Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Functional Adaptive Control for Multi-Input Multi-Output Systems

Ladislav Král and **Miroslav Šimandl**

Department of Cybernetics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 306 14 Plzeň, Czech Republic, email: ladkral@kky.zcu.cz, simandl@kky.zcu.cz

Abstract: A functional adaptive control for nonlinear stochastic systems with Multi-Input Multi-Output is suggested. The systems are modelled using a multilayer perceptron networks. Parameters of the model are estimated by the Gaussian sum method which allows to determine conditional probability density functions of the network weights. Control design is based on bicriterial dual approach that use two separate criterions to introduce one of opposing aspects between estimation and control; caution and probing. The proposed approach is compared with two adaptive non-dual controllers. The quality of the proposed functional adaptive controller is illustrated in a numerical example.

1. INTRODUCTION

In the last decade, functional adaptive control system design of nonlinear systems has received a great deal of attention [Fabri and Kadirkamanathan, 2001, Murray-Smith and Sbarbaro, 2002, Šimandl et al., 2005b, Herzallah and Lowe, 2006]. The title 'Functional adaptive control' refers to the fact that the type of model uncertainty dealt with functional uncertainty, where the nonlinear functions same as parameters of the system are unknown.

Modelling of the nonlinear unknown functions of the system can be approached via functional approximators like diverse types of neural networks (radial basis function (RBF) or multilayer perceptron (MLP) [Haykin, 1999]). Gaussian process (GP) technique is used as alternative tool as well [Murray-Smith and Sbarbaro, 2002].

Typically, aim of the methods of the functional adaptive control should be both simultaneously optimizing control performance and reducing uncertainty. In differing from methods using well known equivalence principle, this final control system generates action signal that represents compromise between control and identification of the system. In addition, it is possible to avoid time consuming process of off-line identification of the model. Typically, such methods are either adaptive critic [Herzallah and Lowe, 2006]) or dual control methods [Filatov and Unbehauen, 2004].

It should be pointed out that the above mentioned results of functional adaptive control are limited to single-input single-output (SISO) systems. However, many control systems are multivariable [Narendra and Mukhopadhyay, 1994, Fu and Chai, 2007, te Braake et al., 1998]. The control problems for multi-input multi-output (MIMO) systems are more difficult and very different from those for SISO systems. The result for SISO systems cannot be simply extended to MIMO systems in general. Hence, problems of representation, identification or control of the system with MIMO is significantly more challenging than in case of the SISO systems. Even in case of linear system with known parameters a task of design control is difficult, especially due to existing coupling between individual inputs and outputs. The control problem is more complicated in case of stochastic nonlinear system with containing uncertainties about nonlinear functions of the systems. It is a task of the functional adaptive control of the MIMO systems. However, this area of functional adaptive control have been addressed only minor attention so far.

[Narendra and Mukhopadhyay, 1994] represents a first attempt to deal with the theoretical aspects of both the representation and control of nonlinear multivariable dynamical systems, as well as the development of a design methodology for their control using neural networks. But an intensive off-line training of the neural network is needed. In [Sbarbaro et al., 2004] a comparison neural networks and gaussian process model is performed. The certainty equivalence principle is used in control design and so there is no reduction of future uncertainties of the model. Further, the GP techniques requires off-line process of identification. In [Fu and Chai, 2007], the technique of multiple models is used for MIMO control design, but only system with no disturbances is considered. The functional adaptive control for the multivariable stochastic systems discrete in time has not been studied yet.

Hence, main goal of the paper is to design functional adaptive control for non-linear stochastic MIMO systems by using an idea of bicriterial dual control, and thus, to provide an extension of authors previous work for SISO nonlinear stochastic systems [Šimandl et al., 2005b].

The paper is organized as follows: In Section 2 the problem of dual stochastic adaptive control for non-linear MIMO systems is formulated. Section 3 concentrates on MIMO system identification by neural networks. The derivation of the bicriterial dual controller is shown in Section 4. In Section 5 the proposed approach is demonstrated in a numerical example.

2. PROBLEM STATEMENT

The dynamical system to be controlled is nonlinear stochastic discrete time-invariant system with m inputs and n outputs given by

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_{k-1}) + \mathbf{G}(\mathbf{x}_{k-1})\mathbf{u}_{k-1} + \mathbf{e}_k, \tag{1}$$

where $\mathbf{u}_k = [u_k^{(1)}, \ldots, u_k^{(m)}]^T$ are inputs of the system, $\mathbf{y}_k = [y_k^{(1)}, \ldots, y_k^{(n)}]^T$ describes outputs of the system, $\mathbf{f}(\mathbf{x}_{k-1}) = [f^{(1)}(\mathbf{x}_{k-1}), \ldots, f^{(n)}(\mathbf{x}_{k-1})]^T$ is the n dimension vector of nonlinear unknown functions, further

$$\mathbf{G}(\mathbf{x}_{k-1}) = \begin{bmatrix} g^{(11)}(\mathbf{x}_{k-1}) & \ldots & g^{(1m)}(\mathbf{x}_{k-1}) \\ \vdots & \ddots & \vdots \\ g^{(n1)}(\mathbf{x}_{k-1}) & \ldots & g^{(nm)}(\mathbf{x}_{k-1}) \end{bmatrix} \tag{2}$$

is the matrix of nonlinear *unknown* functions with dimensions $n \times m$, $\mathbf{x}_{k-1} \triangleq [\mathbf{y}_{k-p}^T, \ldots, \mathbf{y}_{k-1}^T, \mathbf{u}_{k-1-s}^T, \ldots, \mathbf{u}_{k-2}^T]^T$ is state of the system and $\mathbf{e}_k = [e_k^{(1)}, \ldots, e_k^{(n)}]^T$ contains sequences of the additive noises. It is required the following of the chosen reference signals $\mathbf{r}_k = [r_k^{(1)}, \ldots, r_k^{(n)}]^T$ and it is assumed accomplishment of the following conditions:

Assumption 1. Parameters p and s of the system state are known.

Assumption 2. The system has a globally uniformly asymptotically stable zero dynamics and nonlinear functions in the matrix $\mathbf{G}(\mathbf{x}_{k-1})$ are bounded away from zero for all $\mathbf{x}_k$ ([Chen and Khalil, 1995]).

Assumption 3. The sequences $\{\mathbf{e}_k\}$ are mutually independent Gaussian noises with zero mean and covariance matrix $\boldsymbol{\Xi} = \text{diag}\left(\sigma_e^{(i)}\right)$, where $\sigma_e^{(i)}$ are known variances of the $e_k^{(i)}$ for $i = 1, \ldots, n$.

Assumption 4. The relative order of the system is the same for all outputs.

The goal of the control is to design the functional adaptive dual controller for the system (1) in such a way that the output of the system $y_k^{(i)}$ for $i = 1, 2, \ldots, n$ will follow appropriate reference signal $r_k^{(i)}$ chosen by designer, otherwise to provide the control law by minimization of properly chosen criterion.

Design of the functional adaptive control will be made analogical to [Šimandl et al., 2005b]. In terms of solution design is necessary to deal with suitable representation of the MIMO system (1) by neural networks and controller design. The controller design will be based on the bicriterial dual control approach ([Filatov and Unbehauen, 2004]). Attention will be focused on the MLP networks, because they can approximate nonlinear function at the same accuracy as RBF networks with significantly less number of neurons for real time applications. One issue of system identification by MLP networks is estimation of network parameters. In this case, the parameter estimation represents nonlinear optimization problem. It is known that parameter estimation methods are based either on minimization of prediction error [Nørgaard et al., 2000] or on nonlinear filtering methods [de Freitas et al., 2000, Fabri and Kadirkamanathan, 2001, Šimandl et al., 2005a]. The designer choice of the estimation method affects accuracy of obtained model.

3. MODEL OF THE MIMO SYSTEM BY NEURAL NETWORKS

Firstly, a suitable model of the system (1) have to be specified. MIMO systems are mostly characterized by high dimension of the system state. Hence, multilayer perceptron (MLP) network is suitable type of neural network for model of the MIMO system.

An approximation of the nonlinear system (1) can be made by several ways [Narendra and Mukhopadhyay, 1994]. A proper compromise between complexity of the model and synthesis of the dual adaptive controller can be model pictured in the Figure 1. This alternative uses two neural networks $\hat{f}^{(i)}$ and $\hat{g}^{(i\cdot)}$ where $i = 1, \ldots, n$ for every of n output of the system (1). Every network $\hat{f}^{(i)}$ has single output and network $\hat{g}^{(i\cdot)}$ has m outputs. Total number of the networks is $2n$. Although some difficulties are connected with this technique, as design of the $2n$ neural networks and nonlinear estimation of the parameters, this model will be prefered further.

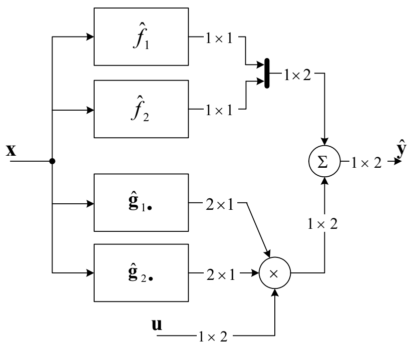

Fig. 1. The model structure design for system with two inputs and two outputs. The blocks introduce individual neural networks. Signals are described by corresponding dimensions.

Therefore, the model of the system is described by the equation

$$\hat{\mathbf{y}}_k = \hat{\mathbf{f}}(\mathbf{x}_{k-1}, \mathbf{w}_k^f, \mathbf{c}_k^f) + \hat{\mathbf{G}}(\mathbf{x}_{k-1}, \mathbf{w}_k^g, \mathbf{c}_k^g)\mathbf{u}_{k-1}, \tag{3}$$

where the i^{th} output of the model is given as

$$\hat{y}_k^{(i)} = \hat{f}^{(i)} + \sum_{j=1}^{m} \hat{g}^{(ij)}(\cdot) u_{k-1}^{(j)}, \quad \text{for } i = 1, \ldots, m \tag{4}$$

$$\hat{f}^{(i)} = \hat{f}^{(i)}(\mathbf{c}_k^{f_i}, \mathbf{x}_{k-1}^a, \mathbf{w}_k^{f_i}) = (\mathbf{c}_k^{f_i})^T \phi^{f_i}(\mathbf{x}_{k-1}^a, \mathbf{w}_k^{f_i}), \tag{5}$$

$$\hat{g}^{(ij)} = \hat{g}^{(ij)}(\mathbf{c}_k^{g_{ij}}, \mathbf{x}_{k-1}^a, \mathbf{w}_k^{g_i}) = (\mathbf{c}_k^{g_{ij}})^T \phi^{g_i}(\mathbf{x}_{k-1}^a, \mathbf{w}_k^{g_i}), \tag{6}$$

where $\mathbf{x}_{k-1}^a = [\mathbf{x}_{k-1}^T \ , \ 1]^T$ is the state vector augmented by constant bias input, $\mathbf{c}_k^{f_i}$, $\mathbf{c}_k^{g_{ij}}$ are vectors of the unknown parameters of the output layer of the network with lengths nf_i, resp. ng_{ij} approximated the nonlinear function $f^{(i)}$, resp. $g^{(ij)}$, $\mathbf{w}_k^{f_i}$ and $\mathbf{w}_k^{g_i}$ are vectors of the unknown parameters of the hidden layer of the i^{th} network with length $(n+p+1)nf_i$, resp. $(n+p+1)ng_{ij}$. Scalar functions $\phi^{f_i}(\cdot)$ and $\phi^{g_i}(\cdot)$ are sigmoidal activation functions of the neurons in the hidden layers.

Equations (3)–(6) describe the model of the system (1). Before an application of an estimation method for the parameters estimation a suitable estimation model of the identified system have to be defined. First, all parameters of the model (3) will be included to one parameter vector

$$\boldsymbol{\Theta}_k = \Big[(\mathbf{c}_k^{f_1})^T, (\mathbf{w}_k^{f_1})^T, (\mathbf{c}_k^{g_{11}})^T, \ldots, (\mathbf{c}_k^{g_{1m}})^T, (\mathbf{w}_k^{g_1})^T, \ldots, (\mathbf{c}_k^{f_n})^T, (\mathbf{w}_k^{f_n})^T, (\mathbf{c}_k^{g_{n1}})^T, \ldots, (\mathbf{c}_k^{g_{nm}})^T, (\mathbf{w}_k^{g_n})^T\Big]^T, \tag{7}$$

where length of the vector $\boldsymbol{\Theta}_k$ is marked n_Θ.

The parameters of the networks are considered as t-invariant in time

$$\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k. \tag{8}$$

It is assumed that it is possible to approximate system with sufficient precision by chosen neural network. Thereafter, it is possible to obtain the equation of measurement from (1) by its rewrite as

$$\mathbf{y}_k = \mathbf{h}_k(\boldsymbol{\Theta}_k, \mathbf{x}_{k-1}^a, \mathbf{u}_{k-1}) + \mathbf{e}_k, \tag{9}$$

where

$$\mathbf{h}_k(\cdot) = \hat{\mathbf{f}}(\mathbf{c}_k^f, \mathbf{x}_{k-1}^a, \mathbf{w}_k^f) + \hat{\mathbf{G}}(\mathbf{c}_k^g, \mathbf{x}_{k-1}^a, \mathbf{w}_k^g)\mathbf{u}_{k-1}, \tag{10}$$

and

$$\begin{aligned}
\mathbf{w}_k^f &= [(\mathbf{w}_k^{f_1})^T, \ldots, (\mathbf{w}_k^{f_n})^T]^T,\\
\mathbf{w}_k^g &= [(\mathbf{w}_k^{g_1})^T, \ldots, (\mathbf{w}_k^{g_n})^T]^T,\\
\mathbf{c}_k^f &= [(\mathbf{c}_k^{f_1})^T, \ldots, (\mathbf{c}_k^{f_n})^T]^T,\\
\mathbf{c}_k^g &= [(\mathbf{c}_k^{g_{11}})^T, \ldots, (\mathbf{c}_k^{g_{1m}})^T, \ldots, (\mathbf{c}_k^{g_{n1}})^T, \ldots, (\mathbf{c}_k^{g_{nm}})^T]^T.
\end{aligned} \tag{11}$$

Equations (8) and (9) define the estimation model of the system (1). Unfortunately, dependence of $\hat{y}_k$ on the parameters of the neural network is nonlinear in Equations (3)–(6). Therefore, it is possible to exploit nonlinear estimation method for finding of the unknown parameters. Gaussian sum (GS) approach [MacKay, 1992, Šimandl et al., 2005a,b] represents a suitable estimation method because of high quality estimation and feasible computational demands. Unknown parameters $\hat{\boldsymbol{\Theta}}$ described by Equation (8) are considered as random variables with initial conditions in the form of Gaussian sums

$$p(\boldsymbol{\Theta}_0|\mathbf{I}^{-1}) = \sum_{\ell=1}^{N_{0|-1}} \alpha_{0|-1}^{(\ell)} \mathcal{N}\left\{\boldsymbol{\Theta}_0 : \hat{\boldsymbol{\Theta}}_{0|-1}^{(\ell)}, \mathbf{P}_{0|-1}^{(\ell)}\right\}, \tag{12}$$

where $\sum_{\ell=1}^{N_{0|-1}} \alpha_{0|-1}^{(\ell)} = 1$, $\alpha_{0|-1}^{(\ell)} > 0$. Points $\hat{\boldsymbol{\Theta}}_{0|-1}^{(\ell)}$ are chosen in order to cover space in which the true parameters are expected. Noises $e_k^{(i)}$ and initial condition $\boldsymbol{\Theta}_0$ are considered as mutually independent.

Analytic solution of Bayessian relations will be obtained by linearization of the function $\mathbf{h}_k(\cdot)$ using the Taylor expansion at the points $\hat{\boldsymbol{\Theta}}_{k|k-1}^{(\ell)}$ representing predictive point estimates of the parameters $\boldsymbol{\Theta}_k$ from the time $k-1$ for $\ell = 1, \ldots, N_{k|k-1}$. For notational convenience the arguments $\mathbf{x}_{k-1}$ and $\mathbf{u}_{k-1}$ of the function $\mathbf{h}_k(\cdot)$ are omitted below. Thus

$$\mathbf{h}_k(\boldsymbol{\Theta}_k) \approx \mathbf{h}_k(\hat{\boldsymbol{\Theta}}_{k|k-1}^{(\ell)}) + \boldsymbol{\nabla}_k^{(\ell)}[\boldsymbol{\Theta}_k - \hat{\boldsymbol{\Theta}}_{k|k-1}^{(\ell)}],$$

where $\boldsymbol{\nabla}_k$ represents the first derivative of the function $\mathbf{h}_k(\cdot)$ with respect to parameters of the network modelling function $\mathbf{f}_k(\cdot)$ and $\mathbf{G}_k(\cdot)$ and has following form

$$\boldsymbol{\nabla}_k \triangleq \left.\frac{\partial \hat{\mathbf{h}}(\boldsymbol{\Theta})}{\partial \boldsymbol{\Theta}}\right|_{\boldsymbol{\Theta}=\hat{\boldsymbol{\Theta}}_k} = \begin{bmatrix} \boldsymbol{\nabla}_k^{(1)} & & & & \mathbf{0} \\ & \ddots & & & \\ & & \boldsymbol{\nabla}_k^{(i)} & & \\ & & & \ddots & \\ \mathbf{0} & & & & \boldsymbol{\nabla}_k^{(m)} \end{bmatrix}, \tag{13}$$

where

$$\boldsymbol{\nabla}_k^{(i)} = \left[\boldsymbol{\nabla}_k^{f_i},\, \boldsymbol{\nabla}_k^{g_{i1}} u_{k-1}^{(1)},\, \ldots,\, \boldsymbol{\nabla}_k^{g_{im}} u_{k-1}^{(m)}\right]. \tag{14}$$

Non-diagonal parts in (12) are equal to zero because of the independency of corresponding derivations on elements of (7). This fact results from the equation of the model (3)–(6) and definition of the vector of the unknown parameters $\boldsymbol{\Theta}_k$ in (7).

Then, the filtering pdf $p(\boldsymbol{\Theta}_k|\mathbf{I}^k)$ is given as follows

$$p(\boldsymbol{\Theta}_k|\mathbf{I}^k) = \sum_{\ell=1}^{N_{k|k}} \alpha_{k|k}^{(\ell)} \mathcal{N}\left\{\boldsymbol{\Theta}_k : \hat{\boldsymbol{\Theta}}_{k|k}^{(\ell)}, \mathbf{P}_{k|k}^{(\ell)}\right\}, \tag{15}$$

where

$$\hat{\boldsymbol{\Theta}}_{k|k}^{(\ell)} = \hat{\boldsymbol{\Theta}}_{k|k-1}^{(\ell)} + \mathbf{K}_{k|k}^{(\ell)}\left[\mathbf{y}_k - \hat{\mathbf{y}}_k^{(\ell)}\right], \tag{16}$$

$$\mathbf{P}_{k|k}^{(\ell)} = \mathbf{P}_{k|k-1}^{(\ell)} - \mathbf{K}_{k|k}^{(\ell)} \boldsymbol{\nabla}_k^{(\ell)} \mathbf{P}_{k|k-1}^{(\ell)}, \tag{17}$$

$$\mathbf{K}_{k|k}^{(\ell)} = \mathbf{P}_{k|k-1}^{(\ell)} [\boldsymbol{\nabla}_k^{(\ell)}]^T \left[\boldsymbol{\nabla}_k^{(\ell)} \mathbf{P}_{k|k-1}^{(\ell)} [\boldsymbol{\nabla}_k^{(\ell)}]^T + \boldsymbol{\Xi}\right]^{-1}, \tag{18}$$

$$\alpha_{k|k}^{(\ell)} = \alpha_{k|k-1}^{(\ell)} \zeta_{k|k}^{(\ell)} \Big/ \sum_{s=1}^{N_{k|k}} \alpha_{k|k-1}^{(s)} \zeta_{k|k}^{(s)}, \tag{19}$$

$$\zeta_{k|k}^{(\ell)} = \mathcal{N}\left\{y_k : \hat{y}_k^{(\ell)}, \boldsymbol{\nabla}_k^{(i)} \mathbf{P}_{k|k-1}^{(\ell)} [\boldsymbol{\nabla}_k^{(\ell)}]^T + \boldsymbol{\Xi}\right\}, \tag{20}$$

$$\hat{y}_k^{(\ell)} = \hat{\mathrm{h}}_k(\hat{\boldsymbol{\Theta}}_{k|k-1}^{(\ell)}), \tag{21}$$

for $i = 1, 2, \ldots, N_{k|k-1}$ and $N_{k|k} = N_{k|k-1}$.

The conditional predictive pdf is given as a mixture of normal distributions:

$$p(\boldsymbol{\Theta}_{k+1}|\mathbf{I}^k) = \sum_{i=\ell}^{N_{k+1|k}} \alpha_{k+1|k}^{(\ell)} \mathcal{N}\left\{\boldsymbol{\Theta}_k : \hat{\boldsymbol{\Theta}}_{k+1|k}^{(\ell)}, \mathbf{P}_{k+1|k}^{(\ell)}\right\}, \tag{22}$$

where

$$\hat{\boldsymbol{\Theta}}_{k+1|k}^{(\ell)} = \hat{\boldsymbol{\Theta}}_{k|k}^{(j)}, \tag{23}$$

$$\mathbf{P}_{k+1|k}^{(\ell)} = \mathbf{P}_{k|k}^{(j)}, \tag{24}$$

$$\alpha_{k+1|k}^{(\ell)} = \alpha_{k|k}^{(j)}, \tag{25}$$

for $j = 1, 2, \ldots, N_{k|k}$.

Relations (15)-(20) and (22)-(24) represent a bank of $N_{k|k}$ extended Kalman filters (EKF) working in parallel. The results could be also interpreted as multi model approach with $N_{k|k}$ neural networks modelling the system and trained by EKF from several different initial points. Using more EKF's for parameter estimation makes multiple linearization of a function $\hat{\mathbf{h}}_k(.)$ at several points and ensures better stability of algorithm as well.

The GS estimator provides the filtering and the predictive pdf of parameters, however the control system based on bicriterial method requires a point estimate of the parameters and a matrix describing uncertainty of the parameters estimate. One possibility is to choose predictive mean $\hat{\mathbf{\Theta}}_{k+1}$ and covariance matrix $\mathbf{P}_{k+1}$ using predictive pdf (22):

$$\hat{\mathbf{\Theta}}_{k+1} \triangleq E[\mathbf{\Theta}_{k+1}|\mathbf{I}^k] = \sum_{\ell=1}^{N_{l|k}} \alpha_{k+1|k}^{(\ell)} \hat{\mathbf{\Theta}}_{k+1|k}^{(\ell)} \tag{26}$$

$$\mathbf{P}_{k+1} = \sum_{\ell=1}^{N_{k+1|k}} \alpha_{k+1|k}^{(\ell)} [\mathbf{P}_{k+1|k}^{(\ell)} + + (\hat{\mathbf{\Theta}}_{k+1|k}^{(\ell)} - \hat{\mathbf{\Theta}}_{k+1})(\hat{\mathbf{\Theta}}_{k+1|k}^{(\ell)} - \hat{\mathbf{\Theta}}_{k+1})^T] \tag{27}$$

Now, it is possible to obtain both the estimate and the covariance matrix of the parameters of the system in every step of estimation algorithm which are necessary in computation of a control action $\mathbf{u}_k$. Due to chosen neural network structure, the covariance matrix $\mathbf{P}_{k+1}$ with dimensions $n_\Theta \times n_\Theta$ is possible write in diagonal block form as

$$\mathbf{P}_{k+1} = \begin{bmatrix} \mathbf{P}_{k+1}^{(1,1)} & & & & \mathbf{0} \\ & \ddots & & & \\ & & \mathbf{P}_{k+1}^{(i,i)} & & \\ & & & \ddots & \\ \mathbf{0} & & & & \mathbf{P}_{k+1}^{(n,n)} \end{bmatrix}, \tag{28}$$

where

$$\mathbf{P}_{k+1}^{(i,i)} = \begin{bmatrix} \mathbf{P}_{k+1}^{f_i f_i} & \mathbf{P}_{k+1}^{f_i g_{i1}} & \dots & \mathbf{P}_{k+1}^{f_i g_{im}} \\ \mathbf{P}_{k+1}^{g_{i1} f_i} & \mathbf{P}_{k+1}^{g_{i1} g_{i1}} & \dots & \mathbf{P}_{k+1}^{g_{i1} g_{im}} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{P}_{k+1}^{g_{im} f_i} & \mathbf{P}_{k+1}^{g_{im} g_{i1}} & \dots & \mathbf{P}_{k+1}^{g_{im} g_{im}} \end{bmatrix} \tag{29}$$

is an individual sub-matrix having dimensions given by numbers of parameters relevant to the neural networks $\hat{f}^{(i)}$, $\hat{g}^{(ij)}$.

Now, it is available all information necessary for functional adaptive control design.

4. BICRITERIAL DUAL CONTROL DESIGN

In this section functional adaptive control for MIMO system (1) will be designed using the idea of bicriterial dual control (BDC). It can be mentioned that basic idea of bicriterial approach is based on sequence minimization of two critera. These criteria represent two opposite goals of the dual control; identification and control.

First criterion evaluating the control quality is described as

$$J_k^c = E\{(\mathbf{y}_{k+1} - \mathbf{r}_{k+1})^T \mathbf{Q}_{k+1} (\mathbf{y}_{k+1} - \mathbf{r}_{k+1}) + + \mathbf{u}_k^T \mathbf{S}_{k+1} \mathbf{u}_k)|\mathbf{I}_k\}, \tag{30}$$

where $\mathbf{Q}_{k+1}$ is suitable chosen positive semidefinite with dimensions $n \times n$, $\mathbf{S}_{k+1}$ is positive definite matrix $m \times m$ and $\mathbf{I}_k$ describes available information segment until time k.

Remark 4.1. The arguments of nonlinear functions $\mathbf{f}$, $\mathbf{G}$, $\hat{\mathbf{f}}$ and $\hat{\mathbf{G}}$ will be omitted in the following derivation for abbreviation of notation.

Criterion J_k^c can be written using substitution (1) in (29) as

$$J_k^c = E\{(\mathbf{f} + \mathbf{G}\mathbf{u}_k + \mathbf{e}_k - \mathbf{r}_{k+1})^T \mathbf{Q}_{k+1} \times \times (\mathbf{f} + \mathbf{G}\mathbf{u}_k + \mathbf{e}_k - \mathbf{r}_{k+1}) + \mathbf{u}_k^T \mathbf{S}_{k+1} \mathbf{u}_k | \mathbf{I}_k\}, \tag{31}$$

where the functions $\mathbf{f}$, $\mathbf{G}$ can be assumed as random variables. Subsequent multiply and partially application of mean operator over information segment $\mathbf{I}_k$ one can obtain

$$J_k^c = E\{\mathbf{f}^T \mathbf{Q}_{k+1} \mathbf{G}\}\mathbf{u}_k + \mathbf{u}_k^T E\{\mathbf{G}^T \mathbf{Q}_{k+1} \mathbf{f}\} + + \mathbf{u}_k^T E\{\mathbf{G}^T \mathbf{Q}_{k+1} \mathbf{G}\}\mathbf{u}_k - \mathbf{r}_{k+1}^T \times E\{\mathbf{Q}_{k+1} \mathbf{G}\}\mathbf{u}_k - -\mathbf{u}_k^T E\{\mathbf{G}^T \mathbf{Q}_{k+1}\}\mathbf{r}_{k+1}^T + \mathbf{u}_k^T \mathbf{S}_{k+1} \mathbf{u}_k + c, \tag{32}$$

where c represents all values of terms of J_k^c that are independent on $\mathbf{u}_k$. They can not influence value of the criteria and need not be considered further.

Now, it is possible to determine control action $\mathbf{u}_k^c$ as extreme of criteria J_k^c

$$\mathbf{u}_k^c = \frac{\partial J_k^c}{\partial \mathbf{u}_k} = E\{\mathbf{G}^T \mathbf{Q}_{k+1} \mathbf{f}\} + E\{\mathbf{G}^T \mathbf{Q}_{k+1} \mathbf{G}\}\mathbf{u}_k - - E\{\mathbf{G}^T \mathbf{Q}_{k+1}\}\mathbf{r}_{k+1} + \mathbf{S}_{k+1}\mathbf{u}_k = \mathbf{0}. \tag{33}$$

In the remaining terms in (32) can be applicated mean operator by using well known relation $E\{\mathbf{a}^T \mathbf{Q}_{k+1} \mathbf{a}\} = \hat{\mathbf{a}}^T \mathbf{Q}_{k+1} \hat{\mathbf{a}} + E\{[\mathbf{a} - \hat{\mathbf{a}}]^T \mathbf{Q}_{k+1} [\mathbf{a} - \hat{\mathbf{a}}]\}$. Then, control action $\mathbf{u}_k^c$ can be written as

$$\mathbf{u}_k^c = [\mathbf{S}_{k+1} + \hat{\mathbf{G}}^T \mathbf{Q}_{k+1} \hat{\mathbf{G}} + \boldsymbol{\nu}_{k+1}^{GG}]^{-1} [\hat{\mathbf{G}}^T \mathbf{Q}_{k+1} \mathbf{r}_{k+1} - - \hat{\mathbf{G}}^T \mathbf{Q}_{k+1} \hat{\mathbf{f}} - \boldsymbol{\nu}_{k+1}^{GF}], \tag{34}$$

where

$\boldsymbol{\nu}_{k+1}^{GG} = \mathbf{Q}_{k+1} \boldsymbol{\nabla}_{k+1}^G \mathbf{P}_{k+1}^G (\boldsymbol{\nabla}_{k+1}^G)^T$ is matrix $m \times m$,
$\boldsymbol{\nu}_{k+1}^{GF} = \mathbf{Q}_{k+1} \boldsymbol{\nabla}_{k+1}^G \mathbf{P}_{k+1}^{GF} (\boldsymbol{\nabla}_{k+1}^F)^T$ is vector with length m. (35)

Matrices $\mathbf{P}_{k+1}^G$ and $\mathbf{P}_{k+1}^{GF}$ can be obtained by choosing from the matrix $\mathbf{P}_{k+1}$ described by Equation (28) and have the following form

$$\mathbf{P}_{k+1}^{GF} = \begin{bmatrix} \mathbf{P}_{k+1}^{f_1 g_{11}} & \dots & \mathbf{P}_{k+1}^{f_1 g_{1m}} \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{k+1}^{f_m g_{m1}} & \dots & \mathbf{P}_{k+1}^{f_m g_{mm}} \end{bmatrix},$$

$$\mathbf{P}_{k+1}^G = \begin{bmatrix} \begin{matrix} \mathbf{P}_{k+1}^{g_{11} g_{11}} & \dots & \mathbf{P}_{k+1}^{g_{11} g_{1m}} \\ \vdots & \vdots & \vdots \\ \mathbf{P}_{k+1}^{g_{11} g_{12}} & \dots & \mathbf{P}_{k+1}^{g_{1m} g_{1m}} \end{matrix} & & 0 \\ & \ddots & \\ 0 & & \begin{matrix} \mathbf{P}_{k+1}^{g_{m1} g_{m1}} & \dots & \mathbf{P}_{k+1}^{g_{m1} g_{mm}} \\ \vdots & \vdots & \vdots \\ \mathbf{P}_{k+1}^{g_{mm} g_{m1}} & \dots & \mathbf{P}_{k+1}^{g_{mm} g_{mm}} \end{matrix} \end{bmatrix} \tag{36}$$

and $\boldsymbol{\nabla}_{k+1}^G$, resp. $\boldsymbol{\nabla}_{k+1}^F$ can be obtained from (12)

$$\boldsymbol{\nabla}_{k+1}^F = \left[\boldsymbol{\nabla}_{k+1}^{f_1}, \dots, \boldsymbol{\nabla}_{k+1}^{f_m}\right]^T,$$

$$\boldsymbol{\nabla}_{k+1}^G = \begin{bmatrix} \boldsymbol{\nabla}_{k+1}^{g_{11}} \dots \boldsymbol{\nabla}_{k+1}^{g_{1m}} & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{\nabla}_{k+1}^{g_{m1}} \dots \boldsymbol{\nabla}_{k+1}^{g_{mm}} \end{bmatrix}. \tag{37}$$

It can be noted that (33) respects uncertainties in knowledge of the unknown functions and it is equal to cautious control as one of integral component of dual control.

Second component of control law should evaluates estimation quality and it is given by

$$J_k^a = -E\left\{(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1})^T \mathbf{W}_{k+1}(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1})|\mathbf{I}_k\right\}, \tag{38}$$

where $\mathbf{W}_{k+1}$ is chosen as positive semidefinite matrix $n \times n$. By substitution from (1) and (3) on $\mathbf{y}_{k+1}$ and $\hat{\mathbf{y}}_{k+1}$, the follow relation can be obtained

$$\begin{aligned} J_k^a = & - E\{(\mathbf{f} + \mathbf{G}\mathbf{u}_k + \mathbf{e}_k)^T \mathbf{W}_{k+1}(\mathbf{f} + \mathbf{G}\mathbf{u}_k + \mathbf{e}_k) - \\ & - (\mathbf{f} + \mathbf{G}\mathbf{u}_k + \mathbf{e}_k)^T \times \mathbf{W}_{k+1}(\hat{\mathbf{f}} + \hat{\mathbf{G}}\mathbf{u}_k) - \\ & - (\hat{\mathbf{f}} + \hat{\mathbf{G}}\mathbf{u}_k)^T \mathbf{W}_{k+1}(\mathbf{f} + \mathbf{G}\mathbf{u}_k + \mathbf{e}_k) + \\ & + (\hat{\mathbf{f}} + \hat{\mathbf{G}}\mathbf{u})^T \mathbf{W}_{k+1}(\hat{\mathbf{f}} + \hat{\mathbf{G}}\mathbf{u}_k)|\mathbf{I}_k\}. \end{aligned} \tag{39}$$

Following multiplying and omitting of the independent articles on $\mathbf{u}_k$, $\bar{J}_k^a$ is a suitable form for optimization

$$\begin{aligned} \bar{J}_k^a = -E\{ & \mathbf{u}_k^T \mathbf{G}^T \mathbf{W}_{k+1}\mathbf{f} + \mathbf{u}_k^T \mathbf{G}^T \mathbf{W}_{k+1}\mathbf{G}\mathbf{u}_k + \mathbf{f}^T \mathbf{W}_{k+1} \times \\ & \times \mathbf{G}\mathbf{u}_k - \mathbf{f}^T \mathbf{W}_{k+1} \times \hat{\mathbf{G}}\mathbf{u}_k - \mathbf{u}_k^T \mathbf{G}^T \mathbf{W}_{k+1}\hat{\mathbf{G}}\mathbf{u}_k - \\ & - \mathbf{u}_k^T \mathbf{G}^T \mathbf{W}_{k+1}\hat{\mathbf{f}} - \hat{\mathbf{f}}^T \mathbf{W}_{k+1}\mathbf{G}\mathbf{u}_k - \mathbf{u}_k^T \hat{\mathbf{G}}^T \mathbf{W}_{k+1}\mathbf{f} - \\ & - \mathbf{u}_k^T \hat{\mathbf{G}}^T \mathbf{W}_{k+1}\mathbf{G}\mathbf{u}_k + \hat{\mathbf{f}}^T \mathbf{W}_{k+1}\hat{\mathbf{G}}\mathbf{u}_k + \\ & + \mathbf{u}_k^T \hat{\mathbf{G}}^T \mathbf{W}_{k+1}\mathbf{f} + \mathbf{u}_k^T \hat{\mathbf{G}}^T \mathbf{W}_{k+1}\hat{\mathbf{G}}\mathbf{u}_k|\mathbf{I}_k\}. \end{aligned} \tag{40}$$

The bicriterial control u_k is then searched as

$$\mathbf{u}_k = \underset{\mathbf{u}_k \in \Omega_k}{\text{argmin}}\, \bar{J}_k^a. \tag{41}$$

Minimization of $\bar{J}_k^a$ is performed on region Ω_k that is specified by $\mathbf{u}_k^c$ and its surrounding symmetrically distributed around the caution control as $\Omega_k = [\mathbf{u}_k^c - \boldsymbol{\delta}_k, \mathbf{u}_k^c + \boldsymbol{\delta}_k]$, whereas $\boldsymbol{\delta}_k = [\delta_k^{(1)}, \ldots, \delta_k^{(m)}]^T$. The choice of the parameter $\boldsymbol{\delta}_k$ stems from reasoning that it is necessary to enrich the caution control with probing in proportional to uncertainty of the unknown functions $\mathbf{f}$, $\mathbf{G}$ in the controlled system (1). A common choice [Filatov and Unbehauen, 2004] for $\boldsymbol{\delta}_k$ is

$$\boldsymbol{\delta}_k = \boldsymbol{\eta}\,\text{tr}(\mathbf{P}_{k+1}), \qquad \boldsymbol{\eta} > 0, \tag{42}$$

where $\boldsymbol{\eta}$ is vector with length m, that provides the amplitude of the probing signal and the matrix $\mathbf{P}_{k+1}$ describes rate of uncertainty of the parameters estimate conditioned by $\mathbf{I}^k$ and can be obtained using a nonlinear filtering method of GS (15) - (24).

Relation (40) can be treated and can be rewritten as

$$\mathbf{u}_k = \mathbf{u}_k^c + \boldsymbol{\delta}_k \,\text{sign}\left[\bar{J}_k^a(\mathbf{u}_k^c - \boldsymbol{\delta}_{\boldsymbol{k}}) - \bar{J}_k^a(\mathbf{u}_k^c + \boldsymbol{\delta}_{\boldsymbol{k}})\right]. \tag{43}$$

Now, assesment of the term in the bracket remains to solve. This term can be obtained by substitution $(\mathbf{u}_k^c + \boldsymbol{\delta}_k)$, resp. $(\mathbf{u}_k^c - \boldsymbol{\delta}_k)$ instead of $\mathbf{u}_k$ in Equation (39). With using of elementary adjustments can be obtained

$$\begin{aligned} \bar{J}_k^a(\mathbf{u}_k^c - \boldsymbol{\delta}_k) - \bar{J}_k^a(\mathbf{u}_k^c + \boldsymbol{\delta}_k) = 4\boldsymbol{\delta}_k^T E\{(\mathbf{G} - \hat{\mathbf{G}})^T \mathbf{W}_{k+1} \times \\ \times (\mathbf{f} - \hat{\mathbf{f}}) + (\mathbf{G} - \hat{\mathbf{G}})^T \mathbf{W}_{k+1}(\mathbf{G} - \hat{\mathbf{G}})\mathbf{u}_k^c|\mathbf{I}_k\}. \end{aligned} \tag{44}$$

Now, it is possible to reuse initiated substitution (34). Together with application of the mean operator and with using assumption of $\mathbf{Q}_{k+1} = \mathbf{W}_{k+1}$, Equation (43) has form

$$\bar{J}_k^a(\mathbf{u}_k^c - \boldsymbol{\delta}_k) - \bar{J}_k^a(\mathbf{u}_k^c + \boldsymbol{\delta}_k) = 4\boldsymbol{\delta}_k^T(\boldsymbol{\nu}_{k+1}^{GF} + \boldsymbol{\nu}_{k+1}^{GG}\mathbf{u}_k^c). \tag{45}$$

Final equation of functional adaptive controller for MIMO system based on bicriterial approach can be obtained as combination of (42) and (44) and it possible to write

$$\mathbf{u}_k = \mathbf{u}_k^c + \boldsymbol{\delta}_k \text{sign}\left[\boldsymbol{\delta}_k^T(\boldsymbol{\nu}_{k+1}^{GF} + \boldsymbol{\nu}_{k+1}^{GG}\mathbf{u}_k^c)\right]. \tag{46}$$

5. NUMERICAL EXAMPLE

The discrete-time nonlinear stochastic system with two inputs and two outputs described by following equations is considered:

$$\begin{aligned} y_k^{(1)} = & \frac{0.7 y_{k-1}^{(1)} y_{k-2}^{(1)}}{1 + (y_{k-1}^{(1)})^2 + (y_{k-2}^{(2)})^2} + \frac{0.1 u_{k-1}^{(2)}}{1 + 3(y_{k-2}^{(1)})^2 + (y_{k-1}^{(2)})^2} + \\ & + u_{k-1}^{(1)} + 0.25 u_{k-2}^{(1)} + 0.5 u_{k-2}^{(2)} + e_k^{(1)}, \\ y_k^{(2)} = & \frac{0.5 y_{k-1}^{(2)} \sin y_{k-2}^{(2)}}{1 + (y_{k-1}^{(2)})^2 + (y_{k-2}^{(1)})^2} + 0.5 u_{k-2}^{(2)} + 0.3 u_{k-2}^{(1)} + \\ & + u_{k-1}^{(2)}\left(0.1 u_{k-2}^{(2)} - 1.5\right) + e_k^{(2)}, \end{aligned}$$

where $\mathbf{x}_{k-1} = [\mathbf{y}_{k-1}, \mathbf{y}_{k-2}, \mathbf{u}_{k-2}]$ is the state of the system, $\{e^{(1)}\}$, $\{e^{(2)}\}$ are mutually independent Gaussian noises with zero means and variances $(\sigma_e^{(1)})^2 = (\sigma_e^{(2)})^2 = 0.001$. Reference signals are chosen as

$$\begin{aligned} r_k^{(1)} = & 0.75 \sin \frac{2\pi k}{50} + 0.75 \sin \frac{2\pi k}{10}, \\ r_k^{(2)} = & 0.55 \sin \frac{2\pi k}{30} + 0.55 \sin \frac{2\pi k}{20}. \end{aligned} \tag{47}$$

Initial values of the inputs and the outputs are zero. How it was mentioned in Section 2, each of the nonlinear functions $f^{(i)}$, $g^{(i\cdot)}$ for $i = 1, \ldots, n$ is modelled by individual neural network. Model of the system is composed from four neural network. Each of the neural networks is perceptron neural network with one hidden layer containing 20 neurons. The unknown parameters of the model are estimated by GS method with 3 terms of the mixture, $\mathbf{P}_0 = 10\mathbf{I}$, initialized parameters are generated from uniform distribution from interval $< -0.1; 0.1 >$ (more about estimation of the neural network parameters based on GS method can be found in [Šimandl et al., 2005a]). Finally, parameters of the BDC are chosen as follows: $\mathbf{W}_{k+1} = \mathbf{Q}_{k+1} = \mathbf{I}$, $\mathbf{S}_{k+1} = 0.01\mathbf{I}$ and $\boldsymbol{\eta}^T = [0.00002\ \ 0.00004]$.

Influence of choice of the controller on control performance for the system (46) is shown in Table 1. The BDC is compared with two non-dual adaptive controllers as special cases of BDC: cautious (Equation (33)) and certainty equivalence (Equation (33) with $\boldsymbol{\nu}_{k+1}^{GF} = \boldsymbol{\nu}_{k+1}^{GG} = 0\ \forall k$). Criterion for comparison is set as mean of sums of square errors of the reference and the system output $\mathbf{r}_k$, $\mathbf{y}_k$ over 100 trials: $\hat{V} = \frac{1}{100}\sum_{i=1}^{2}\sum_{j=1}^{100}\sum_{k=1}^{200}(y_{kj}^{(i)} - r_{kj}^{(i)})^2$. It is clear that the best performance was obtained for the BDC. Attained mean and variance of the criterion have significantly lower values.

	certainty eq.	cautious	bicriterial
$\hat{V}$	92.3	58.8	23.6
$\text{cov}(\hat{V})$	650	112	12

Table 1. Influence of choice of controller on quality of control system.

Results of the simulation are illustrated in Figures 2 and 3. In Figure 2 the tracking of the chosen reference signals for the both outputs of the system $y_k^{(1)}$ and $y_k^{(2)}$ is shown. It is clear that quick adaptation of the parameters of the model and very good control quality occur during short simulation time about 200 steps. Goal of the control, the tracking of the chosen reference signals $r_k^{(1)}$ and $r_k^{(2)}$ is fulfilled. In Figure 3 the typical control signals $u_k^{(1)}$ and $u_k^{(2)}$ are pictured. In bottom part the probing components of the control signals (the second term of (45)) that contribute to active identification of the system are figured.

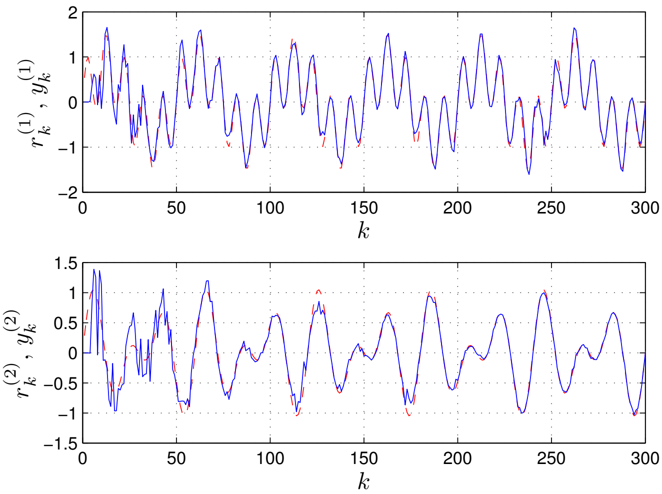

Fig. 2. Typical outputs of the system controlled by bicriterial dual controller (solid line) and following chosen reference signals (dash line).

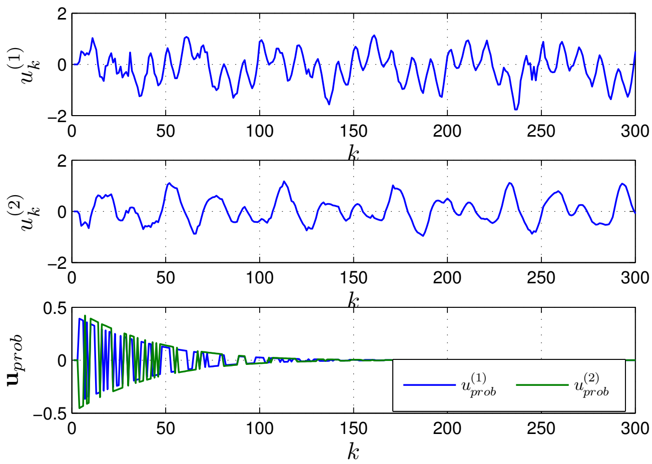

Fig. 3. Control signals of the bicriterial controller and the probing component of the control signals (bottom part).

6. CONCLUSIONS

The bicriterial dual controller for non-linear stochastic MIMO systems was designed as an extension of the known results for SISO nonlinear stochastic systems. The model of the system is given by the multilayer perceptron network. The nonlinear filter Gaussian sum method was applied for the on-line parameters estimate of the derived estimation model. Then the bicriterial approach to dual control design was used. The proposed adaptive controller has computational demands comparable with caution control but with dual control ability.

ACKNOWLEDGEMENT

The work was supported by the Czech Science Foundation, project GP102/08/P080 and project GA102/08/0442.

REFERENCES

Fu-Chuang Chen and Hassan K. Khalil. Adaptive control of a class of nonlinear discrete-time systems using neural networks. *IEEE Transaction on Automatic Control*, 40 (5):791–801, 1995.

J.F.G. de Freitas, M. Niranjan, A.H. Gee, and A. Doucet. Sequential Monte Carlo methods to train neural network models. *Neural computation*, 12(4):955–993, 2000.

Simon G. Fabri and Visakan Kadirkamanathan. *Functional Adaptive Control: An intelligent Systems Approach*. Springer-Verlag, 2001. ISBN 1-85233-438-X.

N. K. Filatov and H. Unbehauen. *Adaptive Dual Control*. Springer-Verlag, 2004.

Yue Fu and Tianyou Chai. Nonlinear multivariable adaptive control using multiple models and neural networks. *Automatica*, 43(7):1101–1110, 2007.

S. Haykin. *Neural Networks: A comprehensive foundation*. Prentice-Hall, Upper Saddle River, NJ, 2nd edition, 1999.

R. Herzallah and D. Lowe. Bayesian adaptive control of nonlinear systems with functional uncertainty. *In: proceedings of the 7th Portuguese Conference on Automatic Control*, 2006. Lisbon, Portugal.

D. J. C. MacKay. A practical bayesian frameworkfor backpropagation networks. *Neural computation*, 4(3): 448–472, 1992.

R. Murray-Smith and D. Sbarbaro. Nonlinear adaptive control using non-parametric gaussian process prior models. *In Proceedings of 15th IFAC World Congress, Barcelona, Spain*, pages 21–26, 2002.

K. S. Narendra and S. Mukhopadhyay. Adaptive control of nonlinear multivariable systems using neural networks. *Neural Networks*, 7(5):737–752, 1994.

M. Nørgaard, O.E. Ravn, N.K. Poulsen, and L.K. Hansen. *Neural Networks for Modelling and Control of Dynamic Systems: A Practitioner's Handbook*. Advanced Textbooks in Control and Signal Processing. Springer-Verlag, 2000. ISBN 1-85233-227-1.

D. Sbarbaro, R. Murray-Smith, and A. Valdes. Multivariable generalized minimum variance control based on artificial neural networks and gaussian process models. *Proceedings of International Symposium on Neural Networks - ISNN 2004*, pages 52–58, 2004.

H. te Braake, E. van Can, J. Scherpen, and H. Verbruggen. Control of nonlinear chemical processes using neural models and feedback linearization. *Computers Chem. Eng.*, 22(7-8):1113–1127, 1998.

M. Šimandl, P. Hering, and L. Král. Identification of nonlinear non-gaussian systems by neural networks. *In: Preprints of the 6th IFAC Symposium on Nonlinear Control Systems 2004 (NOLCOS 2004)*, pages 1307–1312, 2005a.

M. Šimandl, L. Král, and P. Hering. Neural network based bicriterial dual control of nonlinear systems. *In: Preprints of the 16th IFAC world congress*, 2005b. Prague: IFAC.