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Repetitive and Iterative Learning Controllers Designed by Duality with Experimental Verification

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Abstract: A duality theory existing between iterative learning and repetitive control for linear time-invariant systems has been reported. This paper considers the application of this duality in the design of such controllers for a non-minimum phase experimental facility and a three-axis gantry robot, where the task performed can be configured in either mode of operation. The models used in the design work have been obtained using frequency domain tests conducted on the physical plants. The control design has been performed for both systems, and verified using simulation studies in the case of the gantry robot and experimentally obtained test results in the case of the non-minimum phase plant.

1. INTRODUCTION

Many tracking systems have to operate with periodic reference/disturbance signals and are encountered in a wide range of practical applications, examples of which include robotic manipulators that are required to repeat a given task to a high level of accuracy and chemical batch processes. Iterative learning control (ILC) is one technique especially developed for controlling systems operating in a repetitive (or pass-to-pass) mode with the requirement that a reference trajectory $r(t)$ defined over a finite interval $0 \leq t \leq T$ is followed to a high precision.

Since the original work of Arimoto et al. [1984] in the mid 1980's, the general area of ILC has been the subject of intense research effort. A possible initial source for the literature in this field is the survey paper Bristow et al. [2006]. One approach in ILC is to construct the input to the plant or process from the input used on the last trial plus an additive increment which is typically a function of the past values of the measured output error, where on any trial this latter quantity is the difference between the achieved output and the desired reference trajectory. As such, it places the analysis of ILC schemes firmly outside standard (or 1D) control theory — although it still has a significant role to play in certain cases of practical interest. Instead, ILC must be seen in the context of fixed-point problems or, more precisely, repetitive processes (see the references in Rogers et al. [2007]) which are a distinct class of 2D systems where information propagation in one of the two independent directions only occurs over a finite duration.

A periodic signal can be generated by an autonomous system operating in a positive feedback loop where the plant model is a pure time delay. Hence by the internal model principle it is to be expected that these periodic signals can be controlled by duplicating this model within a feedback control loop. This has led to the ILC approach and also to repetitive controllers (RC). For background on RC see, for example, Hara et al. [1988], Tomizuka et al. [1989].

These two approaches differ in the way the periodic compensation is applied but they are not equivalent. It has, however, been shown in de Roover and Bosgra [1997], de Roover et al. [2000] that they are related by duality as a consequence of the difference in location of the internal model inside the controller. In particular, the repetitive controller has the structure of a servo compensator with the internal model located at the system output whereas the iterative learning controller has the structure of a disturbance observer where the internal model is located at the system input. These results have led to a general framework for the design of multiple-input multiple-output (MIMO) ILC and RC controllers where a number of existing schemes in both cases appear as special cases on making suitable modifications to the internal model.

In this paper, controllers designed using this duality are applied to both a gantry robot, and a non-minimum phase electromechanical system, both of which can be configured to operate in ILC and RC modes. The high level of performance achieved using these controllers is then established using both experimental and simulation test results. The next section gives the required background. This is followed by a description of the test facilities, both having been extensively used to evaluate a wide variety of ILC and RC control laws. After this, the controller designs and accompanying results for the gantry robot and non-minimum phase system are detailed.

2. BACKGROUND

Any periodic signal can, with appropriate boundary conditions, be generated by an autonomous system consisting of a positive feedback control loop with a pure time delay in the forward path and appropriate initial conditions. In

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particular, a discrete-time periodic signal of length N can be generated from

$$x_w(t_{k+1}) = A_w x(t_k),\ x_w(t_0) = x_{w0}$$
$$w(t_k) = C_w x(t_k) \tag{1}$$

where the $N \times N$ matrix A_w is given by

$$A_w = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

and the $1 \times N$ row vector C_w as

$$C_w = [\,1\ 0\ 0 \cdots 0\,]$$

Now consider a discrete linear time-invariant system with m outputs and l inputs described by the $m \times l$ transfer-function matrix $G(z)$ with input vector $u = u_p + u_w$ and output signal $y = Gu$. Then given a desired periodic output signal $r(t_k) = r(t_{k+N})$, $t_k = 0,\ \Delta T, 2\Delta T, \cdots$, with sampling time ΔT, let $e = r - y$ denote the tracking error. Then the robust periodic control problem is to find a controller $K(z)$ (where z denotes the discrete-time delay operator) such that the resulting closed loop system is 1) asymptotically stable, 2) the tracking error tends to zero exponentially for all periodic reference vectors r and periodic disturbances satisfying (1), and 3) properties 1) and 2) are robust to perturbations in the plant dynamics.

The solution to this robust periodic control problem is given by the internal model principle Francis and Wonham [1975]. In particular, suppose that the controller $K(z)$ contains in each channel a realization of the disturbance generating system driven by the error $E(z)$. Also let $K(z)$ be such that the feedback connection of $K(z)$ and $G(z)$ is internally stable. Then $K(z)$ solves the robust periodic control problem.

Both ILC and RC attempt to solve the robust periodic control problem and hence it follows that the internal model principle provides a solution for these cases. Also this principle can be formulated as a servo compensator where the disturbance model is realized in each channel of the output space (or vector) or, dually, in each channel of the input space (or vector). The first case here corresponds to implementation of RC and the second uses the structure of a disturbance observer and corresponds to ILC.

Following on from the work in de Roover and Bosgra [1997], de Roover et al. [2000], the design algorithms for RC and ILC can be stated as follows.

RC. Introduce the following $N \times 1$ vector

$$B_w = [\,0 \cdots 0\ 0\ 1\,]^T$$

and

$$A_r = \text{diag}\{A_w\}$$
$$B_r = \text{diag}\{B_w\}$$
$$C_r = \text{diag}\{C_w\}$$

where each diagonal block is repeated m times. In a similar way we define $\{A_l, B_l, C_l\}$ where each diagonal block is repeated l times. Also

$$C_r(zI_{Nm} - A_r)^{-1}B_r = z^{-N}I_m(I_m - z^{-N}I_m)^{-1}$$
$$C_l(zI_{Nl} - A_l)^{-1}B_l = z^{-N}I_l(I_l - z^{-N}I_l)^{-1}$$

An RC implementation with current error feedback based on state feedback of memory variables and estimated state feedback of the plant $G(z) = C(zI - A)^{-1}B + D$ is as follows

System

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k + Du_k$$

Observer

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L\epsilon_k$$

Feedback Error

$$e_k = r_k - y_k$$

Observer Error

$$\epsilon_k = e_k + C\hat{x}_k + Du_k$$

Disturbance Memory

$$x_{r,k+1} = A_r x_{r,k} + B_r u_k$$

Control Input

$$u_k = K_r x_{r,k} + K\hat{x}_k \tag{2}$$

where L is the plant state observer gain matrix and K_r and K are the state feedback law matrices for the disturbance memory and the plant respectively.

ILC. The underling structure here is generated by a disturbance observer (which has been shown to be the exact dual of a servo compensator de Roover and Bosgra [1997]). Assume at this stage there is a periodic disturbance at the plant input generated by a system $\{A_l, C_l\}$ with non-zero initial conditions. An observer for this system now results from duplicating the system and applying feedback, with gain matrices L_r and L respectively to the estimated states from an observer error. The estimate of the disturbance $\hat{d}_k$ is used to compensate the disturbance d_k and also the estimated plant state feedback control is applied using the feedback gain matrix K. Finally, the assumed error d at the plant input is replaced by an actual error resulting from the reference input vector r. Hence the disturbance estimator compensates for an input disturbance which is equivalent to the control error in the output space.

We now have

System As in RC above.

Observer

$$\hat{x}_{k+1} = A\hat{x}_k + B(u_k + \hat{d}_k) + L\epsilon_k$$

Observer Error

$$\epsilon_k = e_k + C\hat{x}_k + D(u_k + \hat{d}_k)$$

Feedback Error

$$e_k = r_k - y_k$$

Disturbance Memory

$$x_{l,k+1} = A_l x_{l,k} + L_l\epsilon_k$$
$$\hat{d}_k = C_l x_{l,k}$$

Control Input

$$u_k = K\hat{x}_k - C_l x_{l,k} \tag{3}$$

The design of the RC (2) and the ILC (3) involves the selection of the gain matrices $\{L, K_r, K\}$ and $\{K, L_l, L\}$

respectively. The following results give necessary and sufficient conditions for their existence in each case.

Theorem 1. Consider the RC law (2) and suppose that L is chosen such that $A + LC$ is asymptotically stable. Suppose also that $\{K_r, K\}$ is chosen such that the linear system with state matrix

$$\begin{bmatrix} A_r & -B_r C \\ 0 & A \end{bmatrix}$$

is asymptotically stable. Then (2) solves the problem under consideration if, and only if,

$$\operatorname{rank}\left(\begin{bmatrix} \lambda I - A & B \\ -C & D \end{bmatrix}\right) = n_x + m$$

for all λ in the spectrum of the matrix A_w, where n_x denotes the state vector dimension.

Theorem 2. Consider the ILC law (3) and suppose that K is chosen such that $A + BK$ is asymptotically stable. Suppose also that $\{L_l, L\}$ is chosen such that the linear system with state matrix

$$\begin{bmatrix} A_l & 0 \\ BC_l & A \end{bmatrix}$$

is asymptotically stable. Then (2) solves the problem under consideration if, and only if,

$$\operatorname{rank}\left(\begin{bmatrix} \lambda I - A & B \\ -C & D \end{bmatrix}\right) = n_x + l$$

for all λ in the spectrum of the matrix A_w and

$$\operatorname{rank}(B) + n_y = \operatorname{rank}\left(\begin{bmatrix} B \\ D \end{bmatrix}\right)$$

The first result here requires that the plant transfer-function matrix does not have transmission zeros which are also eigenvalues of the matrix A_w. Also the transfer-function must be square. The second result places the same restriction on transmission zeros and also the plant must have at least as many outputs as inputs. A routine argument also shows that the ILC will only give asymptotic tracking of the reference vector when the plant transfer-function is square and invertible.

Turning now to the actual design of RC for given data, it can be shown de Roover et al. [2000] that the separation principle allows $\{K_r, K\}$ and L to be designed independently. Also the case for ILC follows by duality. Next we describe the gantry robot and non-minimum phase facilities and then proceed to design controllers for each system by both routes.

3. EXPERIMENTAL FACILITIES

A non-minimum phase experimental test facility (see Figure 1) has been constructed to evaluate ILC and RC schemes (see Freeman et al. [2005],Cai et al. [2007] for details). The presence of a right-half plane zero has led to this type of system traditionally presenting difficulties in ILC, due to problems associated with obtaining a stable plant inverse. The facility consists of a rotary mechanical system of inertias, dampers, torsional springs, a timing belt, pulleys and gears. A further spring-mass-damper system is connected to the input in order to increase the relative degree and complexity of the system. A 1000 pulse/rev encoder records the output shaft position and a standard squirrel cage induction motor drives the load. The sampling frequency used during the tests conducted with this system is 100Hz. The nominal continuous time plant transfer function has been identified from frequency response data (using the Bode gain plot approximation and some data conditioning) as

$$G(s) = \frac{1.202(4 - s)}{s(s + 9)(s^2 + 12s + 56.25)} \tag{4}$$

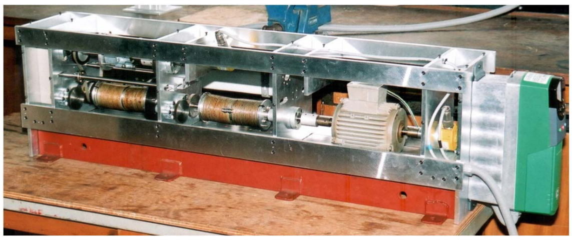

Fig. 1. The non-minimum phase plant experimental test facility.

A multi-axis test facility, see Figure 2, has also been constructed in order to enable ILC and RC schemes to be practically assessed on a realistic industrial application. The apparatus consists of a three-axis gantry robot which is supported above one end of a 6m long industrial chain conveyor. A description of the test facility can be found in Ratcliffe [2005]. Experimental tests using a sample frequency of 100Hz have been conducted to calculate models for the dynamic response of each axis. The combined displacement reference trajectories for each axis (shown in Figure 3) produce a 'pick and place' action, designed to collect a payload from a dispenser, synchronize position and velocity with the conveyor and place the payload on the conveyor. The reference trajectories fix the time taken for each iteration at 2 seconds, which, using a 100Hz sampling frequency, results in there being 200 sample instants per trial (or iteration).

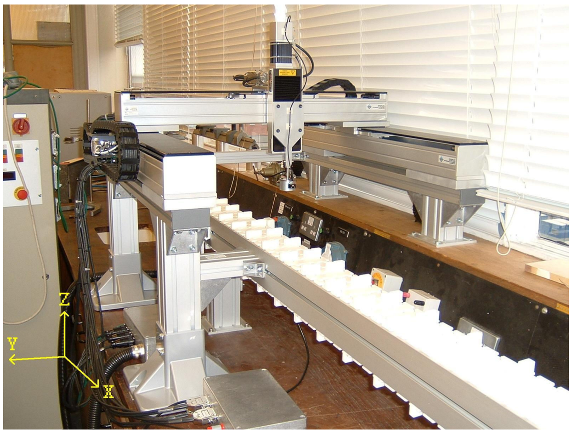

Fig. 2. Gantry robot system

Each axis of the gantry robot has been modelled using data from frequency response tests, and, because the axes are orthogonal, minimal interaction has been found to exist between them. Results for both RC and ILC controllers have been obtained for the complete system but due to space limitations only one axis is considered here — the

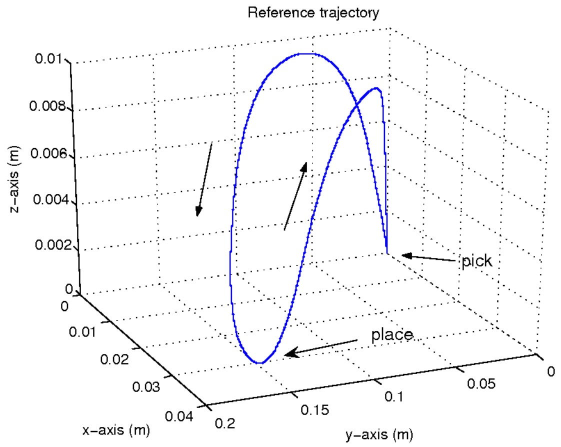

Fig. 3. 3D reference trajectories for the gantry robot

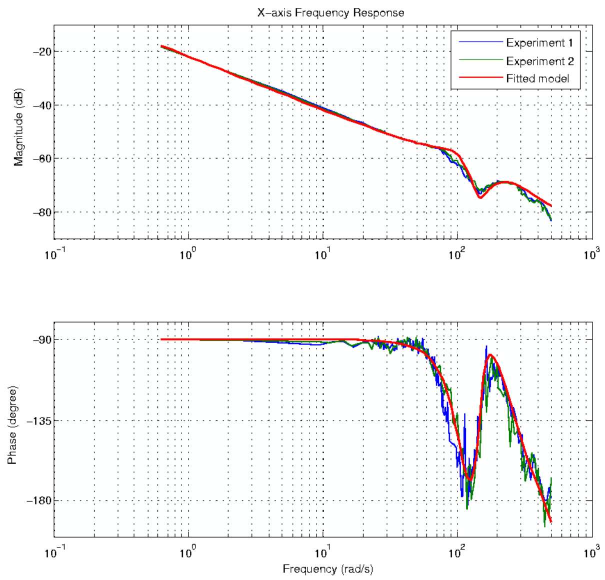

Fig. 4. Frequency response test results and fitted model

X-axis (this axis is parallel to the conveyor in Figure 2). Frequency response tests (via Bode approximate gain plots, see Figure 4) result in the following 7th order continuous-time transfer-function in Equation (5) as an accurate model of the dynamics on which to base control systems design.

$$G(s) = \frac{13077183.4436(s+113.4)}{s(s^2+61.57s+1.125\cdot 10^4)} \cdot \frac{(s^2+30.28s+2.13\cdot 10^4)}{(s^2+227.9s+5.647\cdot 10^4)(s^2+466.1s+6.142\cdot 10^5)} \quad (5)$$

4. DESIGN RESULTS

4.1 Simulations using the gantry robot

The control law matrices in all cases have been designed using a linear quadratic regulator setting. In the case of ILC there is a trade-off required when selecting the location of the poles of the closed loop system in terms of speed of convergence in the error from trial to trial,

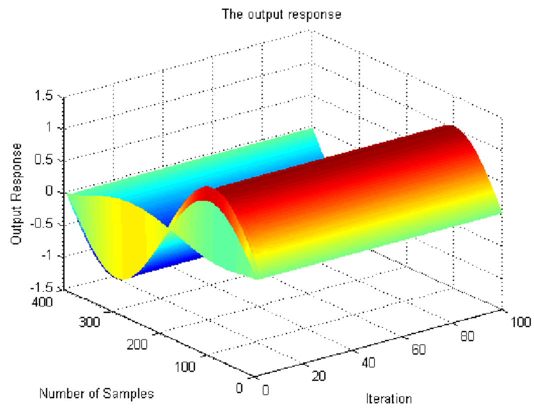

Fig. 5. Output of gantry robot using ILC

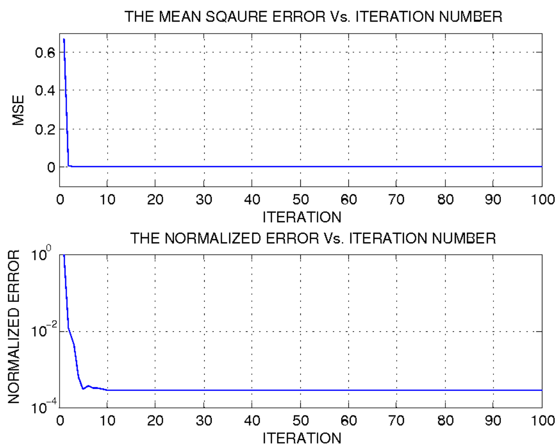

Fig. 6. Normalized Mean Squared Error for gantry robot using ILC

and the transient response along the trial. In particular, fast error convergence can cause significant controller effort which in the practical case may lead to damage of the robot actuators. Simulations for the gantry robot system have clearly shown that the designs here can limit the transient response such that there is no overshoot when the input demand is sinusoidal — see Figure 5. Figure 6 gives the mean squared error against trial number and shows that within 5 trials a high level of performance has been achieved and the error on the fifth trial is more than three orders of magnitude less than that recorded on the first trial.

By duality, an RC design can be applied to the same system using an identical choice of parameters. Typical simulation results are shown in Figure 7. Comparison with the ILC results shows that the repetitive controller exhibits much slower convergence but achieves a minimum error which is substantially less.

4.2 Experiments using the non-minimum phase system

Both RC and ILC designs have been implemented on the non-minimum phase plant in order to verify the duality approach experimentally. A sinewave with a period of 3 seconds has been used as the reference trajectory for the RC experiments, and this has modified for the ILC implementation by preceding it with a zero input of 1

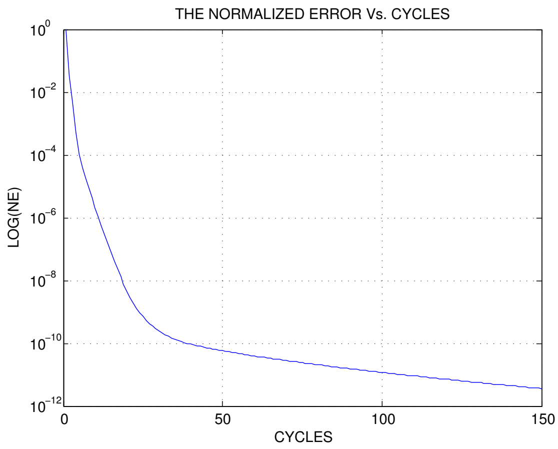

Fig. 7. Normalized Mean Squared Error for gantry robot using RC

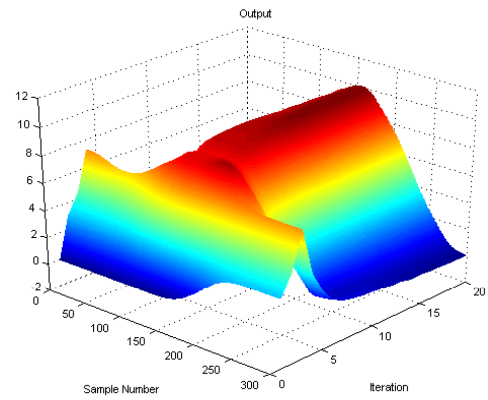

Fig. 8. Experimental output for the non-minimum phase system using RC

second duration (in order to avoid an excessive initial control input on each trial). Figure 8 shows the tracking output produced by the non-minimum phase plant using the repetitive control scheme over the first 20 trials in one of the experiments undertaken, and Figure 9 shows the corresponding error evolution. Figure 10 and Figure 11 show the tracking output and error respectively using the dual ILC scheme. In order to confirm the high level of tracking accuracy that has been achieved using the non-minimum phase system, Figure 12 and Figure 13 show the mean squared error results corresponding to the RC and ILC designs respectively.

5. CONCLUSIONS AND FUTURE WORK

Results have been presented to verify the efficacy of using a duality approach to design RC and ILC schemes. Simulated and experimental test results have shown that a high level of tracking accuracy can be achieved, and future work will examine the effect of the design parameters used on the performance and robustness properties of the controllers.

Focus will then concentrate on applying this approach in order to produce new controllers through the dualisation of existing ILC or RC designs. These will then be im-

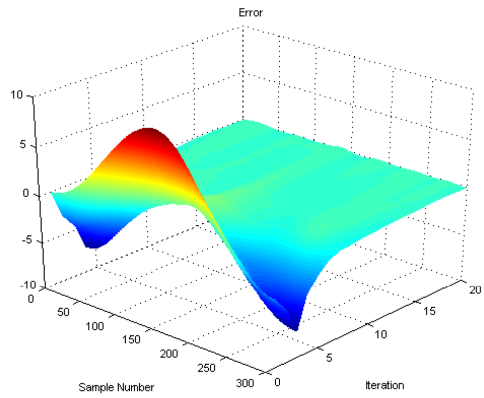

Fig. 9. Experimental error for the non-minimum phase system using RC

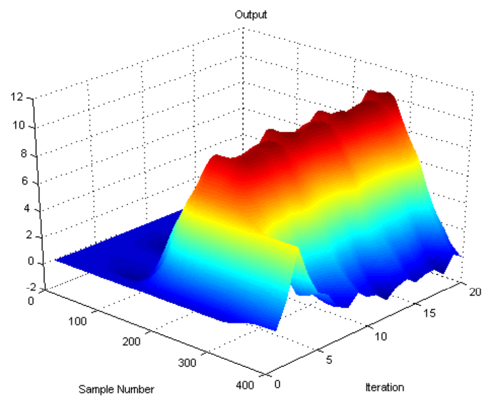

Fig. 10. Experimental output for non-minimum phase system using ILC

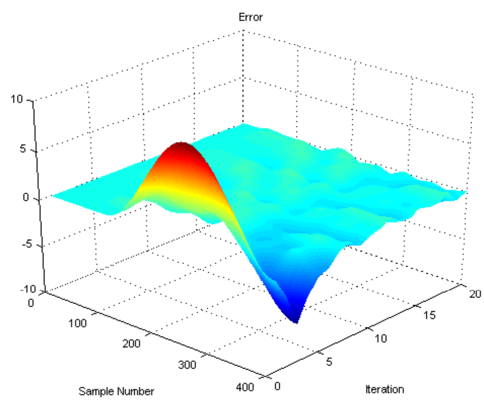

Fig. 11. Experimental error for non-minimum phase system using ILC

plemented and evaluated using the non-minimum phase system and gantry robot.

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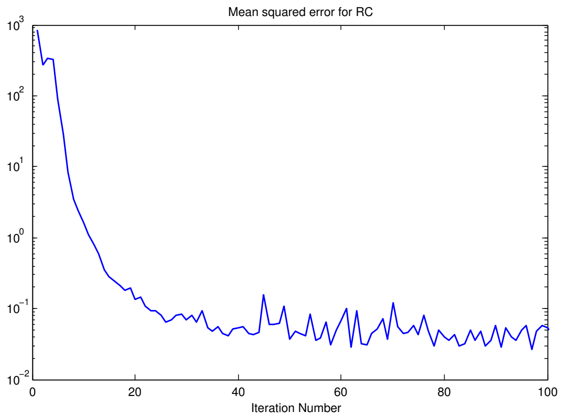

Fig. 12. Mean Squared Error for non-minimum phase system using RC

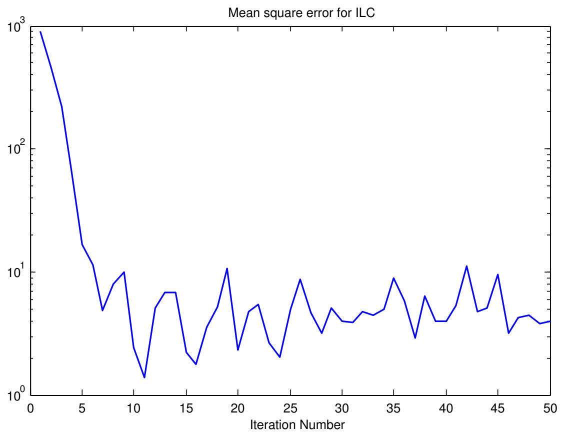

Fig. 13. Mean Squared Error for non-minimum phase system plot using ILC

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