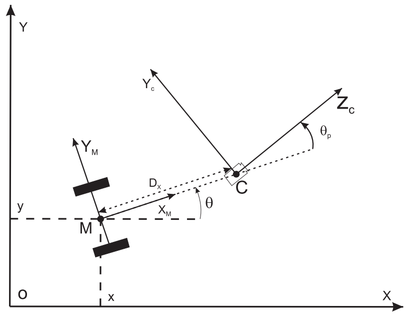

Fig. 2. Robot Model

$R(0, x, y, z)$ is the referential linked to the origin 0, therefore, to the environment; $R_C(C, x_C, y_C, z_C)$ is the referential linked to the camera, having as optical center the point C, z_C coincident with the optical axis; $R_M(M, x_M, y_M, z_M)$ is the referential linked to the mobile robot base; the angle θ_p is the orientation angle of the axis z_C related to the axis x_M; the vector $[x\ y\ \theta]' \in \Re^3$ gives the configuration of the mobile robot related to the referential R, as the pair (x, y) gives the coordinates of M related to R, and the angle θ is the orientation angle of x_M related to the vector x of R.

The kinematic model of the mobile robot and the relations between the camera and the robot, via kinematic screw, are now presented (for details, the reader can consult Gao [2006]).

Kinematic model of the robot: By considering the robot base configuration $[x\ y\ \theta]$ related to the reference R, the mobile robot model is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos(\theta) & 0 \\ sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\theta} \end{bmatrix} \quad (2)$$

where v and $\dot{\theta}$ are the linear and angular velocities of the robot related to R. The robot's kinematic screw $T^R_{R_M/R}$ related to R is:

$$T^R_{R_M/R} = \begin{bmatrix} V^R_{R_M/R} & \Omega^R_{R_M/R} \end{bmatrix}'$$

given by

$$\begin{cases} V^R_{R_M/R} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}' = \begin{bmatrix} v\cos(\theta) & v\sin(\theta) & 0 \end{bmatrix}' \\ \Omega^R_{R_M/R} = \begin{bmatrix} 0 & 0 & \dot{\theta} \end{bmatrix}' \end{cases} \quad (3)$$

.

Kinematic screw of the camera: The reduced screw, related to R_M, is described by the following relation:

$$T^{R_C}_{red} = J_{red}\dot{q} \quad (4)$$

where J_{red} is given by:

$$J_{red} = \begin{bmatrix} -\sin(\theta_p) & D_x\cos\theta_p & 0 \\ \cos\theta_p & D_x\sin\theta_p & 0 \\ 0 & -1 & -1 \end{bmatrix} \quad (5)$$

with $\dot{q} = \begin{bmatrix} v & \dot{\theta} & \dot{\theta}_p \end{bmatrix}$.

3. PROBLEM FORMULATION

3.1 Definition of a Task Function

The target model is presented in the Fig. 3. This target consists of three aligned points E_i, $i = 1, 2, 3$ with the same distance l between them. In this case, $l = 0.5\ m$. α denotes the angle between E_1E_3 and the optical axis z_C of the camera. η is the angle between the optic axis z_C and CE_2. The distance $d_2 = CE_2$ is the distance between the camera and the point E_2 of the target. The depth of the target points is done by z_i, $i = 1, 2, 3$.

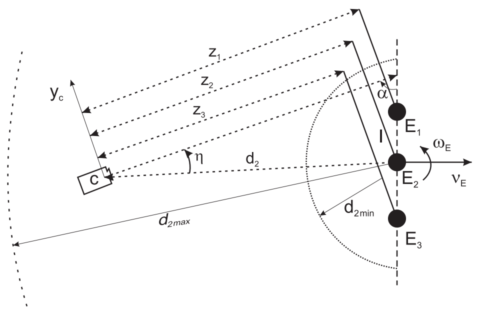

Fig. 3. Target model and its parameters

The position task function is defined as follows:

$$e(t) = \begin{bmatrix} e_{(1)} \\ e_{(2)} \\ e_{(3)} \\ e_{(4)} \\ e_{(5)} \\ e_{(6)} \end{bmatrix} = \begin{bmatrix} S_{yc} \\ S_{zc} - v^*t \\ Y_2 - Y_2^* \\ Y_1 - Y_1^* \\ Y_2 - Y_2^* \\ Y_3 - Y_3^* \end{bmatrix} \in \Re^6 \quad (6)$$

where S_{yc} and S_{zc} are respectively the curvilinear abscissas of the camera in the directions of y_C and z_C; Y_i and Y_i^* are the visual index and the reference visual index, defined in (1) and v^* is the constant reference robot velocity.

The control objective is the regulation of this function to zero.

To find the derivate of $e(t)$, we follow the technique introduced in Samson et al. [1991]:

$$e(r(t), t) = s(r(t), t) - s^* \quad (7)$$

where $s(t)$ is a vector containing visual information, and $r(t)$ is a function that links the camera to the reference R. The derivate becomes:

$$\dot{e}(r(t), t) = \dot{s}(r(t), t) = \frac{\partial s}{\partial r} + \frac{\partial s}{\partial t} = \begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \end{bmatrix} + \begin{bmatrix} \dot{Y}_{E_1} \\ \dot{Y}_{E_2} \\ \dot{Y}_{E_3} \end{bmatrix} \quad (8)$$

where $\dot{Y}_{Ei}$, $\forall\ i = 1, 2, 3$, is the variation of s related to the target movement with respect to the referential R as proposed by Gao [2006].

So, as given by Chaumette [1990]:

$$\dot{s}(t) = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = L_s T_{R_c/R} = \begin{bmatrix} -\frac{1}{z} & 1+Y^2 & -XY \end{bmatrix} T_{R_c/R} \tag{9}$$

where L_s is called Interaction Matrix [Chaumette et al., 1991] and [Chaumette, 1990]. The partial derivatives $\frac{\partial s}{\partial r}$ and $\frac{\partial s}{\partial t}$ are described by:

$$\frac{\partial s}{\partial r} = \tilde{B}_1 T + \tilde{B}_2 v^*; \ \frac{\partial s}{\partial t} = \tilde{B}_3 \omega$$

where $\omega(t)$ is defined as:

$$\omega(t) = \begin{bmatrix} -v_E \cos(\alpha) \\ -v_E \sin(\alpha) \\ -l\omega_E \cos(\alpha) \\ -l\omega_E \sin(\alpha) \end{bmatrix} \in \Re^4$$

with v_E and ω_E begin the linear and angular velocities of the moving target, and $\tilde{B}_1$, $\tilde{B}_2$ and $\tilde{B}_3$ are obtained from (1) and (9):

$$\tilde{B}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/z_2 & (e_{(3)}+Y_2^*)/z_2 & 1+(e_{(3)}+Y_2^*)^2 \\ -1/z_1 & (e_{(4)}+Y_1^*)/z_2 & 1+(e_{(4)}+Y_1^*)^2 \\ -1/z_2 & (e_{(5)}+Y_2^*)/z_2 & 1+(e_{(5)}+Y_2^*)^2 \\ -1/z_3 & (e_{(6)}+Y_3^*)/z_2 & 1+(e_{(6)}+Y_3^*)^2 \end{bmatrix}; \ \tilde{B}_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\tilde{B}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/z_1 & -(e_{(4)}+Y_1^*)/z_1 & 1/z_1 & -(e_{(4)}+Y_1^*)/z_1 \\ 1/z_2 & -(e_{(5)}+Y_2^*)/z_2 & 0 & 0 \\ 1/z_3 & -(e_{(6)}+Y_3^*)/z_3 & -1/z_3 & (e_{(6)}+Y_3^*)/z_3 \end{bmatrix}$$

The objective is to find a control law and a stability region associated with the task function, taking into account:

C1: the depth of the target points E are bounded but unknown;

C2: the errors $e_{(i)}$ $i = 3, 4, 5, 6$ must be bounded during the task, to ensure the visibility;

C3: the kinematic screw of the camera related to R must be bounded to satisfy the actuators constraints as:

$$\begin{array}{c} -u_1 \preceq T \preceq u_1 \\ -u_0 \preceq \dot{T} \preceq u_0 \end{array} \tag{10}$$

C4: the velocity vector of the target is square integrable but unknown.

Let us consider the augmented state

$$x(t) = \begin{bmatrix} e(t)' & T(t)' \end{bmatrix}' \in \Re^9$$

and its derivate:

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & \tilde{B}_1 \\ 0 & 0 \end{bmatrix}}_{\mathbb{A}} x(t) + \underbrace{\begin{bmatrix} 0 \\ I_3 \end{bmatrix}}_{\mathbb{B}_1} \dot{T}(t) + \underbrace{\begin{bmatrix} \tilde{B}_3 \\ 0 \end{bmatrix}}_{\mathbb{B}_2} \omega + \begin{bmatrix} \tilde{B}_2 \\ 0 \end{bmatrix} v^* \tag{11}$$

By defining the tracking error as $\varepsilon(t) = x(t) - r$, where $r = [0\ 0\ 0\ 0\ 0\ 0\ 0\ v^*\ 0]'$, the complete system reads as follows:

$$\dot{\varepsilon}(t) = \mathbb{A}\varepsilon(t) + \mathbb{B}_1\dot{T}(t) + \mathbb{B}_2\omega + \mathbb{A}r + \begin{bmatrix} \tilde{B}_2 \\ 0 \end{bmatrix} v^* \tag{12}$$

$$\dot{\varepsilon}(t) = \mathbb{A}\varepsilon(t) + \mathbb{B}_1\dot{T}(t) + \mathbb{B}_2\omega + \mathbb{B}_3 v^* \tag{13}$$

The part $\mathbb{A}\varepsilon$ can be written as follows:

$$\mathbb{A}\varepsilon = \left[\mathbb{R}'\bar{B}_1\mathbb{C} + \mathbb{R}'T_{(2)}\bar{B}_2\mathbb{R} + \mathbb{B}T_{(3)}\left(\bar{B}_3 + D\right)\mathbb{R}\right]\varepsilon - \mathbb{B}_3 v^* \tag{14}$$

where $\mathbb{B}_3$, $\mathbb{R} = [I_6\ 0] \in \Re^{6\times 9}$ and $\mathbb{C} = [0\ I_6] \in \Re^{6\times 9}$ are given by

$$\mathbb{B}_3 = \begin{bmatrix} 0 \\ 0 \\ (e_{(3)}+Y_2^*)/z_2 \\ (e_{(4)}+Y_1^*)/z_1 \\ (e_{(5)}+Y_2^*)/z_2 \\ (e_{(6)}+Y_3^*)/z_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} v^*$$

and,

$$\bar{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/z_2 & 0 & 1 \\ 0 & 0 & 0 & -1/z_1 & 0 & 1 \\ 0 & 0 & 0 & -1/z_2 & 0 & 1 \\ 0 & 0 & 0 & -1/z_3 & 0 & 1 \end{bmatrix}; \ \bar{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/z_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/z_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/z_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/z_3 \end{bmatrix} \tag{15}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e_{(3)} & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{(4)} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{(5)} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{(6)} \end{bmatrix}; \ \bar{B}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_2^* & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_1^* & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_2^* & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_3^* \end{bmatrix} \tag{16}$$

The system is now written as:

$$\dot{\varepsilon}(t) = \left[\mathbb{R}'\bar{B}_1\mathbb{C} + \mathbb{R}'T_{(2)}\bar{B}_2\mathbb{R} + \mathbb{B}T_{(3)}\left(\bar{B}_3 + D\right)\mathbb{R}\right]\varepsilon + \mathbb{B}_1\dot{T} + \mathbb{B}_2\omega \tag{17}$$

3.2 Problem Statement

Find the region sets $\mathcal{S}_\infty$ and $\mathcal{S}_\prime$ and the control gain $\mathbb{K} \in \Re^{3\times 9}$ that guarantees stability, even with uncertainties, and respect the visual index and velocities constraints, satisfying:

$$\dot{T} = sat_{u_0}(\mathbb{K}\varepsilon) \tag{18}$$

4. CONTROL SYNTHESIS

4.1 Preliminaries

Consider the memoryless and descentralized nonlinearity $\phi(\mathbb{K}\varepsilon)$ satisfying [da Silva Jr. and Tarbouriech, 2004]:

$$\phi(\mathbb{K}\ \varepsilon) = sat_{u_0}(\mathbb{K}\varepsilon) - \mathbb{K}\varepsilon \tag{19}$$

where each component of $\phi(\mathbb{K}\varepsilon)$ is defined as:

$$\phi_i(\mathbb{K}\ \varepsilon) = \begin{cases} u_{0(i)} - \mathbb{K}\varepsilon \text{ if } \mathbb{K}\varepsilon > u_{0(i)} \\ 0 \text{ if } |\mathbb{K}\varepsilon| \leq u_{0(i)} \\ -u_{0(i)} - \mathbb{K}\varepsilon \text{ if } \mathbb{K}\varepsilon < -u_{0(i)} \end{cases} \tag{20}$$

This form of defining the nonlinearity permits to transform the saturation problem into a deadzone. As a consequence, the classical methods that envolve sectors condition can be used to provide the LMI formulation.

The deadzone nonlinearity $\phi(\mathbb{K}\ \varepsilon)$ satisfies the sector condition described by the following lemma:

Lemma 1. Consider a matrix $\mathbb{G} \in \Re^{m\times n}$. If $\varepsilon \in S(u_0)$, defined by

$$S(u_0) = \left\{x \in \Re^m; -u_0 \preceq (\mathbb{K} - \mathbb{G})\varepsilon \preceq u_0\right\} \tag{21}$$

then the nonlinearity $\phi(\mathbb{K}\ \varepsilon)$ satisfies the following sector condition

$$\phi(\mathbb{K}\ \varepsilon)' M(\phi(\mathbb{K}\ \varepsilon) + \mathbb{G}\varepsilon) \leq 0 \tag{22}$$

for all positive-definite diagonal matrix $M \in \Re^{m\times n}$.

□

The closed loop system becomes:

$$\dot{\varepsilon}(t) = \left[\mathbb{R}'\bar{B}_1\mathbb{C} + \mathbb{R}'T_{(2)}\bar{B}_2\mathbb{R} + \mathbb{B}T_{(3)}\left(\bar{B}_3 + D\right)\mathbb{R} + \mathbb{B}_1\mathbb{K}\right]\varepsilon + \\ + \mathbb{B}_1\phi(\mathbb{K}_\varepsilon) + \mathbb{B}_2\omega \tag{23}$$

From the constraints **C1**, **C2**, **C3** and **C4**, one has

$$\left|e_{(i)}\right| \leq \beta, \forall\ i = 3,4,5,6 \tag{24}$$

$$-u_1 \preceq T \preceq u_1 \tag{25}$$

Thus ε should belong to the polyhedral set $\Omega(\varepsilon)$

$$\Omega(\varepsilon) = \left\{\varepsilon \in \Re^9; -\begin{bmatrix}\beta\\\beta\\\beta\\\beta\\u_{1(1)}\\u_{1(2)} - v^*\\u_{1(3)}\end{bmatrix} \preceq \begin{bmatrix}\varepsilon_{(3)}\\\varepsilon_{(4)}\\\varepsilon_{(5)}\\\varepsilon_{(6)}\\\varepsilon_{(7)}\\\varepsilon_{(8)}\\\varepsilon_{(9)}\end{bmatrix} \preceq \begin{bmatrix}\beta\\\beta\\\beta\\\beta\\u_{1(1)}\\u_{1(2)} - v^*\\u_{1(3)}\end{bmatrix}\right\} \tag{26}$$

Consider the uncertain parameters:

$$z = \begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix} = \begin{bmatrix}z_2 + l\cos(\alpha)\\z_2\\z_2 - l\cos(\alpha)\end{bmatrix}$$

The following uncertain parameter can also be defined:

$$\frac{1}{z_2} = p_1 \tag{27}$$

where p_1 satisfies $\alpha \in [-\pi + \alpha_{min}, -\alpha_{min}]$, $|\eta| \leq \eta_{max} = \arctan(\beta) < \pi/2$, $d_2 \in [d_{2\ min}, d_{2\ max}]$ and $z_2 \in [d_{2\ min}\cos(\eta_{max}), d_{2\ max}]$. Hence

$$\frac{1}{z_1} = \frac{1}{z_2\left(1 + \frac{l\cos(\alpha)}{z_2}\right)};\ \frac{1}{z_3} = \frac{1}{z_2\left(1 - \frac{l\cos(\alpha)}{z_2}\right)}$$

Since $l \ll z_2$, the relations above can be rewritten:

$$\frac{1}{z_1} \approx \frac{1}{z_2}\left(1 - \frac{l\cos(\alpha)}{z_2}\right) = p_1 - p_2$$
$$\frac{1}{z_3} \approx \frac{1}{z_2}\left(1 + \frac{l\cos(\alpha)}{z_2}\right) = p_1 + p_2$$

By considering the intervals between z_2, α, p_1 and p_2, satisfying

$$p_1 \in \left[\frac{1}{d_{2\ max}}, \frac{1}{d_{2\ min}}\cos(\eta_{max})\right] \tag{28}$$

$$p_2 \in \left[\frac{l\cos(-\pi + \alpha_{min})}{(d_{2\ min}\cos(\eta_{max}))^2}, \frac{l\cos(-\alpha_{min})}{(d_{2\ min}\cos(\eta_{max}))^2}\right] \tag{29}$$

where $\bar{B}_k$ with $k = 1,2,4,5$ is dependant of the uncertain parameter p_1, described by:

$$\bar{B}_k \in \{B_{kj};\ j = 1,2,3,4\},\ k = 1,2,4,5 \tag{30}$$

The closed-loop system can be described by the following polytopic system:

$$\dot{\varepsilon}(t) = \sum_{j=1}^{4}\lambda_j\left[\left(\mathbb{R}'B_{1j}\mathbb{C} + \mathbb{R}'T_{(2)}B_{2j}\mathbb{R} + \mathbb{R}T_{(3)}(\bar{B}_3 + D)\mathbb{R} + \mathbb{B}_1\mathbb{K}\right)\varepsilon + \\ + \mathbb{B}_1\phi(\mathbb{K}\varepsilon) + \mathbb{B}_2\omega\right] \tag{31}$$

with $\sum_{j=1}^{4}\lambda_j = 1$, $\lambda_j \geq 0$.

4.2 Theoretical Issues

Theorem 2. If there exists a positive-definite function $V(\varepsilon)$ ($V(\varepsilon) > 0\ \forall\varepsilon \neq 0$ *and* $V(0) = 0$) a gain $\mathbb{K}$, a diagonal positive-definite matrix M, a matrix $\mathbb{G}$ and two positive scalars ζ and δ_1, satisfying, for all admissible depth z:

$$\frac{\partial V}{\partial\varepsilon}\left[\left(\mathbb{R}'\bar{B}_1\mathbb{C} + \mathbb{R}'T_{(2)}\bar{B}_2\mathbb{R} + \mathbb{R}'T_{(3)}(\bar{B}_3 + D)\mathbb{R} + \right.\right. \\ \left.\left. + \mathbb{B}_1\mathbb{K}\right)\varepsilon + \mathbb{B}_1\phi(\mathbb{K}\varepsilon)\right] - 2\phi(\mathbb{K}\varepsilon)'M(\phi(\mathbb{K}\varepsilon) + \mathbb{G}) + \\ - \omega'\omega < 0 \tag{32}$$

$$V(\varepsilon) - \varepsilon'(\mathbb{K}_{(i)} - \mathbb{G}_{(i)})'\frac{\frac{1}{\zeta} + \frac{1}{\delta_1}}{u_{0(i)}^2}(\mathbb{K}_{(i)} - \mathbb{G}_{(i)})\varepsilon \geq 0 \tag{33}$$

$$V(\varepsilon) - \varepsilon'\mathbb{R}_{(i)}'\frac{\frac{1}{\zeta} + \frac{1}{\delta_1}}{\beta^2}\mathbb{R}_{(i)}\varepsilon \geq 0 \tag{34}$$

$$V(\varepsilon) - \varepsilon'\mathbb{C}_{(i)}'\frac{\frac{1}{\zeta} + \frac{1}{\delta_1}}{u_{1(i)}^2}\mathbb{C}_{(i)}\varepsilon \geq 0 \tag{35}$$

$$V(\varepsilon) - \varepsilon'\mathbb{C}_{(2)}'\frac{\frac{1}{\zeta} + \frac{1}{\delta_1}}{u_{1(2)}^2 - v^*}\mathbb{C}_{(2)}\varepsilon \geq 0 \tag{36}$$

Hence the gain $\mathbb{K}$ and the sets

$$\mathcal{S}_1(V,\zeta,\delta_1) = \left\{\varepsilon \in \Re^9;\ V(\varepsilon) \leq \frac{1}{\zeta} + \frac{1}{\delta_1}\right\}$$
$$\mathcal{S}_0(V,\zeta) = \left\{\varepsilon \in \Re^9;\ V(\varepsilon) \leq \frac{1}{\zeta}\right\}$$

are solutions of the stated problem.

□

The demonstration of this theorem is similar to that shown in Gao [2006].

The difficulty is to choose a Lyapunov function $V(\varepsilon)$ in order to obtain the constructive conditions. An adequated choice is the quadratic:

$$V(\varepsilon) = \varepsilon' P \varepsilon \tag{37}$$

with $P = P' > 0$.

By considering the Theorem 2, (37) and the polytopic system (31), with the objective of set the conditions as LMIs, the following proposition can be written:

Proposition 3. If there are two symmetric positive-definite matrices $W \in \Re^{9\times 9}$, $R_1 \in \Re^{6\times 6}$, a diagonal positive matrix $S \in \Re^{3x3}$, two matrices $Y \in \Re^{3\times 9}$ and $Z \in \Re^{3\times 9}$, three positive scalars ϵ, ζ and δ_1, satisfying:

$$\begin{bmatrix} \begin{matrix} W(\mathbb{R}B_{1j}\mathbb{C})' + (\mathbb{R}B_{1j}\mathbb{C})W + \\ \mathbb{B}_1 Y + Y'\mathbb{B}_1' + \mathbb{R}'R_1\mathbb{R} + \\ \epsilon(u_{1(3)}^2(1+\beta^2+\beta^2))\mathbb{R}'\mathbb{R} \end{matrix} & * & * & * & * & * \\ u_{1(2)}B_{2j}\mathbb{R}W & -R_1 & * & * & * & * \\ [B_3[I0]']\mathbb{R}W & 0 & -\epsilon I & * & * & * \\ S\mathbb{B}_1' - Z & 0 & 0 & -2S & * & * \\ B_{4j}'\mathbb{R} & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & B_{5j} & -\epsilon I \end{bmatrix} < 0 \tag{38}$$

$$\begin{bmatrix} W & * & * \\ Y_{(i)} - Z_{(i)} & \zeta u_{0(i)}^2 & * \\ Y_{(i)} - Z_{(i)} & 0 & \delta_1 u_{0(i)}^2 \end{bmatrix} \geq 0 \quad \forall i = 1,2,3 \tag{39}$$

$$\begin{bmatrix} W & * & * \\ \mathbb{R}_{(i)}W & \zeta\beta^2 & * \\ \mathbb{R}_{(i)}W & 0 & \delta_1\beta^2 \end{bmatrix} \geq 0 \quad \forall i = 1,2,3 \tag{40}$$

$$\begin{bmatrix} W & * & * \\ \mathbb{C}_{(i)}W & \zeta u_{1(i)}^2 & * \\ \mathbb{C}_{(i)}W & 0 & \delta_1 u_{1(i)}^2 \end{bmatrix} \geq 0 \quad \forall i = 1,3 \tag{41}$$

$$\begin{bmatrix} W & * & * \\ \mathbb{C}_{(2)}W & \zeta u_{1(2)}^2 & * \\ \mathbb{C}_{(2)}W & 0 & \delta_1(u_{1(2)} - v^*)^2 \end{bmatrix} \geq 0 \tag{42}$$

The gain $\mathbb{K} \in \Re^{3\times 9}$, is given by $\mathbb{K} = YW^{-1}$.

(1) when $\omega \neq 0$, the trajectories of the closed loop system done by (23) stay bounded in the set

$$\varphi_1(W,\zeta,\delta_1) = \left\{\varepsilon \in \Re^9;\ \varepsilon' W^{-1}\varepsilon \leq \frac{1}{\zeta} + \frac{1}{\delta_1}\right\} \tag{43}$$

for all $\varepsilon(0) \in \varphi_0(W,\zeta)$

$$\varphi_0(W,\zeta) = \left\{\varepsilon \in \Re^9;\ \varepsilon' W^{-1}\varepsilon \leq \frac{1}{\zeta}\right\} \tag{44}$$

and all perturbation $\omega(t)$ satisfying:

$$\|\omega(t)\|_2^2 = \int_0^\infty \omega'(\tau)\omega(\tau)d\tau = \int_0^\infty (V_E^2(\tau)) + l^2\omega_E^2 d\tau \leq \frac{1}{\delta_1} \tag{45}$$

(2) when $\omega = 0$, the sets $\varphi_0(V,\zeta) = \varphi_1(V,\zeta,\delta_1)$ is an asymptotic stability region for the closed loop system showed in (23).

□

The demonstration of this proposition is similar to that shown in Gao [2006].

4.3 Optimization

With the objective of maximizing the size of the sets φ_0 and φ_1, the following convex optimization problem can be formulated by using the LMI condition of the proposition 3.

$$\min_{W,R_1,Y,Z,S,\zeta,\delta_1,\epsilon} \zeta + \delta_1 + \delta + \sigma \tag{46}$$

under the conditions (38), (39), (40), (41), (42), $\begin{bmatrix} \sigma I & * \\ Y & I \end{bmatrix}$ and $\begin{bmatrix} \delta I & * \\ I & W \end{bmatrix}$. These two last conditions are used to minimize the trace of W^{-1} and the norm of Y. By this approach, the value of δ_1 is obtained by minimization.

The distance between the camera and the target is in the interval $d_2 \in [2.226m, 6m]$. The reference visual indices are $Y_1^* = 0.2$, $Y_2^* = 0$ and $Y_3^* = -0.2$. To guarantee the visibility of the target, $\beta = 0.4$ and $\alpha \in [-\pi + \pi/6, -\pi/6]$.

5. NUMERICAL AND SIMULATION RESULTS

By considering the uncertain parameters p_1 and p_2 as showed (28) and (29), the matrices (15), (47) and (48) the polytopes $\bar{B}_{kj}$, with $k = 1,2,4,5$ and $j = 1,2,3,4$ can be determined, using α, η and $d_2 \in [d_{2\ min}; d_{2\ max}]$.

$$\bar{B}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/z_1 & -Y_1^*/z_1 & 1/z_1 & -Y_1^*/z_1 \\ 1/z_2 & -Y_2^*/z_2 & 00 & \\ 1/z_3 & -Y_3^*/z_3 & -1/z_3 & -Y_3^*/z_3 \end{bmatrix} \tag{47}$$

$$\bar{B}_5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1/z_1 & 0 & -1/z_1 \\ 0 & -1/z_2 & 0 & 0 \\ 0 & -1/z_3 & 0 & 1/z_3 \end{bmatrix} \tag{48}$$

Setting kinematic screw limits as $u_1 = [1\ 1\ 0.5]'$ and $u_0 = [4\ 4\ 5]'$ and solving the optimization problem (46), the following gain $\mathbb{K}$ was obtained by the **lmitool Scilab** package:

$$\mathbb{K} = \begin{bmatrix} -19.64 & -0.06 & 1.00 & -2.44 & -0.65 & 4.29 & -5.42 & 0 & 0.18 \\ 0.06 & -31.71 & 0.21 & 0.38 & 0.12 & -0.63 & 0 & -3.73 & 0 \\ -4.84 & 0.16 & -15.86 & -5.20 & 2.65 & -7.41 & 0.50 & 0 & -4.34 \end{bmatrix}$$

Simulation results verified the theoretical results presented in the paper. With respect to the absolute frame of R, the initial coordinates of the target are: $E_1(9.2929, 20.7071)$, $E_2(10, 20)$ and $E_3(10.7071, 19.2929)$. The distance admissible between the target and the camera is $d_2 \in [2.226, 6]$. The reference visual index are $Y_1^* = 0.2$, $Y_2^* = 0$ and $Y_3^* = -0.2$. To guarantee the visibility, $\beta = 0.4$ was considered. The initial configuration of the robot is given by: $x_0 = -5\ m$, $y_0 = 15\ m$, $\theta_0 = 0\ rad$ and $\theta_{p0} = 0.5\ rad$. The bounds on the camera velocity and acceleration are $u_0 = [4\ 4\ 5]'$ and $u_1 = [1\ 1\ 0.5]'$. The reference velocity used was $v^* = 0.8$.

Figure 4 shows the robot's trajectory evolution, tracking a moving 3-point target in linear trajectory. Figure 5 shows the evolution of the orientation angle θ of the robot and θ_p of the camera. The visual index and the task function are illustrated in Fig. 6. The kinematic screw evolution and the control signal are shown in Fig. 7.

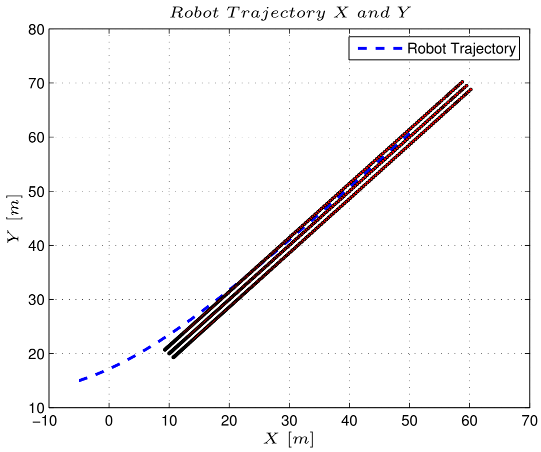

Fig. 4. Trajectory of the robot and the mobile target

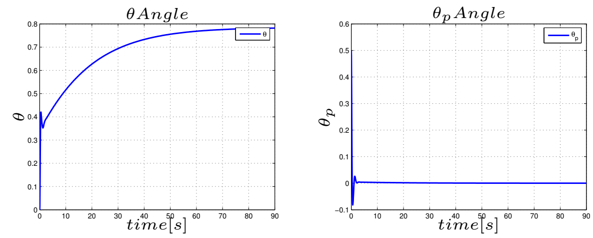

Fig. 5. Orientation angle θ of the robot and θ_p of the camera

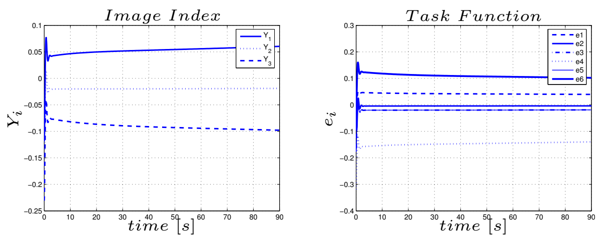

Fig. 6. Visual index and task function

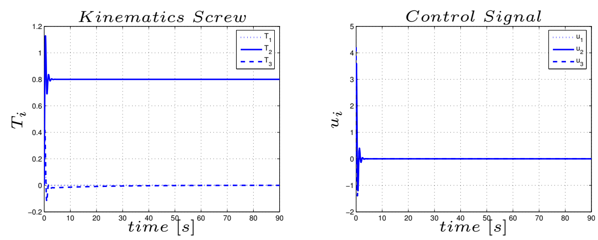

Fig. 7. Kinematic screw and control signal

6. CONCLUSION

This paper proposed the design of a state feedback control for a task function, for the case where both camera and robot principal axis must be perpendicular to the target. In addition to the stability guarantee, several constraints were considered during the design, such as target points depth, actuator saturation, visibility and target velocity. Simulation results show that this task controlled the robot and the camera, achieving the reference velocity of $v^* = 0.8\ m/s$, satisfying the actuator's dynamic constraints. Further works should consider practical applications of the proposed method.

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