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Decentralized Formation Control of Multi Vehicles Systems with Non-Holonomic Constraints Using Artificial Potential Field

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Abstract: In this work, decentralized formation control of a multi vehicle system is investigated. Each vehicle model considers kinematic constraints of differential drives which is a principal approach for various application for mobile robotics in 2D space. A virtual leader is assigned to navigate the whole cluster in a certain formation via predefined paths. Each vehicle produces its own control signal via communication with other vehicles and interactions with virtual leader and the obstacles around. These interactions and communications are modeled with artificial potential fields. Also formation shape is given as global minimums of artificial potentials between vehicles. The proposed system can successfully form any shape and navigate even in the presence of unknown obstacles. Simulations of flocking and formation navigation with kinematics constraints are presented.

Keywords: Multi Vehicles, Decentralized Control, Formation, Artificial Potentials, Non-holonamic Constraint

1. INTRODUCTION

Control of multiple vehicles raises new challenges that do not exist in single vehicle systems. Some examples can be given in areas of communication, coordinated path planning, sensor fusion and formation control. Among these areas, the formation control is possibly the most attractive one for researchers in recent years, since flocking and formation navigation are essential operations for multi vehicle systems, Murray et al. [1997].

In this work a system for formation control of arbitrary number of autonomous vehicles is presented. The presented system uses a kinematic model with nonholonamic constraints and double integrator model for vehicle dynamics which are previously investigated by Jonathan R. T. Lawton [2003] and Pedro V. Fazenda [2007]. This study also improves the method for obstacle avoidance of the vehicle group by means of an artificial vector rotation field defined on obstacle surfaces. Interactions of vehicles are modeled via artificial potentials similar to the gravitational and magnetic potentials. The presented system can flock and navigate in any formation. A virtual leader is used to navigate the vehicles and safe group navigation in formation is ensured in an environment with obstacles while non of the vehicles have the a-priori knowledge about the place and positioning of obstacles.

The article is organized as follows: We begin by presenting basic concepts of artificial potential and movement dynamics in section 2 and explain the vehicles' interactions with leader, other vehicles and obstacles in section 4. In section 5 we present the simulations. The conclusions and future work is presented in the last section.

2. BASIC CONCEPTS

Vehicles are thought to be in various artificial potential fields assumed exist in between the vehicles and between the vehicles and obstacles. These potentials are similar to magnetic or gravitational potentials and create forces. Every vehicle moves under the influence of these virtual forces. Potential fields are constructed to have a global minimum at a desired position q_f of a vehicle. $q_f \in \mathbb{R}^2$ is a two dimensional position vector in cartesian coordinates. Generally a potential field consists of combinations of attractive and repulsive fields. An attractive field is a field that pulls the vehicle towards a desired position, and a repulsive field is a field that pushes the robot away from a non-desired position.

$$U = U_{att} + U_{rep} \tag{1}$$

According to this formulation, finding where to move is an optimization problem of finding the global minimum of U starting from the initial position q_i. Gradient descent can be used in order to solve this problem. In this case the negative gradient of U gives the force acting on the vehicle.

$$F = -\nabla U \tag{2}$$

Just like in gravitational potentials, artificial potentials' gradients are forces, and these forces are used to calculate the control signal u which can be thought as the acceleration of the vehicle. In this case p is speed and q is position of the vehicle, which is in accordance with the movement dynamics given in equation in the next chapter.

3. KINEMATIC AND DYNAMIC MODEL OF VEHICLES

Each vehicle in the vehicle groups has the state vector $x_i = (q_{x_i}\ q_{y_i}\ \theta_i\ p_i\ \omega_i)^T$, $x \in \Re^5$,where q_{x_i} and q_{y_i} represent position vector, $\theta_i$represents orientation angle and p_i, ω_i represents linear and angular velocities respectively for the vehicle i and this can be shown in figure 1. Dynamical equations the single vehicle can derived by the following nonlinear equation set:

$$\begin{pmatrix} \dot{q_{x_i}} \\ \dot{q_{y_i}} \\ \dot{\theta_i} \\ \dot{p_i} \\ \dot{\omega_i} \end{pmatrix} = \begin{pmatrix} p_i cos(\theta_i) \\ p_i sin(\theta_i) \\ \omega_i \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/m_i & 0 \\ 0 & 1/J_i \end{pmatrix} \begin{pmatrix} F_i \\ \tau_i \end{pmatrix} \quad (3)$$

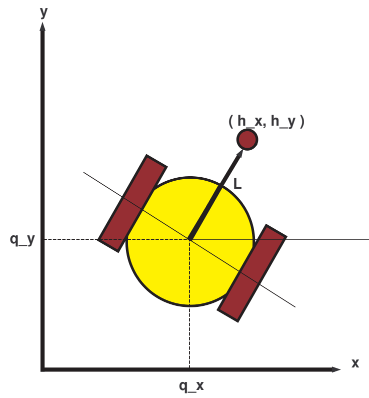


Fig. 1. A single vehicle.

where F_i and $\tau_i$are force and torque inputs affecting center of the vehicle i. As it is seen in the state equation, state transition terms are nonlinear while the input terms in linear relationship. Hence the state equation, in general, can be given as $x_i = f(x_i) + g_i u_i$. It is obvious that the input vector u for vehicle i is formed as $u_i = (\ F_i\ \tau_i\ )$. Due to constraints in the wheels of the vehicles, the input variable F may not cause a motion along some directions. In our general approach those forces are derived from gradient operator of the potential fields. Hence, lets assume that the external forces are applied a specific point instead of the center of the vehicle, namely, handle point, as it is shown in figure 1. The distance of the point from the center is given by L parameter. Then point h_i is defined by:

$$h_i = q_i + L_i \begin{pmatrix} cos(\theta_i) \\ sin(\theta_i) \end{pmatrix} \quad (4)$$

by taking time derivation of both sides of the above equation:

$$h_i = \begin{pmatrix} cos(\theta_i) & -L_i sin(\theta_i) \\ sin(\theta_i) & L_i cos(\theta_i) \end{pmatrix} \begin{pmatrix} p_i \\ \omega_i \end{pmatrix} \quad (5)$$

It is possible to define a closed map such that $\Sigma(x_i) : \Re^5 \rightarrow \Re^5$ which maps the state vector x_i into a state vector assigned at handle point h_i:

$$\beta_i = \Sigma(x_i) = \begin{pmatrix} q_{x_i} + L_i cos(\theta_i) \\ q_{y_i} + L_i sin(\theta_i) \\ p_i cos(\theta_i) - L_i \omega_i sin(\theta_i) \\ p_i sin(\theta_i) - L_i \omega_i cos(\theta_i) \\ \theta_i \end{pmatrix} \quad (6)$$

$$\beta_i = (\ \beta_{1_i}\ \ \beta_{2_i}\ \ \beta_{3_i}\ \ \beta_{4_i}\ \ \beta_{5_i}\ )$$

The mapping Σ between x_i and β_i is diffeomorphism Jonathan R. T. Lawton [2003] and its inverse is given by:

$$\Sigma^{-1}(\beta_i) = \begin{pmatrix} \beta_{1_i} - L_i cos(\beta_{5_i}) \\ \beta_{2_i} - L_i sin(\beta_{5_i}) \\ \beta_{5_i} \\ \beta_{3_i} cos(\beta_{5_i}) + \beta_{4_i} sin(\beta_{5_i}) \\ (-1/L_i)\beta_{3_i} sin(\beta_{5_i}) + (1/L_i)\beta_{4_i} cos(\beta_{5_i}) \end{pmatrix}$$

Thus, the inverse mapping supply position and velocity vectors for the handling point. The orientation of a vehicle θ_i is uncontrollable as a result of the inverse mapping, however the orientation will always be aligned with the velocity vector in translational motion. For the sake of simplicity, the vehicle dynamic model will then be assumed as a double integrator in order to get rid of inertial parameters such as m_i and J_i. Thus, the dynamic of vehicle i is represented by:

$$\ddot{h} = \dot{p} \quad (7)$$

formation control of these vehicles is explained in the following chapter.

4. VEHICLE FORMATION & CONNECTIONS

As mentioned before every vehicle is modeled as double integrator dynamics is given in equation (**??**). In a flocking motion three types of potential fields exist: One between a vehicle and the virtual leader, the other between a vehicle and its neighbors and the last one is between a vehicle and the obstacles around. These potentials are explained in the following sections.

4.1 Interactions between vehicle and leader

The field defined between a vehicle and a leader is an attractive field that attracts the vehicle towards the leader. For some or all vehicles a field having a global minimum at a desired distance from the virtual leader is defined. This field create a force at the direction of its gradient which attracts the vehicle at that desired location as illustrated in figure 2.

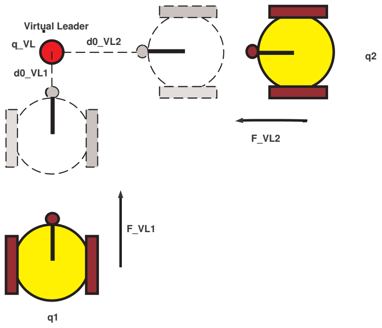


Fig. 2. Interaction between a vehicle and the virtual leader.

Let q_i be the position of the vehicle, q_{VL} be the position of the virtual leader, d_{i,VL_0} be the desired distance of ith vehicle to the virtual leader and k_{VL} is a scalar coefficient. The potential field is defined in equation (8):

$$U_{VL,i} = \frac{1}{2} k_{VL} ||q_{VL} - q_i - d_{i,VL_0}||^2 \tag{8}$$

The attractive force that acts on the vehicle is thus:

$$F_{VL} = k_{VL}(d_{i,VL} - d_{i,VL_0}) \tag{9}$$

$$d_{i,VL} = q_{VL} - q_i \tag{10}$$

4.2 Communication between neighboring vehicles

In a desired formation, a neighboring relation between the vehicles are defined. In such a case, vehicle i is required to align its position according to a limited set of vehicles $j \in J_i$. Here J_i is the set of neighboring vehicles for ith vehicle. For example in the triangular shape in figure 3, vehicles 2 and 3 are neighbors for vehicle 1, thus vehicle 1 is required to align its position according to vehicles 2 and 3. The neighboring relationship is a unidirectional relationship.

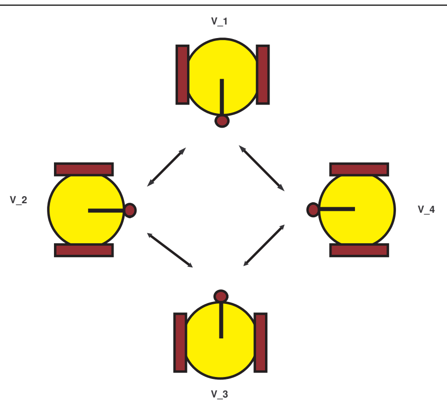


Fig. 3. A circular formation

This alignment is maintained by an artificial potential fields defined in between neighboring vehicles. This field is formulated as below:

$$U_{ij} = \frac{1}{2} k_{ij} ||q_j - q_i - d_{ij_0}||^2 \tag{11}$$

Say q_i and q_j are positions of ith and jth vehicles respectively and k_{ij} is a scalar coefficient for inter-vehicle force between ith and jth vehicles. The attractive force that acts on the vehicle is as expressed in equation (12).

$$F_{ij} = k_{ij}(d_{ij} - d_{ij_0}) \tag{12}$$

$$d_{ij} = q_j - q_i \tag{13}$$

To simplify the expression we may take all k_{ij} coefficients equal $k_{ij} = k_{IV}$ for all vehicle connections, and express the total inter-vehicle force for a vehicle as in equation (14).

$$F_{IV} = k_{IV} \sum_{j \in J_i} F_{ij} \tag{14}$$

It is important to note that neighboring relation is defined by the final formation shape. For a running system, from the beginning to the end neighbors of a vehicle are the same regardless of their position in the space. This means a constant and stable communication link is assumed to exist in between the neighboring vehicles.

4.3 Interactions between vehicles and obstacles

Say a vehicle is moving within a formation and aligning itself to the virtual leader and neighboring vehicles according to the formation constraints specified by d_{i,VL_0} and d_{ij_0}. In order to avoid obstacles a vehicle detects the obstacles around itself by sensing the smallest distance to that obstacle which is expressed as d_{OB}. The idea is illustrated in figure 4.

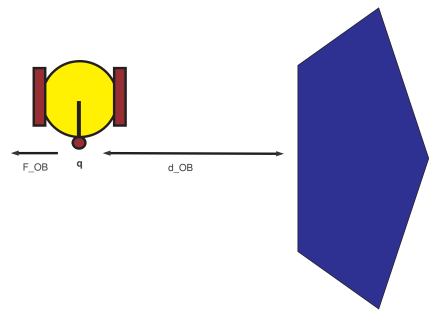


Fig. 4. Vehicle - obstacle interaction.

Say the closest point of an obstacle to the vehicle is q_{OB}. So the obstacle force on a vehicle is as expressed in equation (15). Here n is the number of obstacles the vehicle is sensing, d_{OB_m} is the distance to the mth obstacle and k_{OB} is a scalar coefficient for obstacle force.

$$F_{OB} = \sum_{m=1}^{n} \frac{k_{OB}}{d_{OB_m}} \tag{15}$$

$$d_{OB_m} = q_i - q_{OB_m} \tag{16}$$

In order to increase obstacle avoidance we increase the leader following behavior by rotating the obstacle force in the direction of virtual leader. To do this, a vector field Φ is defined around obstacles. This field takes a vector F which is directed at a direction α as its argument and turns it halfway towards a desired direction θ. The resulting direction of vector F is then β as expressed in equations (17) and (18).

$$F_{new} = \Phi(F) = |F|(\mathbf{i}cos\beta + \mathbf{j}sin\beta) \tag{17}$$

$$\beta = \begin{cases} \frac{\alpha+\theta}{2} & if\, \alpha - \theta < \pi \\ \frac{\alpha+\theta}{2} + \pi & if\, \alpha - \theta > \pi \end{cases} \tag{18}$$

When the vehicle is moving under close to the obstacles we want to improve the leader following behavior. This is achieved by moving to be closer to the leader when an obstacle is detected. In other words, by turning the effect of obstacle forces towards the leader's direction. Say θ_{OB} is the direction of F_{OB} and θ_{VL} is the direction of the virtual leader. We apply the vector field Φ to the obstacle forces. The resulting force is and its direction is given in equation (19) and (21).

$$F_{OB_{new}} = \Phi(F_{OB}) \tag{19}$$
$$\Phi(F_{OB}) = |F_{OB}|(\mathbf{i}cos\theta_{OB_{new}} + \mathbf{j}sin\theta_{OB_{new}}) \tag{20}$$

$$\theta_{OB_{new}} = \begin{cases} \frac{\theta_{OB}+\theta_{VL}}{2} & if\, \theta_{OB} - \theta_{VL} < \pi \\ \frac{\theta_{OB}+\theta_{VL}}{2} + \pi & if\, \theta_{OB} - \theta_{VL} > \pi \end{cases} \tag{21}$$

The rotation of obstacle force is illustrated in figure 5. This modification shows greater performance for cornerings. In corners when virtual leader turns sharp corners F_{VL} and F_{OB} acts on opposite directions for the vehicles in the back of the group. Since F_{VL} is proportional to distance to leader and F_{OB} is inversely proportional to distance to obstacle, vehicle in this figure ends up moving back and forth towards the obstacle surface. The rotation prevents this and makes the vehicle follow the leader more when an obstacle exists.

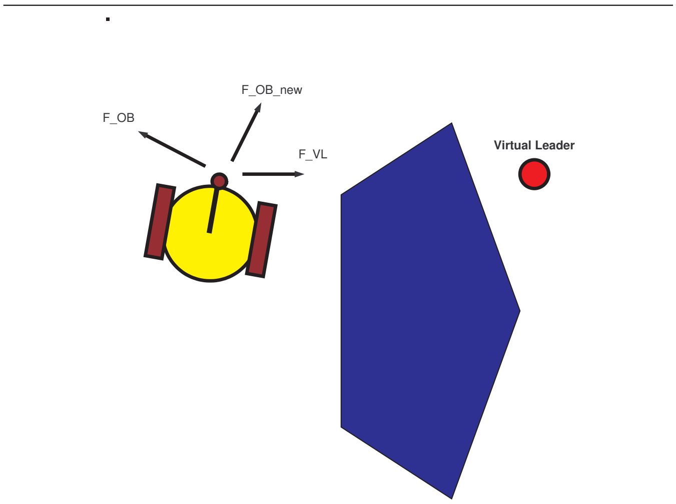


Fig. 5. Rotation of obstacle force.

4.4 Shape of The Formation

Formation shape is given by desired intervehicle distances. Say a vehicle i has neighbors in the set J_i and desired distances $d_{ij_0}$for each element in J_i where intervehicle potential U_{ij} is 0. The set J_i determines the positioning of vehicle i according to its neighbors in the specified formation. The set $J = \{J_i; i = 1, .., n\}$ where n is the number of vehicles gives the shape of the formation.

Since every vehicle is connected to a non-empty set of neighboring vehicles, circle, line, triangle, or any arbitrary shape can be given by distance constraints.

As a geometrical example, in the circular formation in figure 3, vehicle 2 desired to position itself at d_{21_0}, $d_{24_0}$and d_{25_0} where for example $d_{21} = (-a, -b)$ and $d_{24} = (a, b)$. The vehicle set q and the neighboring set J for this shape is given in equation (22) and (23) respectively.

$$q = \{q_1, q_2, q_3, q_4, q_5, q_6\} \tag{22}$$

$$J = \begin{matrix} \{\{q_2, q_3\}, \\ \{q_1, q_4\}, \\ \{q_1, q_6\}, \\ \{q_2, q_5\}, \\ \{q_2, q_3, q_4, q_6\} \\ \{q_3, q_5\}\} \end{matrix} \tag{23}$$

Distances between neighbors can be expressed as a distance set $D = \{D_i; i = 1, .., n\}$ where each element D_i corresponds to J_i and contains distances between ith vehicle and vehicles in J_i. The D set for triangular shape in figure 8 is given in equation (24).

$$D = \begin{matrix} \{\{(b, a), (-b, a)\}, \\ \{(-b, -a), (b, a)\}, \\ \{(b, -a), (-b, a)\}, \\ \{(-b, -a), (-2b, 0)\}, \\ \{(b, -a), (-b, -a), (2b, 0), (-2b, 0)\} \\ \{(b, -a), (2b, 0)\}\} \end{matrix} \tag{24}$$

4.5 Movement

The vehicles move by the double integrator dynamics given in equation (7). Each vehicle calculates its own separate control signal u using the net force acting on itself. According to the theorem in Olfati-Saber and Murray [2002] a damping component is added to ensure stability. The damping component expressed in equation (25) is similar to the frictional force, acting in the opposite direction of speed. In this equation k_f is the damping coefficient and p is the speed.

$$F_D = -k_f \frac{p}{\sqrt{1 + ||p||}} \tag{25}$$

Net force F_{NET} is the sum of virtual leader force F_{VL}, intervehicle forces F_{IV} and obstacle forces F_{OB} and expressed in equation (26). The control signal for a vehicle is expressed in equation (27).

$$F_{NET} = F_{VL} + F_{IV} + F_{OB} \tag{26}$$

$$u = F_{NET} - F_D \tag{27}$$

Since forces are proportional to distances, when vehicles are too far apart forces may grow to be too large. In this case the value of the control signal increases unrealistically. In order to prevent this, an upper bound $\lambda > 0$ to the control signal is set and u is hard clipped in that upper bound. The resulting control signal is given in equation (28).

$$\overline{u} = \begin{cases} +\lambda & if\, u > \lambda \\ u & if\, -\lambda < u < \lambda \\ -\lambda & if\, u < -\lambda \end{cases} \tag{28}$$

5. APPLICATIONS

To demonstrate the flocking and navigation properties of our approach we present two simulations. In the first simulation four vehicles starting from arbitrary initial positions in 2D space forms a diamond around a virtual leader as shown in figure 6.

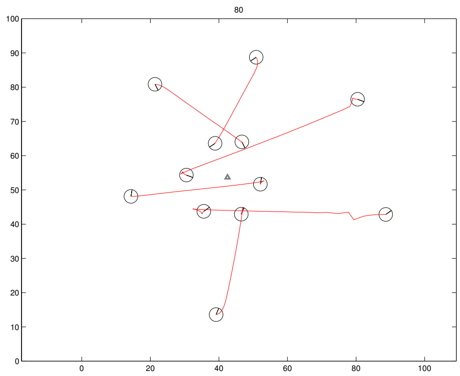


Fig. 6. Flocking simulation

In figure 6 filled circles show the starting positions of vehicles. k parameters are selected as: $k_{VL} = 0.01$, $k_{IV} = 0.005$ and $k_f = 0.2$. Trajectories of vehicles are shown as lines in this figure. For all vehicles horizontal and vertical distances to virtual leader are also given in figure 7.

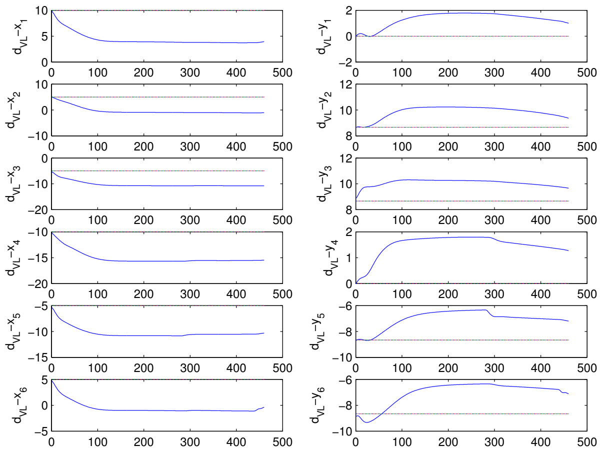
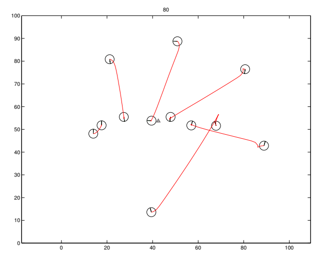
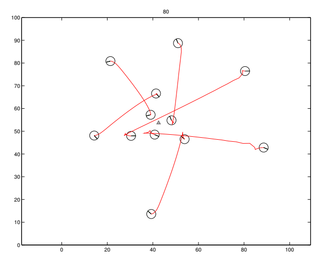


Fig. 7. Horizontal and vertical distances to virtual leader.

Different shapes are also possible. In figure 8 a line formation and a triangular formation is shown. In these figures, vehicles starting from arbitrary locations form desired shapes around the virtual leader. In (a) virtual leader is positioned at (54,19) and vehicles form a line around the virtual leader. For example, the constraints for vehicle q_2 are $d_{2,VL_0} = (15, 0)$, $d_{21_0} = (10, 0)$ and $d_{23_0} = (-10, 0)$.



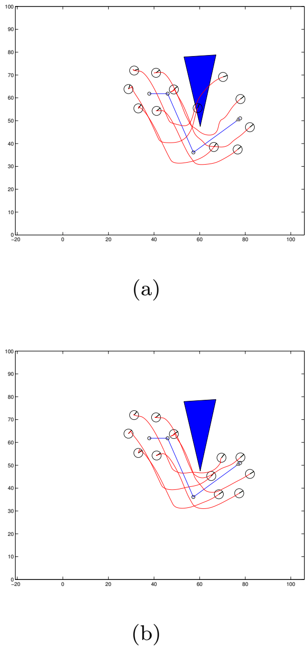
(a) Line



(b) Triangle

Fig. 8. Different formations.

To demonstrate the improved obstacle avoidance performance of the system due to artificial rotational vector fields, two simulations are presented in figure 9. In (a) obstacle forces are not rotated as in Elkaim and Siegel [2005]. Here the vehicles at the back of the group cannot follow the leader while it turns a sharp corner around the obstacle and eventually they hit the obstacle. In (b) obstacle force is rotated as expressed in equation (19) and as seen in figure, all of the vehicles in the group can successfully follow the leader in the same configuration.



(a)

(b)

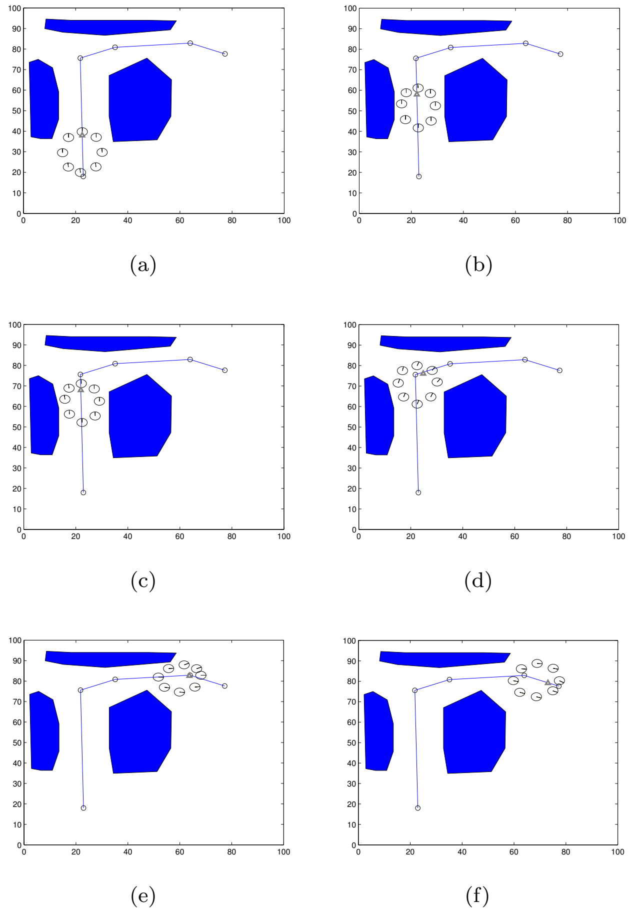
Fig. 9. Improvement on the obstacle avoidance.

As we said before this method can handle any number of vehicles and ensures collision free navigation in an environment with obstacles. In the second simulation given in figure 10 we show 12 vehicles in a circle formation navigating successfully between obstacles.

In figure 10 the blue straight lines show the virtual leaders path and vehicles are navigating in circle formation around the leader. In (a) vehicles are narrow the circle as they enter the passage, and in (b) vehicles can be seen as they change the shape of the circle according to the shape of the environment. Note that vehicles have no a-priori knowledge about the shape and positioning of obstacles. In (c-e) it is observed that the group can handle sharp corners also. In (f) as the vehicles get out of the passage the original circle is formed again. Note that all the vehicles kept the formation and followed the virtual leader successfully.

6. CONCLUSION

We have investigated the flocking and navigation of multiple vehicle systems using a decentralized control approach. Each vehicle uses kinematic model with nonholonamic constraints and double integrator dynamical models for differential drives types of mobile robots. The control laws are derived from the artificial potentials and a damping factor is used to ensure asymptotic stability. For group navigation a virtual leader is used. The presented system is realized in simulation level and it is seen that a multi vehicle system can flock and navigate in any formation successfully independent of obstacles in the environment. It is important to note that for safe navigation the virtual leader is required to follow a collision free path. Further study includes leaderless navigation models and safe group navigation when virtual leader is passing through obstacles.



(a) (b)

(c) (d)

(e) (f)

Fig. 10. 8 vehicles in circle passing through a passage through arbitrary obstacles.

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