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Fault Tolerant, Scalable Multi-Agent Control Under Medium Access Constraints ⋆

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Abstract: This paper extends our earlier work presented at the last IFAC World Congress that concerned the fault tolerant, distributed, scalable control of a group of agents that must move in a formation specified by relative positions between agents and a constant formation velocity. The control law we had proposed naturally accommodated various levels of fault tolerance and scalability and required an amount of inter-agent communication that was commensurate with a designated level of fault tolerance. The control law assumed, however, that this exchange of information occurred simultaneously. In practice communications must occur under Medium Access Control (MAC) constraints. Thus no agent can transmit and receive at the same time, and cannot transmit to another agent who is receiving information from yet another. We modify our earlier control algorithm so that such MAC constraints are respected, and provide a stability analysis of this modified control law.

Keywords: Cooperative control, Stability, Fault tolerance, Decentralized control, Automated guided vehicles

1. INTRODUCTION

Spurred by major advances in computing, wireless communications and networking, and an ever expanding application domain, there has been a growing interest in the cooperative control of networks of mobile autonomous agents, Vicsek et al. [1995]-Anderson et. al. [2006]. Such networks involve multiple mobile objects that cooperate to achieve any number of objectives. Thus they may achieve a formation, perform collective tasks, gather data, avoid collisions and obstacles, and be robust to malicious and hostile environments. Cooperation is effected through limited exchange of information between the agents over wireless media with little or no centralized intervention.

We are concerned with agents modeled as double integrators in each cartesian dimension that must organize themselves in to formations prescribed by the relative positions between the agents. As in Abel et al [2005] our goal is to devise control laws that, require minimal information exchange between the agents and minimal knowledge on the part of each agent of the overall formation objective, are fault tolerant, scalable, and easily reconfigurable in the face of the loss or arrival of an agent, and the loss of a communication link.

A major drawback of Abel et al [2005] is that it assumes that all agents can exchange information at will. This is fine if agents acquire each others state information through straightforward sensing. If however, state information is exchanged through broadcast communication, this assumption is highly unrealistic. In particular when agents broadcast their state information they must compete with each other for access to the communication medium and are constrained by Media Access Control (MAC) protocols. Specifically, if agent A must listen to the broadcast of agent B, then no other agent that has A in its broadcast range can broadcast at that instant. Further in many instances no agent can simultaneously transmit and receive. These requirements limit (often severely) the number of transmissions that can occur at a given time, and the full schedule of information exchange *can only occur over several time slots*. Consequently information available to a given agent as it executes its control law may not be the most upto date. *The principal contribution of this paper is to modify Abel et al [2005] so that MAC protocols are accomodated.*

Significant work in this area has been conducted in the robotics community, and also in the string stability literature, Swaroop et al [March 1996], Khatir et al [December 2004]. The biologically motivated flocking literature, Yamaguchi et al [1996], seeking to mimic flocks of bird, seeks to organize coherent group movement as opposed to manitaining specified relative positions. To induce a set of agents with same speed to move in the same direction Vicsek et al. [1995] proposes a simple algorithm that is rigorously analyzed in Jadbabie et al. [June 2003]. The rendezvous problem, where agents are induced to converge to a single unspecified location, is studied in Lin et al [2003]. Consensus forming or synchronization are also instructive examples, Olfati-Saber et al [September 2004], Moreau [February 2005].

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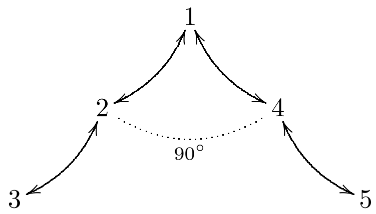

Fig. 1. agent Formation Topology with no Redundancy

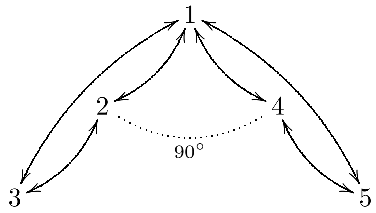

Fig. 2. agent Formation Topology with Redundancy

We are particularly interested in organizing agents into formations defined by desired relative positions and trajectories, Fax et al [July 2002] -Fax et al [September 2004]. The closest approach is in Fax et al [September 2004] which seeks to organize a network of agents according to specified relative positions, and focuses on the communication topology. Further discussion on this paper and our own work in Abel et al [2005] is below.

A related issue is to treat networks with time varying communication links, Olfati-Saber et al [September 2004], Moreau [February 2005] Again results in this area are restricted to the easier synchronization based problems referred to in the foregoing.

Papers like Fax et al [July 2002] and Fax et al [September 2004] separately propose a desired formation and a state exchange architecture and ask whether the latter suffices to achieve a formation. Abel et al [2005] reverse the question and ask given a desired formation, what state exchange architecture suffices to achieve it. They also focus on control laws that incorporate redundancies that permit the formation to survive the loss of agents and/or communication links.

Since the take off point of this paper is Abel et al [2005], we briefly reprise its salient points. Abel et al [2005] recognize that the same geometry can be described in multiple ways. Thus if the desired geometry is that depicted in fig. 1 it can be described by specifying the relative positions between agents joined by arrows. Thus in this figure relative positions and/or relative velocities of the pairs $(1, 2)$, $(1, 4)$, $(2, 3)$ and $(4, 5)$ are specified. One may also specify the same geometry by adding redundant information, as in fig. 2, where the additional constraints are added between the pairs $(1, 3)$ and $(1, 5)$. Such a redundant structure adds fault tolerance to the geometric description. Thus, while the loss of agent 4 in fig. 1, implies that 5 is isolated, in fig. 2, 5 retains its position relative to agent 1 and the new topology remains viable.

Here on we will call this the *Formation Topology*, as opposed to the *Communication Topology* which defines the state information flow required to implement a cooperative control law. We explore here the relation between these two topologies and argue that issues of fault tolerance, scalability and communication derive from the correct design of the formation topology.

To this end Abel et al [2005] proposes a cost function that incorporates the formation topology. A one step ahead optimal control law obtained on its basis has many features. Foremost among them is the fact that the communication topology required to implement it is *identical* to the underlying formation topology.

The key attractive properties of the approach of Abel et al [2005] are as follows: In the sequel we will call a pair of agent neigbors if they appear in the same geometric constraint. Thus in fig. 1 agent 1 has the neighbors 2, and 4, while in fig. 2 it has the additional neighbors 3 and 5.

(a) Agent i needs the state information of only its neighbors in the formation topology. (b) A given agent only *needs to know the constraints imposed on itself* by the formation topolgy. (c) Should the loss of an agent still permit an acceptable topology, then only the neighbors of the lost agent need to reconfigure their control law. (d) Should the loss of a communication channel still permit an acceptable topology, then only the agent at the end points of the lost arc need to reconfigure their control law. (e) If a new agent joins the fleet by establishing a geometric position with respect to a subset of the agents, then only these agents need to reconfigure their control law. (f) Relative position constraints can be augmented by compatible, potentially redundant velocity and/or relative velocity constraints.

Thus (a) indicates the communication topology highlighted in the foregoing. Item (b) has the added attraction of permitting the control to be implemented by a given agent with only a *local knowledge* of the formation topology. Scalability comes from (e) as a new agent 6 in fig. 1 with only 5 as a neighbor would require that only 5 readjust its control law. *Reconfigurability* under the loss of an agent is greatly facilitated.

In this paper, we propose an alternative control law that retains these attractive properties while respecting MAC requirements. A few points of note are as follows: First Abel et al [2005] have an undirected communication architecture, i.e. if agent i must convey its state to j, then j must convey its state to i. Though over a period of time this paper also has this requirement, as no agent can simultaneously transmit and receive, in any given sampling interval the architecture here is directional. This contrasts though from the directional control of Anderson et. al. [2007], Anderson et. al. [2006] where if agent i must sense the state of j, then agent j will not know the sense of i at all. Second as will be evident in the sequel, the control law employs a communication architecture that varies from one sampling interval to the next. However, unlike Moreau [February 2005] this architecure is periodically varying. Of course Moreau [February 2005] is confined to the synchronization problem, as opposed to the harder formation control problem studied here.

2. DYNAMICS AND THE FORMATION TOPOLOGY

When considering the problem of an n-agent formation our focus here is on a two dimensional formation topology, even though the ideas trivially extend to three dimensional formations as well. We shall partition the global, $4n \times 1$ state vector x of the network as

$$x = [x_1^T, x_2^T]^T, \tag{1}$$

where x_1 and x_2 contain the positions and velocities respectively. In particular, denoting $x_{l,j}$ as the j-th element of x_l, we will have

$x_{1,i}$ is the x position of agent i,
$x_{2,i}$ is the x velocity of agent i,
$x_{1,n+i}$ is the y position of agent i, and
$x_{2,n+i}$ is the y velocity of agent i

We shall further assume that each agent has been internally controlled to represent a double integrator with elements u_i and u_{n+i} of the control input vector u representing normalized force variables acting on the i-th agent, in the x and y directions respectively. For notational simplicity we will assume that the sampling interval is 1-second. The ideas trivially extend to nonunity sampling intervals. Thus, to within a suitable force normalization the system of agents can be described by:

$$x(k+1) = \Phi x(k) + \Gamma u(k) \tag{2}$$

where

$$\Phi = \begin{bmatrix} I_{2n} & I_{2n} \\ 0 & I_{2n} \end{bmatrix}, \text{ and } \Gamma = \begin{bmatrix} I_{2n} \\ 2I_{2n} \end{bmatrix}. \tag{3}$$

To ease notation we will often denote

$$\Phi x[k] = \theta(k). \tag{4}$$

The formation topology is alternatively characterized graphically and algebraically. In the former case it is described by an *undirected graph* with agents as nodes. An arc exists between two agents if their relative position constraint explicitly appears in the description of the formation topology.

Algebraically, the formation topology will be characterized in the following way. Observe that the relative positions between two agents i and j can be completely specified, for suitable f and g by the pair of equations

$$x_{1,i} - x_{1,j} = f \text{ and } x_{1,n+i} - x_{1,n+j} = g. \tag{5}$$

Assume that there are L such pairs of constraints. Then with an $L \times n$ matrix A,

$$\mathcal{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 0_{2n\times 2n} \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ 0_{2n\times 1} \end{bmatrix} \tag{6}$$

the topology can be represented by the following equation:

$$\mathcal{A}x = b, \tag{7}$$

Here x is the target state vector. In all there are $2L$ position constraints. Further A is a a matrix with each row having all but two elements zero and the remaining two being ± 1.

Formally, we make the following assumption.

Assumption 2.1. Suppose the formation topology has L arcs. Then the matrix A is $L \times n$. Further if an arc exists between agents i and j then there exists a row of A which has all but the i-th and j-th elements zero and among the remaining two one is 1, and the other -1. Further b_1, b_2 are each in the range space of A, and

$$\text{rank}[A] = n - 1. \tag{8}$$

Note that (8) implies that the graph representing the fomation topology is connected, i.e. there is a path joining any two nodes that can be traversed by moving from one nearest neighbor to the next.

Recall that while figures 1 and 2 describe the same geometry the latter represents a formation topology with redundancies. Observe if the formation topologies in figure 1 and figure 2 are respectively defined by the pairs $[\mathcal{A}^{(1)}, b^{(1)}]$ and $[\mathcal{A}^{(2)}, b^{(2)}]$, then $[\mathcal{A}^{(1)}, b^{(1)}]$ is a *submatrix* of $[\mathcal{A}^{(2)}, b^{(2)}]$. Moreover, should the loss of an agent result in a topology that remains acceptable, e.g. the loss of 4 in figure 2, then this new topology characterized by $[\mathcal{A}^{(3)}, b^{(3)}]$ obtained by removing the rows corresponding to the constraints featuring 4 and columns corresponding to the states of 4, is itself a submatrix of $[\mathcal{A}^{(2)}, b^{(2)}]$. This feature forms a core property to be exploited in fault tolerant design. Scalability is likewise incorporated rather easily. Thus if a new agent 6 appears in figure 2 with an arc between it and 5, then the new pair $[\mathcal{A}^{(4)}, b^{(4)}]$ characterizing it has $[\mathcal{A}^{(2)}, b^{(2)}]$ as a submatrix, and involves just the addition of rows and columns, and aumenting rows in $[\mathcal{A}^{(2)}, b^{(2)}]$ that feature in $[\mathcal{A}^{(4)}, b^{(4)}]$ by zero column entries.

Thus the loss of an agent/communication channel requires working with a submatrix of the original $[A, b]$, and the addition of an agent requires a supermatrix of $[A, b]$.

3. CONTROL LAW WITHOUT MAC CONSTRAINTS

We first recount the control law of Abel et al [2005], that assumes that communication occurs without access control constraints. It involves a one step ahead optimization law using the cost function

$$J(k) = [\mathcal{A}x(k+1) - b]^T [\mathcal{A}x(k+1) - b] + \\ + u^T(k)Qu(k) \tag{9}$$

Where $Q = Q^T > 0$ penalizes the input. The key step in achieving the control law with the desired characteristics described in the introduction is to appropriately select Q.

Since $x(k+1)$ is dependent on $u(k)$ we begin by substituting (2, 4) into the cost function defined in (9). Taking the partial derivative of the resultant expression with respect to $u(k)$, we obtain:

$$\left[\Gamma^T \mathcal{A}^T \mathcal{A}\Gamma + Q\right] u(k) = \Gamma^T \mathcal{A}^T [b - \mathcal{A}\theta(k)]$$

Choose, for some $\alpha > 0$,

$$Q = \alpha I - \Gamma^T \mathcal{A}^T \mathcal{A}\Gamma > 0.$$

The resulting control law is:

$$u(k) = \alpha^{-1}\Gamma^T \mathcal{A}^T b - \alpha^{-1}\Gamma^T \mathcal{A}^T \mathcal{A}\theta(k) \tag{10}$$

It has been shown in Abel et al [2005] that stability is guaranteed if Q is positive definite. Thus α must be chosen so that

$$\alpha I - \Gamma^T \mathcal{A}^T \mathcal{A}\Gamma > 0. \tag{11}$$

Now we reprise the arguments from Abel et al [2005] that show that the communication topology resulting from (10) is identical to the geometric topology and further that only a local knowledge of the formation is required by each agent. Observe that the control inputs to agent i are u_{2i} and u_{2i-1}.

We have the following result that establishes the various properties of the communication topology.

Theorem 3.1. Consider (10) under (1), (3), and (6). Then finding $u_{2i-1}(k)$ and $u_{2i}(k)$ requires: (A) The states of agent l only if there is an arc between agents l and i in the formation topology. (B) The l-th row of A only if for some $j \in \{2i-1, 2i, 2i-1+n, 2i+n\}$ $a_{lj} \neq 0$. (C) The l-th element of $b(k)$ only if for some $j \in \{2i-1, 2i, 2i-1+n, 2i+n\}$ $a_{lj} \neq 0$. (D) The gain λ_i.

(A) shows that the communication topology is the same as the formation topology. (B) and (C) show that agent i need only know those rows of A and elements of b which define the arcs emanating from it. Thus i must only know its place in the formation topology and a distributed knowledge of the formation topology suffices. This in particular has *security implications* as even if an agent is compromised the global objective is not.

4. CONTROL UNDER MAC CONSTRAINTS

The control law in 3 assumes that all agents can communicate at will. In practice when broadcast communication is used MAC constraints must be used to avoid message collisions. At the minimum this requires that when an agent is receiving state information from a neighbor all others in whose broadcast range it resides, must be silent. Nor can an agent receive and broadcast simultaneously.

Further, depending on the circumstance one of the following three situations may hold. (a) All agents are in each others mutual broadcast range. (b) Only agents having an arc between them are in each other's mutual broadcast range. (c) While any two agents that have an arc between them are in each other's mutual broadcast range, other agents without an arc to them may be in their broadcast range.

The control law we propose accommodates all these three settings. At any rate the following assumption will hold.

Assumption 4.1. If an arc exists between i and j in the formation topology, then i and j are in each others broadcast range. Further each agent always knows its position and velocity.

As is customary in *ad hoc* networks, we assume *a priori* that the agents have settle on a broadcast schedule, that is consistent with the MAC constraints noted above. We note that efficient algorithms for determining such a schedule, that involve only local exchange of information are available in the literature.

This schedule must be implemented over K sampling intervals, in each of which certain agents broadcast in a manner consistent with MAC requirements. Each interval is assumed for simplicity to be one. This transmission pattern is repeated after every K-samples. We further assume that while every input is updated in every sampling interval, the agent effecting that update does so by modifying (10), by replacing the instantaneous state information by the latest value it has access to. We make the following assumption capturing MAC.

Assumption 4.2. Every agent broadcasts only once in every K sampling intervals, and when it transmits, all agents it has an arc with receive that information. Further no agent can receive while it is broadcasting, and an agent cannot broadcast if an agent it has an arc to is receiving from another source. Moreover, all communication is instantaneous, in that if a broadcast occurs over an interval $[a, b)$, then the receipent knows the information at time a.

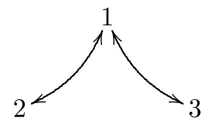

Fig. 3. Desired formation for a three agent system.

As an example consider the setting of (3). Suppose the transmission schedule uses $K = 3$, and is as follows: 1 broadcasts to 2 and 3 at all instants $3k$, 2 transmits to 1 at $3k+1$, and 3 to 1 at $3k+2$. Note that this accords with assumptions, 4.1 and 4.2, regardless of whether 2 and 3 are in each others broadcast range.

Define

$$D_i = e_i e_i' \tag{12}$$

where e_i is a $n \times 1$ vector that has 1 in its i-th element, and zeros in all others. Also denote:

$$D_{ij} = I_j \otimes D_i. \tag{13}$$

Suppose

$$V = \{1, 2, \cdots, n\}.$$

Define $V_i \subset V$ as the set of all agents that have an arc to agent i in the formation topology. Then the schedule comprises a sequence of sets

$$V(l) \subset V, \quad \forall l \in \{0, 1, \cdots, K-1\},$$

where each agent in $V(l)$ broadcasts in every sampling interval starting with $kK + l$. In keeping with assumption 4.2 we obtain the following control law, which we note *retains the attractive properties of (10)*: For all integer k, and $l \in \{0, 1, \cdots, K-1\}$

$$\begin{aligned} u(kK+l) = {} & \frac{\Gamma'}{\alpha}\mathcal{A}'b - \left(\sum_{i=1}^{n} D_{i2}\frac{\Gamma'}{\alpha}\mathcal{A}'\mathcal{A}\Phi D_{i4}\right)x(kK+l) \\ & - \sum_{m=0}^{l}\Big(\sum_{i\in V(m)}\sum_{j\in V_i} D_{j2}\frac{\Gamma'}{\alpha}\mathcal{A}'\mathcal{A}\Phi D_{i4}\Big)x(kK+m) \\ & - \sum_{m=l+1}^{K-1}\left(\sum_{i\in V(m)}\sum_{j\in V_i} D_{j2}\frac{\Gamma'}{\alpha}\mathcal{A}'\mathcal{A}\Phi D_{i4}\right) \\ & x(k(K-1)+m) \end{aligned} \tag{14}$$

The term involving

$$\sum_{i=1}^{n} D_{i2}\frac{\Gamma'}{\alpha}\mathcal{A}'\mathcal{A}\Phi D_{i4}$$

recognizes that each agent always has its state information. The second term captures the fact that all agents have access to their own states at all times. The resulting closed loop system is of course *K-periodic.*

To formalize the underlying rules governing the MAC protocol, that directly impact the stability proof to be presented in the next section, we make the following assumptions.

Assumption 4.3. The $V(l)$ form a disjoint partion of V, i.e.

$$V(i) \cap V(j) = \{\phi\} \text{ and } \cup_{i=0}^{K-1} V(i) = V.$$

This assumption ensures that in every K-cycle each node broadcasts only once, and is consistent with assumption 4.2.

Assumption 4.4. If for some $l \in \{0, 1, \cdots, K-1\}$, $i \in V(l)$ then for all $j \in V(l)$, $i \notin V_j$.

Since every neighbour of $i \in V(l)$ in the formation topology is in receive mode in the pertinent interval, this ensures that no agent receives and transmits simultaneously.

Assumption 4.5. If for some $l \in \{0, 1, \cdots, K-1\}$, $\{i, j\} \subset V(l)$. Then $V(i) \cap V(j) = \{\phi\}$.

This assumption ensures that no node can receive simultaneously from multiple sources. This is necessitated by the fact that no node can broadcast if a node in its broadcast range is receiving from another node.

5. STABILITY

This section proves the stability of the closed loop system defined by (2) and (14). Since the proofs are long, they are omitted. Define first for $m \in \{0, \cdots, K-1\}$

$$\mathcal{G}_m = -\Gamma \sum_{i \in V(m)} \sum_{j \in V_i} D_{j2} \frac{\Gamma'}{\alpha} (A'A \oplus A'A \oplus 0)(D_{i2} \oplus 0)\Phi, \tag{15}$$

and

$$\mathcal{G} = -\Gamma \sum_{i=1}^{n} D_{i2} \frac{\Gamma'}{\alpha} (A'A \oplus A'A \oplus 0)(D_{i2} \oplus 0)\Phi. \tag{16}$$

where, $\oplus$ denotes the direct sum operation on the matrices. Then because of (6) and (3), the closed loop becomes: for $l \in \{0, \cdots, M-1\}$,

$$x(kK + l + 1) = (\Phi + \mathcal{G})x(kK + l) + \sum_{m=0}^{l} \mathcal{G}_m x(kK + m) + \sum_{m=l+1}^{K-1} \mathcal{G}_m x(k(K-1) + m) + \Gamma \frac{\Gamma'}{\alpha} \mathcal{A}' b.$$

Then it follows that

$$\begin{bmatrix} x(kK + K + 1) \\ x(kK + K) \\ \vdots \\ x(kK + 1) \end{bmatrix} = \mathcal{F} \begin{bmatrix} x(kK) \\ x(kK - 1) \\ \vdots \\ x(kK - K + 1) \end{bmatrix} + \hat{\mathcal{G}} \tag{17}$$

where with $\mathcal{F}_1$ given by

$$\begin{bmatrix} I & -(\Phi + \mathcal{G} + \mathcal{G}_{K-1}) & -\mathcal{G}_{K-2} & \cdots & \cdots & -\mathcal{G}_1 \\ 0 & I & -(\Phi + \mathcal{G} + \mathcal{G}_{K-2}) & -\mathcal{G}_{K-1} & \cdots & -\mathcal{G}_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

and $\mathcal{F}_2$ given by

$$\begin{bmatrix} \mathcal{G}_0 & 0 & \cdots & \cdots & \cdots & 0 \\ \mathcal{G}_0 & \mathcal{G}_{K-1} & 0 & \cdots & 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (\Phi + \mathcal{G} + \mathcal{G}_0) & \mathcal{G}_{K-1} & \mathcal{G}_{K-2} & \mathcal{G}_{K-3} & \cdots & \mathcal{G}_1 \end{bmatrix}$$

$$\mathcal{F} = \mathcal{F}_1^{-1} \mathcal{F}_2 \tag{18}$$

and

$$\hat{\mathcal{G}} = \mathcal{F}_1^{-1} [I, \cdots I]^T \Gamma \Gamma' \mathcal{A}' b / \alpha.$$

We first examine the eigenvalues of $\mathcal{F}$. To this end we provide a result that relates its egienvalues to a lower dimensional matrix. Specifically define:

$$G_m = -\sum_{i \in V(m)} \sum_{j \in V_i} D_j \frac{A'A}{\alpha} D_i, \tag{19}$$

$$G = -\sum_{i=1}^{n} D_i \frac{A'A}{\alpha} D_i, \tag{20}$$

F_1 given by

$$\begin{bmatrix} I & -(I+G+G_{K-1}) & -G_{K-2} & \cdots & \cdots & -G_1 \\ 0 & I & -(I+G+G_{K-2}) & -G_{K-1} & \cdots & -G_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$F_2 = \begin{bmatrix} G_0 & 0 & \cdots & \cdots & \cdots & 0 \\ G_0 & G_{K-1} & 0 & \cdots & 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (I+G+G_0) & G_{K-1} & G_{K-2} & G_{K-3} & \cdots & G_1 \end{bmatrix}$$

and

$$F = F_1^{-1} F_2 \tag{21}$$

Then we have the following Lemma stated without proof.

Lemma 5.1. Suppose (11) and assumptions 2.1-4.5 hold. Suppose that some eigenvalues of F in (21) are at 1, and the rest are inside the unit circle. Then the eigenvalues of $\mathcal{F}$ in (18) are also either 1, or inside the unit circle.

The next Lemma characterizes the eigenvalues of the reduced dimensional matrix F.

Lemma 5.2. Suppose (11) and assumptions 2.1-4.5 hold. Then $(K-1)n$ eigenvalues of F in (21) are at 0, one eigenvalue is at 1, and the remaining $n-1$ are inside the unit circle.

Lemmas 5.1 and 5.2 together show that the eigenvalues of $\mathcal{F}$ are either inside the unit circle or at 1. Our eventual goal is to show that

$$\lim_{k \to \infty} \mathcal{A} x(k) = b. \tag{22}$$

Obseve that this is equivalent to the requirement that

$$\lim_{k \to \infty} (I_K \otimes \mathcal{A}) \begin{bmatrix} x(kK) \\ x(kK-1) \\ \vdots \\ x(kK - K + 1) \end{bmatrix} = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} b. \tag{23}$$

To prove (23), we provide a somewhat stronger result than the implications of Lemmas 5.1 and 5.2.

Lemma 5.3. Suppose (11) and assumptions 2.1-4.5 hold. Then all poles of $(I_K \otimes \mathcal{A})(zI - \mathcal{F})^{-1}$ are inside the unit circle.

Then we have the following main result.

Theorem 5.1. Suppose (11), assumptions 2.1, and 4.3-4.5 hold. Then under (2) and (14), one has

$$\lim_{k \to \infty} \mathcal{A} x(k) = b.$$

6. SIMULATIONS

The initial conditions of the fleet are the same for all the simulations. Figure (1) illustrates the desired formation topology without any built-in redundancy. The communication protocol for such a configuration is as follows,

- $(kT, kT+h)$: $1 \to 2$, $1 \to 4$
- $(kT+h, kT+2h)$: $2 \to 1$, $2 \to 3$, $5 \to 4$
- $(kT+2h, kT+3h)$: $4 \to 1$, $4 \to 5$, $3 \to 2$

where, $T = 3h$ and the direction of the arrow indicates the direction of the information flow. This assumes that none of the pairs (2,4), (5,1) and (3,5) are in each others broadcast range.

Now consider the topology with redundancy built-in. Figure 4 illustrates the desired formation topology with redundancy incorporated via the link between agents 2 and 4. The communication protocol for this configuration is shown below

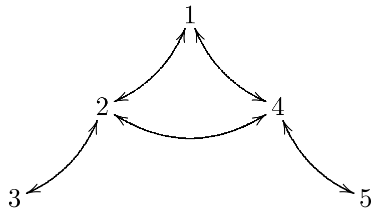

Fig. 4. Desired formation for a five agent system with redundancy.

- $(kT, kT+h)$: $1 \to 2$, $1 \to 4$
- $(kT+h, kT+2h)$: $2 \to 1$, $2 \to 3$, $2 \to 4$
- $(kT+2h, kT+3h)$: $4 \to 1$, $4 \to 5$, $4 \to 2$
- $(kT+3h, kT+4h)$: $3 \to 2$, $5 \to 4$

where $T = 4h$. This does assume that (2,4) are now in each others broadcast range. Effectively, in going from figure 4 to 1 the agents have reduced their broadcast range. This is a device that is commonly employed to ensure a more efficient implementation of the broadcast schedule.

Figure 5 illustrates the position error $\|Ax - b\|$ of the fleet for the redundant formation shown in figures 4.

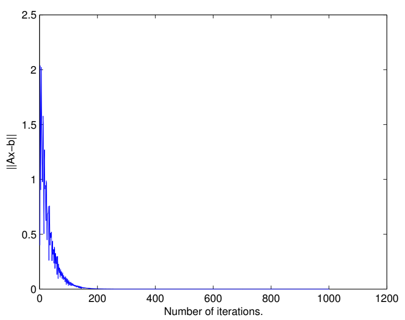

Fig. 5. The formation error in the case of a redundant topology.

7. CONCLUSION

We have examined the cooperative control of a fleet of automomous units that have to achieve arbitrary relative positions. We have proposed a new control strategy that results in distributed control, requiring a communication topology that mirrors exactly the formation topology and respects MAC requirements.

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