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Compressor Surge Suppression by Second-Order Sliding Mode Control Technique ⋆

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Abstract: Surge is one of the two main dynamic instabilities that can occur in compressor systems (the other is the rotating stall) preventing their satisfactory operation and being potential cause of serious damages. In this paper we consider a Close Coupled Valve (CCV) as actuator, and we suggest the use of a second-order sliding-mode (2-SM) technique for the active control of the surge phenomenon. We refer to the Moore-Greitzer model with cubic compressor characteristics, and we show that the proposed technique can drive the system towards a stable operating point by rejecting a significant class of persistent pressure and flow disturbances acting on the system. Furthermore, the perfect knowledge of the compressor and load characteristics is not required to design the controller.

1. INTRODUCTION

Compressors are used in a wide range of applications. These includes turbojet engines used in aerospace propulsion, power generation using industrial gas turbines, turbocharging of internal combustion engines, pressurization of gas and fluids in the process industry, transport of fluids in pipelines and so on. The useful range of operation of turbo-compressors is limited, by choking at high mass flows when sonic velocity is reached in some component, and at low mass flows by the onset of two instabilities known as surge and rotating stall. Surge and rotating stall are dynamic instabilities that can occur in compressor systems of both the axial and centrifugal type. Both these instabilities cause disruption of the normal operating condition which is designed for steady and axisymmetric airflow. Rotating stall is a severely non-axisymmetric distribution of axial flow velocity around the annulus of the compressor, taking the form of a wave or "stall cells" that propagates steadily in the circumferential direction at a fraction of the rotor speed. Pure surge, on the other hand, is an axisymmetric oscillation of the mass flow along the axial length of the compressor. Deep surge is a mostly axisymmetric oscillation with such a large variation of mass flow that during part of the cycle the compressor operates in reversed flow, which is highly undesirable as can cause structural damages. Often surge and stall are coupled although each can occur without the other. Both phenomena are likely to occur in the low range of the mass flow, where the compressor characteristics has positive slope. Powerful methods for "active control" of surge and stall were studied by many researchers. Such methods use feedback to stabilize the unstable regime allowing for both operation in the peak efficiency and pressure rise regions located in the neighborhood of the surge line, as well as an extension of the operating range of the compressor, see Epstein et al. (1989). In the last decade, the literature on feedback stabilization of compression systems has become extensive. This is partly due to the introduction, and success, of the popular Moore-Greitzer model, see Moore and Greitzer (1986), that, despite of its simplicity, captures the essential phenomena underlying the compressor dynamic instabilities.

A variety of actuator devices have been suggested in the literature, among them Inlet Guide Vanes (IGV) (Nakagawa et al. (1994)), Throttle valves (Pinsley et al. (1991)), (Blanchini and Giannatasio (2002)), Close Coupled Valve (CCV) (Gravdahl and Egeland (1999)). In Epstein et al. (1989) an approach with two actuators, inlet valve and plenum with mobile walls, was presented. Among the studies on surge and stall active control using CCVs, it should be cited the work by Simon e Valvani (1991), in which, probably for the first time, the stability analysis was conducted by making reference to the explicit nonlinear model and a Lyapunov-based controller relying on the measure of the mass flow was proposed. The comprehensive book Gravdahl e Egeland (1999) presents a broad overview of the field and also includes a novel compression model which generalizes the standard Moore-Greitzer model by taking into account a possibly time-varying compressor angular velocity. Several adaptive and passivity-based controllers using the CCV actuator were suggested. They considered many control problems with increasing complexity: surge control without disturbances, surge control with constant disturbances, surge control with time varying disturbances, surge and stall control without disturbances, etc.

On the other hand, since the appearance of the Moore Greitzer model, nonlinear analysis and design techniques

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were applied extensively to the field. Some recent interesting contributions used bifurcation methods Nayfeh and Abed (2002) and high-gain adaptive control, Blanchini and Giannatasio (2002).

In this work we focus on the active surge control problem considering a CCV as actuator. We suggest a control technique based on the second-order sliding mode (2-SM) approach that makes use of the airflow measurement only. For the stability analysis we refer to the reduced-order Moore-Greitzer model with the stall state variable set to zero.

The load characteristics, defined by the opening of a fictitious throttle valve, is allowed to vary in a bounded range. We study the practically relevant case in which the compressor is starting near a stable operating point, and, from some time instant on, the throttle valve is closed until the operating point enters the surge zone. The proposed 2-SM CCV control scheme will drive the compressor towards a new, suitably computed, operating regime in the low mass flow region. Furthermore, the presented procedure allows for an imprecise knowledge of the compressor and load characteristics.

The paper is structured as follows. Section 2 recalls the model of Moore and Greitzer and presents, in increasing degree of complexity, the reduced-order (stall-free) model as well as the "actuated" model including the CCV and subject to pressure and flow disturbances. The main standing assumptions are outlined. In Section 3 we derive a constructive procedure to compute a feasible steady-state operating point. In Section 4 it is described the new approach to surge avoidance based on the second-order sliding mode control technique. The results of some computer simulations are discussed in the Section 5, and some concluding remarks are presented in the final Section 6.

2. SYSTEM MODELS

We refer to the well known model suggested by Moore and Greitzer (1986). The basic compression system that forms the basis for the model development comprises an (axial or centrifugal) compressor, working between a large constant pressure reservoir (ambient) and a plenum volume containing compressible gas (Figure 1). The plenum volume discharges through a throttle valve into another large reservoir. **The throttle models the load characteristics.** The third-order system of equations forming the Moore-Greitzer model is as follows

$$\dot{\psi} = \frac{1}{4B^2 l_c}(\phi - \phi_T(\psi)) \tag{1}$$

$$\dot{\phi} = \frac{1}{l_c}(\psi_c(\phi) - \psi - \frac{3H}{4}(\frac{\phi}{W} - 1)J) \tag{2}$$

$$\dot{J} = J(1 - (\frac{\phi}{W} - 1)^2 - \frac{J}{4})\varrho \tag{3}$$

where:

- ψ is the non dimensional plenum pressure or pressure coefficient (pressure divided by density and the square of compressor speed),

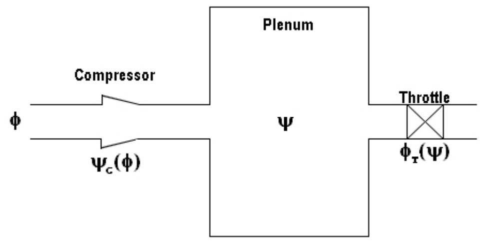

Fig. 1. Basic compression system

- ϕ is the annulus averaged mass flow coefficient (axial velocity divided by compressor speed),
- J is the squared amplitude of rotating stall amplitude
- ψ_c and ϕ_T are the compressor and throttle (i.e., load) characteristics

and the state variables derivatives are understood with respect to the adimensional scaled time variable $\xi = (U/R)t$, with U being the compressor tangential speed and R the mean compressor radius (see Moore and Greitzer (1986)). B, l_c and ϱ are constant parameters, and ψ_c is the compressor characteristics for which a cubic approximation is considered

$$\psi_c(\phi) = \psi_{c0} + H[1 + \frac{3}{2}(\frac{\phi}{W} - 1) - \frac{1}{2}(\frac{\phi}{W} - 1)^3] \tag{4}$$

where ψ_{c0}, W and H are proper constants.

The throttle characteristic is defined by

$$\phi_T(\psi) = \gamma_T \sqrt{\psi} \quad \Leftrightarrow \quad \psi_T(\phi) = \frac{1}{\gamma_T^2}\phi^2 \tag{5}$$

Since the scope of this paper is surge suppression, we disregard the stall dynamics by considering the reduced-order model obtained from (1)-(3) by setting the stall state variable J set to zero. It yields the second-order model

$$\dot{\psi} = \frac{1}{4B^2 l_c}(\phi - \phi_T(\psi)) \tag{6}$$

$$\dot{\phi} = \frac{1}{l_c}(\psi_c(\phi) - \psi) \tag{7}$$

The planar system (6)-(7) allows for a clear graphical interpretation of its behaviour. It admits a single point of equilibrium $P_e \equiv (\phi_e, \psi_e)$ located at the point of intersection between the compressor and load characteristics, whose typical shapes are shown in the Figure 2. From the local stability analysis of the linearized dynamics it follows that **all operating points in correspondence of which the compressor characteristics has negative slope are stable** (see Gravdahl and Egeland (1999)).

2.1 Close Coupled Valve

The CCV actuator is located located close to the compressor outlet duct and causes the additional adjustable pressure drop

$$\psi_v = \frac{1}{\gamma_v^2}\phi^2 = K_{ccv}\phi^2 \tag{8}$$

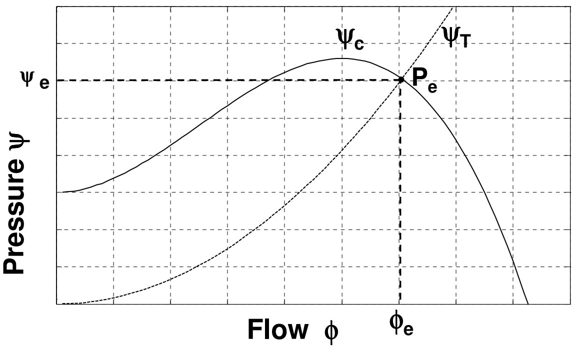

Fig. 2. Typical throttle and compressor characteristics

which leads to the following "controlled" Moore-Greitzer model:

$$\dot{\psi} = \frac{1}{4B^2 l_c}(\phi - \phi_T(\psi)) \tag{9}$$

$$\dot{\phi} = \frac{1}{l_c}(\psi_c(\phi) - K_{ccv}\phi^2 - \psi) \tag{10}$$

The pressure drop across the CCV can be modulated by adjusting the the CCV flow coefficient K_{ccv}, and is therefore treated as the control variable devoted to provide for a surge-free operation of the compressor. In order to better understand the effect of changing K_{ccv}, it is convenient to combine together the compressor and CCV pressure drops by defining the so-called "equivalent" compressor characteristics ψ_E:

$$\psi_E(\phi) = \psi_c(\phi) - K_{ccv}\phi^2 \tag{11}$$

Then we can rewrite system (9)-(10) as

$$\dot{\psi} = \frac{1}{4B^2 l_c}(\phi - \phi_T(\psi)) \tag{12}$$

$$\dot{\phi} = \frac{1}{l_c}(\psi_E(\phi) - \psi) \tag{13}$$

The same graphical analysis done for the original "uncontrolled" model (6)-(7) can be made, which establishes that the unique equilibrium point of (12)-(13) is located at the intersection between the throttle characteristics and the equivalent compressor characteristics (11).

Figure 3 shows the throttle characteristic Ψ_T together with some equivalent characteristics which correspond to different values of K_{ccv}. When $K_{ccv} = 0$, i.e., the CCV is fully open, the equivalent characteristic coincides with the original compressor one.

Again, the local stability of the equilibrium point is affected by the slope of $\psi_E(\phi)$ at that point. By augmenting K_{ccv} the equilibrium point moves to the left and, at the same time, the slope of $\psi_E(\phi)$ at the equilibrium decreases. Then, by taking K_{ccv} sufficiently large, the resulting equilibrium point can be always made stable.

2.2 Perturbed dynamics

We further complicate the model by assuming the possible presence of pressure and flow disturbances

$$\dot{\psi} = \frac{1}{4B^2 l_c}(\phi - \gamma_T\sqrt{\psi} + \eta_\phi(\xi)) \tag{14}$$

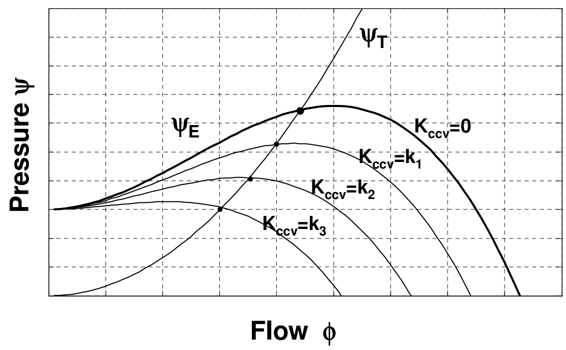

Fig. 3. The throttle characteristic ψ_T and the equivalent characteristics ψ_E for different values of K_{ccv} ($0 < k_1 < k_2 < k_3$).

$$\dot{\phi} = \frac{1}{l_c}(\psi_E(\phi) - \psi + \eta_\psi(\xi)) \tag{15}$$

with the (possibly time-varying) uncertainties η_ϕ and η_ψ bounded, in modulus, by the positive constants a and b:

$$|\eta_\phi| \leq a, \quad |\eta_\psi| \leq b \tag{16}$$

The effect of the disturbances can be interpreted as a rigid translation of the load and equivalent compressor characteristics. The "perturbed" characteristics are

$$\psi_{E\eta}(\phi) = \psi_E(\phi) + \eta_\psi \tag{17}$$

$$\phi_{T\eta}(\psi) = \gamma_T\sqrt{\psi} - \eta_\phi \tag{18}$$

The latter can be reversed as

$$\psi_{T\eta}(\phi) = \frac{1}{\gamma_T^2}(\phi + \eta_\phi)^2 \tag{19}$$

Figure 4 studies the variation of the intersection point caused by the effect of the disturbances on the compressor and load characteristics. It is clear that the intersection point of the perturbed characteristics lies within the shaded region around the "nominal" point of intersection P_2 of the unperturbed system.

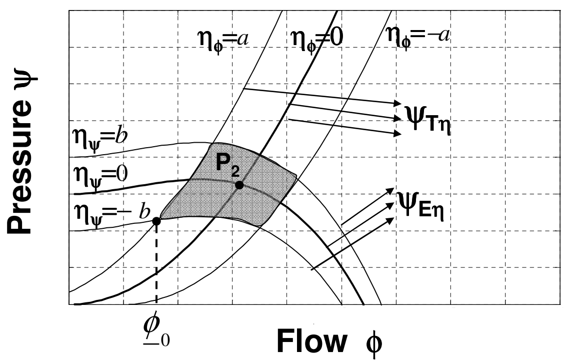

Fig. 4. Effect of the flow and pressure disturbances on the localization of the operating point

2.3 Assumptions

It is assumed that at the beginning the CCV is open, i.e. $K_{ccv} = 0$, and the initial load value $\gamma_T = \overline{\gamma}_T$ is such that the corresponding intersection with the compressor characteristics determines a stable point of operation P_1. Thus, due to the effect of the disturbances, the compressor trajectory starts from some vicinity of the stable equilibrium point $P_1 = (\phi_{e1}, \psi_{e1})$ with the load characterized by the constant opening $\gamma_T = \overline{\gamma}_T$. From this point on, a bounded variation (reduction) of γ_T is allowed such that

$$\gamma_T \in (\underline{\gamma}_T, \overline{\gamma}_T) \tag{20}$$

with a **known lowerbound** $\underline{\gamma}_T$. It is of interest to study the case in which the lower-bound is such that the corresponding point of equilibrium P_2 with the CCV fully opened is **unstable**.

The control task is to close the CCV sufficiently, in order to keep a stable operating regime in spite of the variation of γ_T. The task is feasible according to the Figure 3 and the related considerations previously made.

In the next Section the problem of computing a convenient value for the steady airflow set-point is dealt with. Since the compressor will be driven by the controller towards such operating condition, it is crucial to choose it carefully by taking into account that **it should remain stable in spite of the possible variation of γ_T.** It is shown in next Section that the range of permitted (says, "stabilizable" or "achievable") set-point values is limited. A constructive procedure to select a "good" set-point, namely an operating point that can be stabilized by the CCV, is given. It is desirable to maximize the mass flow in the new operating point, hence the upper bound of the admissible domain will be considered.

3. COMPUTATION OF THE DESIRED STEADY-STATE OPERATING POINT

The active CCV control will drive the compressor towards a suitable airflow set-point value ϕ_0, that needs to be evaluated carefully. The mass flow set-point ϕ_0 cannot be chosen arbitrarily.

3.1 The disturbance-free case ($\eta_\phi = \eta_\psi = 0$)

A set-point ϕ_0 is said to be "***stabilizable***" if, for any load condition in the admissible domain (20), there is $K_{ccv} > 0$ such that the load and equivalent compressor characteristics intersect at $\phi = \phi_0$, and the resulting point of intersection is a locally stable equilibrium.

Consider the "uncontrolled" compressor system ($K_{ccv} = 0$) and the extremal load characteristics at the boundaries of (20) (i.e., $\gamma_T = \underline{\gamma}_T$ and $\gamma_T = \overline{\gamma}_T$), together with the corresponding points of intersection. By assumption the first one (says P_1) is stable, while the second one P_2 is unstable. It is clear that the value $\phi = \phi_{e1}$ cannot be stabilized when $\gamma_T < \overline{\gamma}_T$, since this would require a positive vertical translation of the compressor characteristics, that would be achieved by means of a physically meaningless negative value of K_{ccv}. This is clearly impossible since $K_{ccv} \geq 0$ by construction.

All points in the interval $[\phi_{e2}, \phi_{e1}]$ are subject to similar considerations, i.e, there are admissible load conditions that would prevent the attainment of a stable operation at that point, due to the constraint $K_{ccv} \geq 0$. Then, it follows that the set-point ϕ_0 should satisfy, at least, the condition $\phi_0 < \phi_{e2}$

According to the above considerations, the stabilizable set-points belong to an interval of the type

$$\phi_0 \in (0, \phi_0^*), \qquad \phi_0^* < \phi_{e2} \tag{21}$$

Remember that the stabilizable set-points must fulfill two properties: *i*) there must exist $K_{ccv} > 0$ such that the load and equivalent compressor characteristics intersect at that point, and *ii*) the intersection point is a stable equilibrium. The latter is satisfied if the slope of the equivalent characteristics at that point is negative.

We suggest a simple off-line iterative procedure for computing ϕ_0^*. Define $K_{ccv}^i = i\delta$, with δ being a small positive number and $i = 1, 2, ...$

At the i-th iteration, it is drawn the equivalent characteristics ψ_e^i corresponding to the value of K_{ccv}^i. Then, the corresponding point of intersection ϕ_0^i with the "worst-case" load characteristics ($\gamma_T = \underline{\gamma}_T$) is found. Finally, the stability of the intersection point is studied. If it is unstable the next iteration is made. If it is stable, then the iterations stop and ϕ_0^* is assigned the ϕ_0^i value.

Since P_2 is unstable, the intersection point is certainly unstable for small values of K_{ccv}. Nevertheless, according to Figure 3 and the related considerations previously made in Section 2.1, the iterations will end after a finite number of steps due to the stabilizing effect of increasing K_{ccv}. In particular, because the intersection points decrease along the iterations, the suggested algorithm gives the largest stabilizable set-point as requested.

Analytically, the iterative algorithm is formalized as follows:

***i*-th step**

A. Compute ϕ_0^i such that

$$\psi_c(\phi_0^i) - K_{ccv}^i \left(\phi_0^i\right)^2 = \frac{1}{\underline{\gamma}_T^2} \left(\phi_0^i\right)^2 \tag{22}$$

B. Check the condition

$$\left.\frac{\partial \psi_E}{\partial \phi}\right|_{\phi=\phi_0^i} < 0 \tag{23}$$

which, considering (11) and (4), can be rewritten as follows

$$\phi_0^i > \frac{1}{2K_{ccv}^i} \left.\frac{\partial \psi_c}{\partial \phi}\right|_{\phi=\phi_0^i} = \frac{1}{2K_{ccv}^i} \frac{3}{2} \frac{H}{W} \left[1 - \left(\frac{\phi_0^i}{W} - 1\right)^2\right] \tag{24}$$

C. If (24) is verified then $\phi_0^* = \phi_0^i$ and the iterations stop, otherwise go to the $(i+1)$-th step.

3.2 Model discrepancies in the compressor characteristic

Let us analyze the effect of possible **discrepancies between the nominal and actual compressor characteristics**. In the computation of the intersection points by means of (22), it is permitted that the "wrong" intersection $\tilde{\phi}_0^i$ is less than ϕ_0^i. Then, for example, it can be used any imprecise compressor characteristics $\tilde{\psi}_c$ such that $\tilde{\psi}_c(\phi) < \psi_c(\phi)$ in the flow domain of interest. Indeed if $\tilde{\psi}_c(\phi) < \psi_c(\phi)$ the obtained intersection point $\tilde{\phi}_0^i$ will be smaller than ϕ_0^i.

The stability condition (24) could give unreliable results in the presence of a large uncertainty on H ad W. To enhance the robustness of the stability condition one can implement a more conservative version using the knowledge of some $a_m \geq sup_\phi \frac{\partial \psi_c}{\partial \phi}$ (this approach is followed in Gravdahl and Egeland (1999)). It is obtained

$$2K_{ccv}^i \phi_0^i > a_m \tag{25}$$

It is clear that using condition (25) instead of (24) one obtains a conservatively smaller value of ϕ_0^* at the end of the iterations.

3.3 The perturbed case

Now let us consider the presence of flow and pressure disturbances. As shown in Figure 4, the disturbances alter the compressor and load characteristics leading to an uncertainty region around the "nominal" intersection point in absence of the disturbances.

The rationale and structure of the algorithm are exactly the same, nevertheless, the **formula (22) for computing the intersection points should be modified**. Indeed, actually, instead of finding the unique intersection point solution of (22) we should evaluate the smaller intersection point over the admissible range (which has the form of the shaded region in Figure 4). It is clear from the Figure 4 that we aim to compute the value $\underline{\phi}_0$, thus we have to consider the disturbance values $\eta_\phi(\xi) = a$ e $\eta_\psi(\xi) = -b$.

Considering the perturbed characteristics (14) and (16) into (22) it yields the new formula for computing the lower, worst-case, intersection point.

$$\psi_c(\phi_0^i) - K_{ccv}^i \left(\phi_0^i\right)^2 - b = \frac{1}{\underline{\gamma}_T^2} \left(\phi_0^i + a\right)^2 \tag{26}$$

Clearly the solution of equation (26) is less than that of (22) as it is clear from the Figure 4).

The stability condition (24) is not reliable in the presence of the disturbances. The more roust and restrictive condition (25) should be used in the perturbed case.

4. MAIN RESULT

At the beginning ($\xi = \xi_1$), the compressor is assumed to be working in a vicinity of a stable point of equilibrium ϕ_1.

It is defined a dynamic airflow reference profile ϕ_{0R} that starts from ϕ_1 and converges (decreases) smoothly towards the chosen set-point value $\phi_0 = \phi_0^*$ (i.e., the largest stabilizable set-point) computed in the previous Section. The reference profile is designed as the solution of the following differential equation

$$\tau_\phi \dot{\phi}_{0R} + \phi_{0R} = \phi_0, \quad \phi_{0R}(\xi_1) = \phi_1 \tag{27}$$

The solution ϕ_{0R} of (27) decreases smoothly and monotonically from ϕ_1 to ϕ_0 with an exponential profile depending on the time constant $\tau_{\phi_{0R}}$ (the smaller τ_ϕ, the faster the transient).

Consider the tracking error variable

$$\sigma = \phi - \phi_{0R}(\xi) \tag{28}$$

Introduce the following auxiliary control variable

$$u = \frac{1}{\gamma_v^2} {\phi_\epsilon}^2 = K_{ccv} {\phi_\epsilon}^2 \quad \rightarrow \quad K_{ccv} = {\phi_\epsilon}^2/u \tag{29}$$

with ϕ_ϵ being a lower-saturated version of ϕ ($\epsilon \approx 0$)

$$\phi_\epsilon = \begin{cases} \phi & if\ \phi \geq \epsilon > 0 \\ \epsilon & otherwise \end{cases} \tag{30}$$

As long as condition $\phi \geq \epsilon$ is not violated, the following system dynamics is obtained by considering (30) into (14)-(15)

$$\dot{\psi} = \frac{1}{4B^2 l_c}(\phi - \gamma_T \sqrt{\psi} + \eta_\phi) \tag{31}$$

$$\dot{\phi} = \frac{1}{l_c}(\psi_c(\phi) - u - \psi + \eta_\psi) \tag{32}$$

It is proposed the following control algorithm, referred to as the "Generalized sub-optimal" (G-SO) integral 2-SMC, see Bartolini et al. (2003).

$$\dot{w} = U_d \text{sign}\,(\sigma - \beta \sigma_{Mj}) \tag{33}$$

$$u = \begin{cases} \overline{U} & w > \overline{U} \\ w & 0 \leq w \leq \overline{U} \\ 0 & w < 0 \end{cases} \tag{34}$$

with arbitrary constant $\beta \in [0.5, 1)$, $U_d > 0$ and $\overline{U} > 0$ sufficiently large, and with σ_{Mj} ($j = 1, 2, ...$) being the value of σ at the most recent timer instant t_{Mj} at which $\dot{\sigma}(t_{Mj}) = 0$. σ_{Mj} is then denoted as the "last singular point" of σ.

The performance of the resulting closed loop system is established in the following Theorem:

Theorem 1 *Consider system (31)-(32) starting at the time instant $\xi = \xi_1$ near the stable equilibrium point (ϕ_1, ψ_1). Consider the tracking error variable (28)-(27) and apply the "generalized suboptimal" integral 2-SM control algorithm (33)-(34). Then condition $\sigma = 0$ is achieved after a finite transient, which means that $\phi \rightarrow \phi_0$ exponentially.*

Proof of Theorem 1. See the Appendix

5. SIMULATIONS

For simulation purposes it is considered the compressor model (4), (31)-(32) with the parameters $B = 0.7$, $l_c = 3$, $H = 0.18$, $W = 0.25$, $\psi_{c0} = 0.3$ and the state initial conditions $\phi(0) = 0.4$ and $\psi(0) = 0.3$. For the throttle parameter γ_T we considered the initial and final values 0.65 and 0.5, respectively, and a smooth decrease transition between them starting at $\xi = 250$. Corresponding to the initial throttle parameter $\gamma_T = \overline{\gamma}_T = 0.65$, and assuming no disturbances,it is found the stable equilibrium point (ϕ, ψ)=(0.526,0.657) attracting the compressor state during the first transient. As the throttle parameter start to decrease at $\xi > 250$, the system leaves the equilibrium reaching the surge zone. The appearance of a deep surge phenomenon is seen by performing the simulation TEST

1, with no disturbances ($\overline{\eta}_\psi = \overline{\eta}_\phi = 0$) and no control, i.e., $K_{ccv} = 0$. The phase plane trajectory of the uncontrolled system is shown in the figure 5.

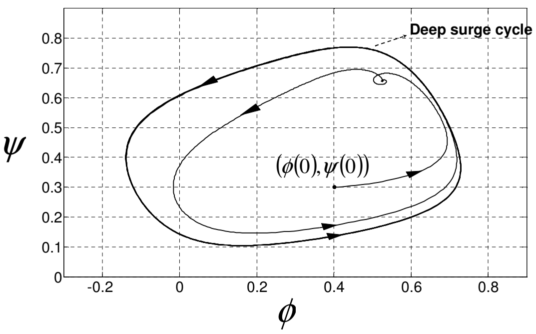

Fig. 5. TEST 1. State plane evolution.

Now let us apply the proposed nonlinear CCV-based controller by disregarding, temporarily, the flow and pressure disturbances. The procedure described in Section 3 for computing the largest stabilizable set point gives rise to $\phi_0^* = 0.3$ for the unperturbed case. The value $\phi_0 = 0.3$ is then selected as the set-point for the steady flow, and the reference profile ϕ_{0R} is defined according to (27) with the time constant $\tau_\phi = 1/20$.

The control algorithm (33)-(34) has been applied with $U_d = 0.1$, $\beta = 0.8$, $\overline{U} = 5$. The CCV flow coefficient K_{ccv} is computed as per (29)-(30), with the parameter $\varepsilon = 0.01$.

In the first controlled test, TEST 2, no disturbances are applied. Figure 6 shows the output u of the suboptimal controller, the CCV flow coefficient K_{ccv}, the pressure ψ and the flow ϕ together with its reference value ϕ_{0R}.

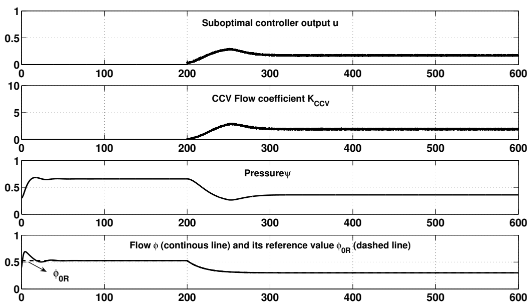

Fig. 6. TEST 2. Time profiles of u, K_{ccv}, pressure and flow.

In the TEST 3 it is investigated the effect of the disturbances. It is applied a pair of disturbing signals satisfying the restrictions (16) with $a = b = 0.02$. To cope with the disturbances the discontinuous magnitude U_d needs to be increased to $U_d = 1$.) The set-point selection criterion suggested in the Section 3 for the perturbed case leads to the value $\overline{\phi}_0^* = 0.25$. Thus the reference profile is changed, as compared to the previous TEST 2, by reducing the desired steady value ϕ_0 to 0.25.

Figure 7 shows the same signals depicted in the previous Figure 6 plus the applied disturbances. Now the control signal u (or equivalently K_{ccv}) cannot be constant in the steady state since it must compensate for the effect of the disturbances. The flow is perfectly regulated along the desired steady state value. The pressure oscillates in the steady state due to the flow disturbance η_ϕ, whose effect cannot be compensated for. If η_ϕ would be zero then ψ would be exactly regulated as well.

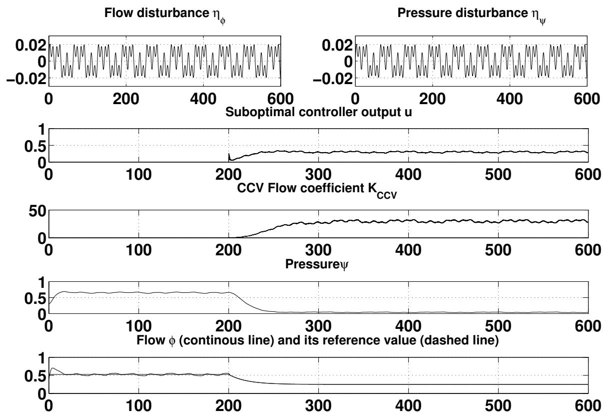

Fig. 7. TEST 3. Time profiles of the flow and pressure disturbances, u, K_{ccv}, pressure and flow.

As a final TEST 4, the complete full-order model (1)-(3) including the stall state variable J has been simulated under the same conditions as those in the TEST 3, and considering the initial condition $J(0) = 0.1$. This test aims to show that the proposed control schemes possesses interesting properties of stall suppression as well. It can be noted that the stall variable tends rapidly to zero. These promising properties will be investigated more thoroughly in next works.

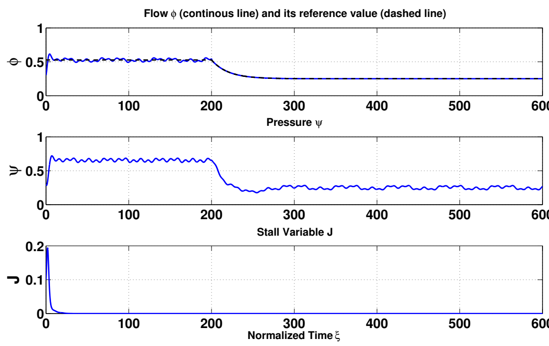

Fig. 8. TEST 4. Simulation with the third-order model including the rotating stall

6. CONCLUSION

A robust surge-avoidance active controller for compressor systems, based on the second-order SMC approach, and using has been proposed. The proposed methodology, that uses a CCV as actuator device, does not require the perfect knowledge of the compressor parameters and characteristics, and it proves to be robust against a significant class of perturbations. The simulative analysis confirms the good performance of the method. Next activities will be targeted to the study of the combined surge/stall avoidance problem, in order to validate the promising simulation results.

REFERENCES

G. Bartolini, A. Pisano, E. Punta and E. Usai. A survey of applications of second order sliding mode control to mechanical systems. *Int. J. of Control*, 76(9/10): 875–892, 2003.

J. T. Gravdahland, O. Egeland. *Compressor Surge and Rotating Stall: Modeling and Control.* Springer Verlag, London, 1999.

F. K. Moore, E. M. Greitzer. *A Theory of Post-Stall Transient in Axial Compression System.* Part I, pp. 68-76, ASME J. Eng. for Gas Turbines and Power, 108, 1986

S. Simon. *Feedback stabilization of compression systems.* Phd thesis. MIT press, 1993.

J. E. Ffowcs Williams, M. F. L. Harper, D. J. Allwright *Active stabilization of compressor instability and surge in a working engine.* Journal of Turbomachinery 115, 68-75, 1993.

A. H. Epstein, J. E. Ffowcs Williams, E. M. Greitzer *Active suppression of aerodynamics instabilities in turbomachines.* Journal of Propulsion and Power 5(2), 204-211, 1989.

Pinsley, J. E., G. R. Guanette, A. H. Epstein and E. M. Greitzer *Active stabilization of centrifugal compressor surge.* Journal of Turbomachinery 113, 723-732, 1991.

Nakagawa, K., M. Fujiwara, T. Nishioka, S. Tanayka and Y. Kashiwabara *Experimental and numerical analysis of active suppression of centrifugal compressor surge by suction-side valve control.* JSME Internationa Journal of, Series B, Fluids and thermal engineering 37(4), 878-885, 1994.

F. Blanchini, P. Giannatasio, P. Pinamonti and D. Micheli *Experimental Evaluation of a High–Gain Control for Compressor Surge Suppression.* ASME Journal of Turbomachinery 124, 1, 27-35, 2002.

M. A. Nayfeh, E. H. Abed *High-gain feedback control of rotating stall in axial flow compressors.* Automatica 38, 995-1001, 2002.

Y. Wang, S. Yeung, R. M. Murray *Bifurcation control of rotating stall with actuators magnitude and rate limits: Part II - control synthesis and comparisons with experiments.* Automatica 38, 611-625, 2002.

Appendix A. PROOF OF THEOREM 1

Let us compute the derivative of the error variable (28). In light of (32) it yields

$$\begin{aligned}\dot{\sigma} = \dot{\phi} - \dot{\phi}_{0R} &= \frac{1}{l_c}(\psi_c(\phi) - u - \psi + \eta_\psi) - \dot{\phi}_{0R} = \\ &= \frac{1}{l_c}(f_1(\phi,\psi,\xi) - u)\end{aligned} \tag{A.1}$$

with $f_1(\phi,\psi,\xi) = \psi_c(\phi) - \psi + \eta_\psi - l_c\dot{\phi}_{0R}$. Now compute also the second derivative of σ. After some manipulations it yields

$$\begin{aligned}\ddot{\sigma} &= \frac{1}{l_c}\left[\frac{\partial\psi_c(\phi)}{\partial\phi}\dot{\phi} - \dot{u} - \dot{\psi} + \dot{\eta}_\psi\right] - \ddot{\phi}_{0R} = \\ &= \frac{1}{l_c}\left[\frac{1}{l_c}\frac{\partial\psi_c(\phi)}{\partial\phi}\left(\psi_c(\phi) - u - \psi + \eta_\psi\right) - \dot{u} - \right. \\ &\qquad \left. - \frac{1}{4B^2 l_c}[\phi - \phi_T(\psi) + \eta_\phi] + \dot{\eta}_\psi\right] - \ddot{\phi}_{0R} \\ &= \frac{1}{l_c}(f_2(\phi,\psi,\xi,u) - \dot{u})\end{aligned} \tag{A.2}$$

with the function f_2 given as follows

$$\begin{aligned}f_2(\phi,\psi,\xi,u) &= \frac{1}{l_c}\frac{\partial\psi_c(\phi)}{\partial\phi}\left(\psi_c(\phi) - u - \psi + \eta_\psi\right) + \\ &- \frac{1}{4B^2 l_c}([\phi - \phi_T(\psi) + \eta_\phi] + \dot{\eta}_\psi - l_c\ddot{\phi}_{0R}\end{aligned} \tag{A.3}$$

The second-order dynamics (A.2)-(A.3) is candidate for the applicability of the suboptimal control algorithm. Note that the actual discontinuous control variable is $\dot{u}$, thus the system input u, obtained by integrating the discontinuous signal $\dot{u}$, will be continuous. This methodology is the so-called “anti-chattering” 2-SMC Bartolini et al. (2003). It must be shown that the uncertain functions f_1 and f_2 fulfill the boundedness requirements required by the 2-SMC applicability conditions.

Assume, temporarily, that the flow will never leave some bounded domain of the type $\phi \in (\underline{\phi}, \overline{\phi})$. It is easy to show that the pressure dynamics (31) is BIBO stable, with the “inputs” being the airflow variable ϕ and the flow disturbance η_ϕ. Then, under the above assumption on ϕ there are $\underline{\psi}$ and $\overline{\psi}$ such that the pressure is bounded too according to the condition $\psi \in (\underline{\psi}, \overline{\psi})$.

Considering (27) one concludes that the generated set-point ϕ_{0R} is smooth, then $|\dot{\phi}_{0R}|$ and $|\ddot{\phi}_{0R}|$ are both bounded by some constant. Thus, by taking into account (16) and according to the boundedness and smoothness of the reference $\phi_{0R}(\xi)$, function f_1 in (A.1) is bounded, in modulus, by some constant F_1.

$$|f_1(\phi,\psi,\xi)| \le F_1 \tag{A.4}$$

The nonnegative control variable u, the pressure drop across the CCV, is also bounded. Assuming that the pressure disturbance η_ψ has bounded derivative than function f_2 is bounded, in modulus, by some constant F_2

$$|f_2(\phi,\psi,\xi,u)| \le F_2 \tag{A.5}$$

The second-order dynamics (A.2), satisfying condition (A.5), can be stabilized in finite time by designing the control variable according to the integral suboptimal algorithm (33)-(34) whose parameters must fulfill the following *sufficient* inequalities

$$\overline{U} > F_1 \qquad U_d > \frac{F_2}{(2\beta - 1)} \qquad \beta \in [0.5, 1) \tag{A.6}$$

According to the stability properties of the suboptimal algorithm, see Bartolini et al. (2003), conditions $\sigma = \dot{\sigma} = 0$ are achieved in finite time. With the above sufficient tuning rules, the convergence transient of σ is monotonical, which means that the flow do not feature transient overshoot or undershoot. After the end of the transient the flow will exactly match the desired profile $\phi_{0R}(\xi)$. Thus, the stable operating condition $\phi = \phi_0$ is reached after an exponential *monotonic* transient. This means that the flow variable ϕ will never leaves the bounded domain $\phi \in (\underline{\phi}, \overline{\phi})$, with $\underline{\phi} > \varepsilon$ as previously assumed. This validates all previous considerations and concludes the proof.