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Connectivity Constrained Multi-UGV Surveillance ⋆

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Abstract: This paper addresses the problem of connectivity constrained surveillance of a given polyhedral area with obstacles using a group of Unmanned Ground Vehicles (UGVs). The considered communication restrictions may involve both line-of-sight constraints and limited sensor range constraints. In this paper, the focus is on dynamic information graphs, $\mathcal{G}$, which are required to be kept *recurrently* connected. The main motivation for introducing this weaker notion of connectivity is security and surveillance applications where the sentry vehicles may have to split temporary in order to complete the given mission efficiently but are required to establish contact recurrently in order to exchange information or to make sure that all units are intact and well-functioning. From a theoretical standpoint, recurrent connectivity is shown to be sufficient for exponential convergence of consensus filters for the collected sensor data.

1. INTRODUCTION

In both civil and military applications, surveillance is performed in order to assist in the prevention, detection and monitoring of intrusion, theft or other safety-related incidents. Facilities that require such supervision are numerous and include airport facilities, storage buildings, power plants, harbors and factories.

The surveillance and security solutions of today are based on a combination of human guards, electronic systems (cameras, intrusion alarms), physical security (fences, gates) and software (verification, logging). However, recent scientific and technological developments enable more autonomous and mobile complementary solutions. From a performance standpoint, the potential benefits of using a group of security or surveillance UGVs include cost savings and reduced risk exposure for human guards. It is therefore not surprising that the research area of surveillance vehicle control is active and growing. In this paper, we extend our previous work, Anisi and Ögren [2008], on cooperative surveillance using multiple UGVs.

Informally, the Connectivity Constrained UGV Surveillance Problem (CUSP) studied in this paper is the following: Given a set of surveillance UGVs and a user defined area to be covered, find waypoint paths such that:

- the so-called information graph is kept *recurrently* connected,
- the area is completely surveyed,
- the cost for performing the search is minimized.

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A more formal statement of CUSP is provided in Section 3 and a typical surveillance mission can be seen in Figure 1.

The main contribution of this paper is to present a concurrent task- and path planning algorithm which can handle connectivity constraints of both line-of-sight and limited sensor range types in the presence of obstacles. Along the way, we introduce the notion of *recurrent connectivity* for graphs and show that it is sufficient for convergence of consensus filters processing the collected sensor data.

The remainder of this paper is organized as follows. Section 2 provides a concise exposition of related work. The considered problem is formally defined in Section 3 and the proposed algorithm for solving it can be found in Section 4. Simulations illustrating the approach are presented in Section 5 and finally, the paper is concluded in Section 6.

2. RELATED WORK

A number of publications have been devoted to different aspects of the communication maintenance problem for mobile platforms. The great majority of these papers have focused on the sensor range constraint and often deal with obstacle-free environments, see *e.g.* Spanos and Murray [2004], Kim and Mesbahi [2006], Zavlanos and Pappas [2005, 2007], De Gennaro and Jadbabaie [2006], Ji and Egerstedt [2007]. To the best of our knowledge Schouwenaars et al. [2006] and Esposito and Dunbar [2006] are the only works that consider communication restrictions involving both limited sensor range and line-of-sight constraints in the presence of obstacles. Below, both these papers will be discussed in detail.

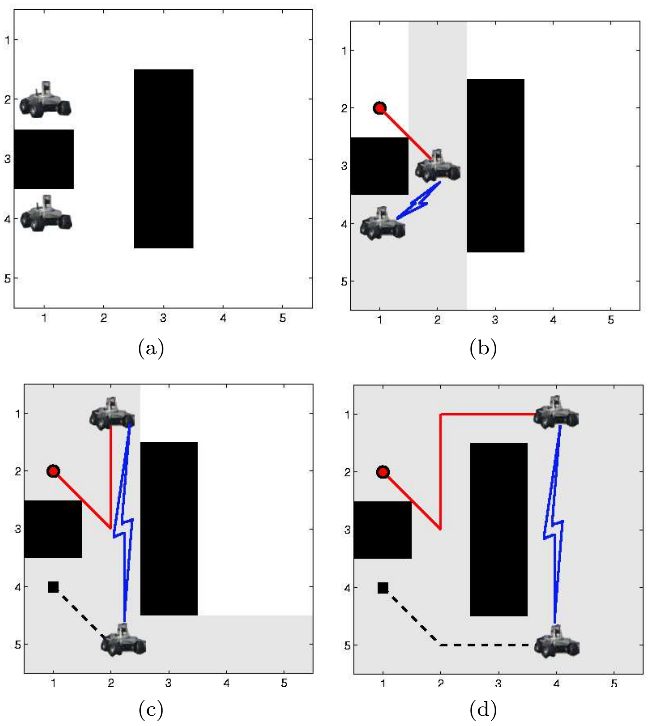


Fig. 1. A four snapshot illustration of a surveillance mission. The two UGVs start off to the left, depicted in Figure 1(a), and maintain connectivity *recurrently* at the three surveillance instances, Figures 1(b)- 1(d). The area already visited is shadowed and the mission is completed once the entire area is surveyed. Notice how the vehicles are allowed to split temporarily in order to pass on different sides of the obstacles.

The problem formulation in Schouwenaars et al. [2006] is reminiscent of the one considered in Nguyen et al. [2003], namely optimal path planning for a number of relay vehicles that have the mission of maintaining a chain of line-of-sight communication links that connect a given leader vehicle to the ground station. In accordance with their previous work in multi-vehicle path planning, Schouwenaars *et al.* use binary variables to capture connectivity between subsequent relay vehicles and end up solving a mixed integer linear program (MILP).

The problem considered in Esposito and Dunbar [2006] is the most closely related one to our work. However, Esposito and Dunbar:

- aim at reaching a final configuration while our high-level objective is to complete the surveillance mission.
- utilize a potential function to synthesize a feasible movement direction for the vehicles.
- consider the case when a *fixed* information graph is given *a priori* and maintain these given links intact *throughout the entire duration of the motion* [1].

Dynamic or time-varying information graphs have also been studied in frameworks other than the one considered in this paper. In particular, results in various applications of information consensus, such as flocking (Olfati-Saber [2006]), rendezvous (Lin et al. [2003]) and formation stabilization (Fax and Murray [2002]), heavily rely on some notion of (joint) connectivity of the underlying information graph. Consequently, a fundamental research issue over the last few years has been the search for a less restrictive notion of connectivity which still renders the consensus control convergent (see Jadbabaie et al. [2003], Moreau [2005], Ren and Beard [2005]). Theorem 1 relates these results to the notion of recurrent connectivity introduced below.

[1] This is equivalent with maintaining 1-hop connectivity of $\mathcal{G}$.

3. PROBLEM FORMULATION

In this section we first informally state the Connectivity Constrained UGV Surveillance Problem (CUSP). A more formal statement is given in Problem 1. Along the way, the terminology used in these formulations is properly defined.

Informally, the problem we are studying is the following: Given a set of surveillance UGVs and a user defined area to be covered, find waypoint paths such that every point of the area can be seen from a waypoint on a path, the information graph is kept *recurrently* connected in the presence of both line-of-sight constraints as well as limited sensor range constraints (see Definitions 5 and 6) and the cost for cooperatively performing the search is minimized.

This problem formulation is an extension of the one considered in Anisi and Ögren [2008] where connectivity constraints where not taken into account.

Remark 1. (Re-connection instances). It is important to note that in CUSP, the re-connection of the information graph does not occur at arbitrary time instances. In fact, $\mathcal{G}$ is required to re-connect at the *surveillance critical time instances, i.e.* exactly when the overall mission is being collectively solved.

The areas to be searched in this paper are all going to be so called orthogonal polygons with holes (obstacles), thus we denote them A, for area. The orthogonality of the polygon is due to the nature of Algorithm 1. It should however be noted, that finding a maximal convex cover of a general environment is not a hard problem, and the rest of the solution proposed in Algorithm 2 is not limited to the orthogonal case.

Definition 1. (Orthogonal polygons with holes). A *polygon* Q in the plane is an ordered sequence of points $q_1, \dots, q_n \in \mathbb{R}^2$, $n \geq 3$, called vertices of Q together with the line segments q_i to q_{i+1} and q_n to q_1, called edges. In the following we assume that none of these edges intersect. A polygon is called *orthogonal* if adjacent edges are orthogonal. Given a polygon Q and a set of m disjoint polygons $Q_1, \dots, Q_m$ contained in Q we call the set $A = Q \setminus \{Q_1 \cup ... \cup Q_m\}$ a *polygon with m holes.*

To be able to require that A is completely searched we state the following definitions and lemma.

Definition 2. (Guardiance). Given two points p and q in A we say that p is *visible* from q if the line segment joining p and q is contained in A, *i.e.*

$$\alpha p + (1-\alpha)q \in A, \quad \forall \alpha \in [0 \ \ 1].$$

A set of points $H = \{h_1, \dots, h_k\} \subset A$ *guards* A if for all $p \in A$ there exists $h_i \in H$ such that p is visible from h_i.

Definition 3. (Maximal convex cover). A *convex cover* C of A is a set of convex sets $C = \{c_i\}$ such that $A \subseteq \cup_i c_i$.

We define a *maximal* convex cover of A to be a convex cover $C = \{c_i\}$ of A, such that for all i, there is no convex set $s \subseteq A$ such that $s \supset c_i$.

Definition 4. (Visiting waypoint path). A *waypoint path* P is an ordered set of points $P = \{p_1, \ldots, p_n\}$. Any convex cover C is said to be *visited* by the waypoint path $P = \{p_1, \ldots, p_n\}$ if $\forall c_i \in C \; \exists p_j \in P : \; p_j \in c_i$.

Given the above definitions, the following lemma can be stated.

Lemma 1. If there exists a convex cover C of A such that the waypoint path P visits C, then P guards A

Proof. See Anisi and Ögren [2008]. ∎

The last concepts needed to make a formal statement of the CUSP are concerned with the information graph.

Definition 5. (Information graph). Let $\mathcal{V} = \{v_1, \ldots, v_N\}$ denote the vertex set representing the N UGVs. The *communication graph*, $\mathcal{G}_c(t) = (\mathcal{V}, \mathcal{E}_c(t))$, is induced by

$$e_{ij} \in \mathcal{E}_c(t) \Leftrightarrow \|p_i(t) - p_j(t)\| \leq R,$$

where R denotes the limited sensor range [2] and $p_i(t)$ is the position of UGV i at time t. The *sensing graph*, $\mathcal{G}_s(t) = (\mathcal{V}, \mathcal{E}_s(t))$, is induced by free line-of-sight, more precisely,

$$e_{ij} \in \mathcal{E}_s(t) \Leftrightarrow \alpha p_i(t) + (1-\alpha) p_j(t) \in A, \quad \forall \alpha \in [0 \; 1].$$

Finally, the *information graph*, $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, is defined as the union of the sensing- and communication graph, *i.e.* $\mathcal{E} = \mathcal{E}_c \cup \mathcal{E}_s$ [3].

In this setting, the sensing graph captures the *passive* information flow among the UGVs gathered by the on-board sensors, while the communication graph represents *active* transmission of inter-vehicle information (*cf.* Fax and Murray [2002]). This distinction is important to make in various applications, *e.g.* military missions where passive sensing is encouraged while active transmission, which might imply enemy exposure and thereby jeopardize the mission, should be avoided.

Definition 6. (Recurrent connectivity). A graph, $\mathcal{G}$, is said to be *recurrently connected* if there exists $T < \infty$ such that

$$\forall t \in \mathbb{R}^+, \exists \, T' \in [t \;\; t+T] : \mathcal{G}(T') \in \mathcal{C}.$$

Here, $\mathcal{C}$ denotes the set of connected graphs.

Definition 6 implies that one never have to wait for more than T time units until $\mathcal{G}$ is re-connected.

Problem 1. (CUSP). Given N vehicles and a polyhedral area A, the Connectivity Constrained UGV Surveillance Problem (CUSP) is to find a set of waypoint paths $P = \{P^1, \ldots, P^N\}$ that solve the following optimization problem

$$\begin{aligned} \min_{P} \sum_{j=1}^{n-1} & \max_{i \in \mathbb{Z}_N^+} ||p_j^i - p_{(j+1)}^i|| \\ \text{s.t.} \quad & \cup_i \, P^i \text{ guards } A \\ & \mathcal{G}_{P(j)} \subseteq \mathcal{C}, \;\; \forall j \end{aligned}$$

[2] Assuming a uniform bound on the range of all sensors, R, is merely a matter of notational convenience. An extension to allow different sensor ranges is straightforward.

[3] Intersection can also be used without any conceptual implications.

Here $P(j) = \{p_j^1, \ldots, p_j^N\}$ denotes the UGV positions at time instance j and $\mathcal{G}_{P(j)}$ is the *induced* information graph when the vehicles are at $P(j)$. Further, $\mathcal{C}$ is the set of connected graphs on N vertices, $P^i = \{p_1^i, \ldots, p_n^i\}$, $\mathbb{Z}_N^+ = \{1, \ldots, N\}$ and the start and finish positions, denoted by $p_1^i, p_n^i, i \in \mathbb{Z}_N^+$ may be given.

Remark 2. (Interdependence). In CUSP, the inter-vehicle dependence stems both from the imposed guarding and connectivity constraints, as well as the non-separable objective function.

3.1 Recurrent connectivity and consensus reaching

So far, our main focus has been on planning waypoint paths for the UGVs. Along these paths, information can be collected by the on-board sensors and propagated through the links of $\mathcal{G}(t)$. In this section, we study the issue of convergence of information filters for the collected sensor data.

We assume that each UGV measures some quantity $y_i \in \mathbb{R}^n$ that is then communicated to the others and used as input to a consensus filter

$$\dot{x}_i = \sum_{j \in N_i} a_{ij} [(x_j(t) - y_j(t)) + (y_i(t) - x_i(t))]. \qquad (1)$$

Let $x = (x_1, \cdots, x_N)^T$, then using standard notations in graph theory, (1) can be rewritten as

$$\dot{x} = -L(x - y), \qquad (2)$$

where L is the Laplacian matrix. Let $e = x - y$, then we have

$$\dot{e} = -Le - \dot{y}.$$

Let us assume $\dot{y}$ is small and focus on

$$\dot{e} = -Le. \qquad (3)$$

This assumption implies that the information filter varies on a faster time-scale than the quantity being agreed upon.

It is well known (see *e.g.* Jadbabaie et al. [2003], Ren and Beard [2005]) that if the graph is (jointly) connected, then a consensus can be reached in (3). The next theorem shows that a consensus can also be reached for (3) if the graph is recurrently connected.

Theorem 1. (Consensus Reaching). Suppose the information graph is recurrently connected with a dwell time τ, then the frequently adopted “Laplacian protocol” (3) will exponentially converge to a consensus in the agreement space provided that the weights a_{ij} are properly tuned.

Proof. Since recurrent connectivity implies that $\mathcal{G}$ is jointly connected over all time intervals $[t \;\; t + T + \varepsilon)$ for any arbitrary constant $\varepsilon > 0$, the result follows directly from Theorem 2 in Jadbabaie et al. [2003].

4. PROPOSED SOLUTION

In this section we will propose a solution to the CUSP described in Section 3. In our previous work, Anisi and Ögren [2008], it was shown that the considered problem is a generalization of the so called Multiple Traveling Salesman Problem (MTSP). It is a well known fact that the MTSP and the closely related Multi Vehicle Routing Problem (MVRP) are $\mathcal{NP}$-hard, and thus represent optimization problems that are in general very hard to solve to optimality, see *e.g.* Bektas [2006], Laporte [1992]. Knowing this, we can not hope to solve all CUSP instances to optimality in reasonable time but must adopt heuristic solution methods. The decomposition method suggested in this paper is reminiscent of the one presented in Anisi and Ögren [2008], in the sense that they both encompass three subproblems as depicted in Figure 2. In fact, it turns

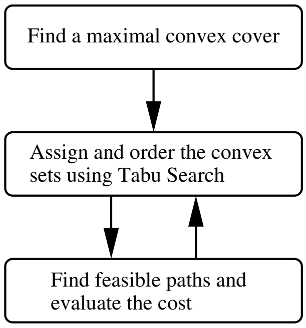


Fig. 2. The solution relies on a three step decomposition method.

out that extension of the results of Anisi and Ögren [2008] in order to capture the connectivity constraints present in CUSP, do not require any modifications to the top two subproblems at all. Consequently, it is only the last subproblem that will be described in detail below.

In the first subproblem, the computationally intractable problem of finding the optimal paths that enable *complete* area surveillance, is turned into a finite dimensional combinatorial optimization problem. This is achieved by finding a *maximal convex cover* of A, as follows:

Algorithm 1. (Maximal convex cover).

(1) Make a discretization of the area A and construct the corresponding graph representation. Since A is orthogonal, a variable sized grid can be created with grid boundaries intersecting all points in the polygon Q and holes $Q_1, \dots, Q_m$.
(2) Find a yet uncovered cell, p.
(3) Start growing a rectangle c_k from p, until it is bounded by the polygon or the holes on all four sides.
(4) While uncovered cells exist, goto 2.

When no more uncovered grid cells can be found the process terminates and A is covered, $A \subseteq \cup_k c_k$.

The second subproblem involves assignment and ordering of the convex sets in the cover. Since this is a modified version of the MVRP and it was established in Cordeau et al. [2002], Laporte [1992], Thunberg et al. [2008] that Tabu Search (TS) is a highly efficient heuristic for a wide range of routing problems, it is reasonable to make TS a part of our solution method as well. To this end, let M denote the number of convex sets used in the cover and assign the id numbers $1, \dots, M$ to these sets $c_k, k \in \mathbb{Z}^+_M$. Let furthermore the N vehicles have id numbers $M + 1, \dots, M + N$. The search space for the TS then consists of all permutations of the id numbers, *i.e.* $\mathbb{Z}^+_{M+N}$. The interpretation of a sequence of id numbers is best explained by means of the following example: Let $M = 7$, $N = 3$, and the final sequence be

$$\pi = (1,\ 8,\ 4,\ 3,\ 9,\ 5,\ 2,\ 10,\ 7,\ 6).$$

This corresponds to the following assignments.

Set with id numbers	assigned to UGV with id number
1 4 3	8
5 2 ★	9
7 6 ★	10

Table 1.

Most of the details of the implementation are identical to those presented in Ögren et al. [2006]. Hence, the interested reader is referred to that paper for a fuller description. We just note that the meaning of the ★ symbol will be clear from Remark 3 below, and that the neighborhood search is performed by pairwise interchanging components in the current permutation. Also, the Tabu condition corresponds to requiring a minimum number of iterations before switching a particular pair again.

The third subproblem, which is called as a subroutine of the second one, addresses two core problems, namely:

- generation of a *feasible* waypoint path,
- evaluation of the objective function in the TS.

In the TS above, each UGV is assigned a number of convex sets to visit and an order of visitation. Let I^π_i denote the set assigned to vehicle $i \in \mathbb{Z}^+_N$ and let $I^\pi_i(j)$ denote the j^{th} component of it. Let further V_k denote the nodes in A associated with the set c_k, $k \in \mathbb{Z}^+_M$. The connectivity constraint imposed on CUSP occurs $\max_{i\in\mathbb{Z}^+_N} |I^\pi_i|$ times [4] (*cf.* Remark 4). At every such re-connection instance, j, we require $p_i \in V_{I^\pi_i(j)}$, *i.e.* each vehicle is inside the appropriate convex set, but also that the information graph is connected. Next, we discuss how this may be achieved in an algorithmic manner.

4.1 An algorithm for generating feasible waypoints

The convex sets that have to be visited at the j^{th} re-connection instance are $\{I^\pi_1(j), \cdots, I^\pi_N(j)\}$ (see Remark 3). The connectivity constraint is imposed by looping through all the nodes of one of these convex sets and start growing a tree having this node as its root. Next, all nodes that are *adjacent* [5] to this root and belong to one of the sets in $\{I^\pi_1(j), \cdots, I^\pi_N(j)\}$, are added as leaves. These new leaves may then give rise to new adjacent nodes that are augmented at the top of the tree. Throughout this process, one must keep lists of all convex sets that have been visited in the current tree. If one of these lists equal a permutation of $\{I^\pi_1(j), \cdots, I^\pi_N(j)\}$, we have found N nodes that respect the communication constraints. This is since only adjacent nodes were added and the chain of nodes are originating from a tree and are hence connected. On the other hand, if the loop comes to an end without having found a tree that contains one node from each of the N sets, $I^\pi_1(j), \cdots, I^\pi_N(j)$, we indicate it by setting the associated objective function to ∞.

Remark 3. (Information relays). In general, there may be UGVs that are assigned to visit strictly less than $\max_{i\in\mathbb{Z}^+_N} |I^\pi_i|$

[4] Here, $|I|$ denotes the cardinality of the set I.
[5] Here, both line-of-sight as well as maximum distance constraints are imposed.

convex sets. This fact has been indicated by the ★ symbol in Table 1. The way we take advantage of this possibility is to utilize those UGVs that have already visited all the convex sets assigned to them, as *information relays*. More precisely, the ★ symbol is interpreted as a fictitious set to be visited. This fictitious set contains *all* free nodes in A and is referred to as a *relay set*. By this construction, the free UGVs may visit any node in A and hence will move to a position that renders the induced information graph connected, *i.e.* act as an information relay.

Remark 4. (Re-connection frequency of $\mathcal{G}$). The above presented algorithm will require $\mathcal{G}$ to be connected between $\lceil \frac{M}{N} \rceil$ and M times. It may be noted that the more "complicated" or challenging the area A is (as defined by a greater number of convex sets required for a complete cover), the more frequently are the vehicles required to reconnect and thereby perform "confirmation checks". This is a very appealing property from a tactical point of view. Nevertheless, if these given limits for connectedness of $\mathcal{G}$ are found to be too sparse in time, one has the opportunity to re-connect $\mathcal{G}$ more frequently by inserting columns of *relay sets* (indicated by ★ in Table 1 and elaborated upon in Remark 3). Obviously, this enhancement of the connectedness of $\mathcal{G}$ occurs at the cost of computational effort. In the limiting case, one may require $\mathcal{G}$ to stay connected at all time-steps. As such, this papers generalizes previous work which aim at keeping the information graph connected throughout the entire duration of motion.

4.2 Functional evaluation

Regarding the evaluation of the objective function, it has already been mentioned that the objective function is set to ∞ for those permutations that are infeasible with respect to the connectivity constraints. In those cases when the algorithm of Section 4.1 successfully returns a feasible configuration, we require the N vehicles to *simultaneously* pass through the set of waypoints that render connectivity maintenance possible. To this end, at each re-connection instance, j, the time of rendezvous is dictated by the UGV which has longest distance to travel. If we let n denote the number of re-connection instances (which may be greater than $\max_{i\in\mathbb{Z}_N^+} |I_i^\pi|$, *cf.* Remark 4) the objective function becomes

$$\sum_{j=1}^{n-1} \max_{i\in\mathbb{Z}_N^+} \|p_j^i - p_{(j+1)}^i\|. \tag{4}$$

4.3 Proposed algorithm

Having described the three subproblems in some detail, we are now ready to state the overall solution algorithm.

Algorithm 2. (Proposed solution). The algorithm consists of the following three steps:

(1) Create a maximal convex cover $C = \{c_1, \ldots, c_M\}$ of A in accordance with Algorithm 1.
(2) Assign and order the convex sets using TS.
(3) Generate a feasible waypoint path in accordance with the algorithm of Section 4.1 and calculate the corresponding objective value according to (4). While the maximal number of TS iterations have not been reached, goto 2.

Proposition 1. Algorithm 2 produces feasible solutions to Problem 1.

Proof. This is clear from Lemma 1 and the following three observations regarding Algorithm 2:

- a convex cover is created in step 1,
- all sets are assigned to different UGVs in step 2,
- the waypoint paths created in step 3 visit all assigned sets and respect the connectivity constraints.

It is now time to run some simulation examples.

5. SIMULATIONS

In this section, a small selection of the simulations made is presented. The objective is to highlight some of the key characteristics of the proposed solution method. Throughout this section, the search area A is chosen to be all of the obstacle free space.

Referring to Figure 3, the area representation is a random matrix with obstacle density $\rho = 0.3$. The starting positions of the two UGVs are chosen randomly while the final positions have been optimized by Algorithm 2. The most important aspect to notice is that the two UGVs are not restricted to pass on the same "side" of the obstacles but are nevertheless *recurrently* connected at the four surveillance instances, Figure 3(a)- 3(d). Also, notice that the area is *completely* surveyed and that the solution is merely locally optimal. This is since both TS and the proposed path planning algorithm are heuristic methods.

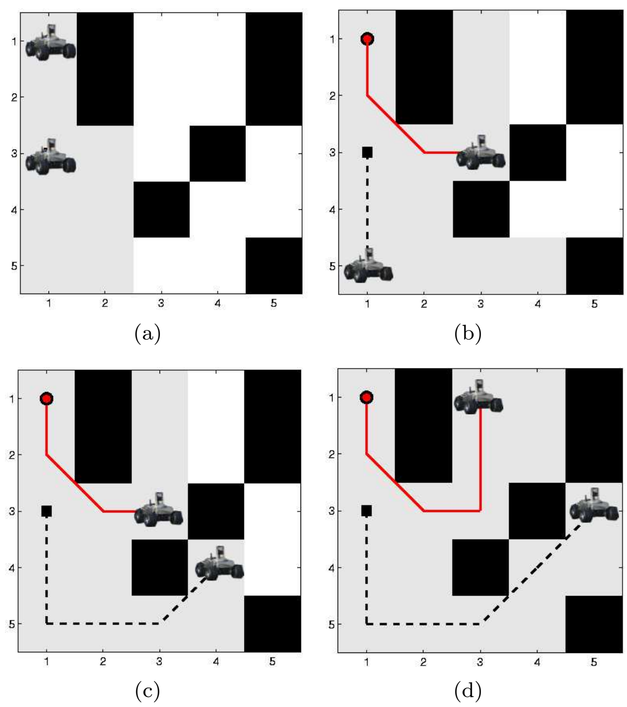
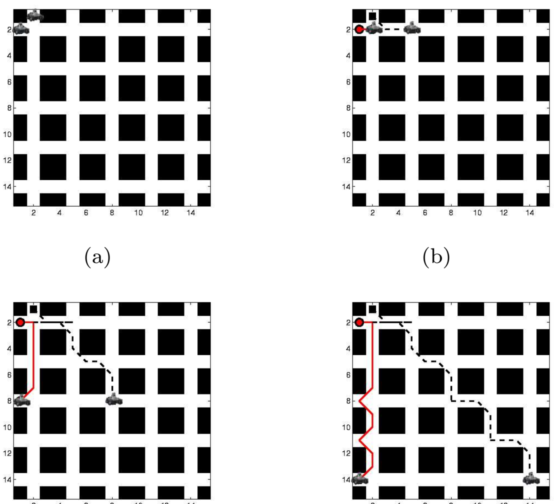


Fig. 3. The UGVs are free to pass on different "sides" of the obstacles but are nevertheless *recurrently* connected at the four surveillance instances.

Figure 4 illustrates surveillance of the so called "Manhattan grid". In this example, the cooperative nature of the solution becomes even more apparent. The two UGVs are dropped off at the upper left corner in A and move downwards in order to fulfill their common goal of complete coverage. In essence, the UGV whose waypoint path has been depicted in dashed/black surveys the vertically aligned streets while the other one (solid/red) covers the others. Notice how the inter-vehicle connectivity is maintained cooperatively as the UGVs timely pass the horizontally aligned streets. Also in this example, the fact that the final solution is merely locally optimal is apparent from the dashed/red waypoint path, which could rather be a straight line segment.



(a) (b) (c) (d)

Fig. 4. Complete surveillance of the so called "Manhattan grid". Notice how the inter-vehicle connectivity is maintained cooperatively as they timely pass the horizontally aligned streets.

6. CONCLUDING REMARKS

An important problem in cooperative UGV surveillance is to make sure that the sensor data can reach all team members but also be transmitted back to the operator. In this paper, we presented a cooperative path and task planning algorithm that made sure that the whole surveillance area was covered, and at the same time the entire UGV group was *recurrently connected* in order to exchange information and upload it to the operator. We defined recurrently connected to mean that there in an upper bound on the time intervals during which the information graph is not connected. From a theoretical standpoint, it was shown that recurrent connectivity is sufficient for exponential convergence of consensus filters.

In the proposed approach, the frequency of the connectedness of the information graph was a design parameter (see Remark4). As such, this paper generalizes previous work which aim at keeping the information graph connected throughout the entire duration of motion.

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