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A 2D Systems Approach to Iterative Learning Control with Experimental Validation

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Abstract: In this paper we use a 2D systems setting to develop new results on iterative learning control for linear plants. It is well known in the subject area that a trade-off exists between speed of convergence and transient response. Here we give new results in this area by designing the control scheme using a strong form of stability for repetitive processes/2D linear systems known as stability along the pass (or trial). The resulting design computations are in terms of Linear Matrix Inequalities (LMIs) and they are also experimentally validated on a gantry robot.

1. INTRODUCTION

Iterative learning control (ILC) is a technique for controlling systems operating in a repetitive, or trial-to-trial, mode with the requirement that a reference trajectory $r(t)$ defined over a finite interval $0 \leq t \leq \alpha$, where α denotes the trial length, is followed to a high precision. Examples of such systems include robotic manipulators that are required to repeat a given task to high precision, chemical batch processes or, more generally, the class of tracking systems.

Since the original work Arimoto et al. (1984) in the mid 1980's, the general area of ILC has been the subject of intense research effort. An initial source for the literature here is the survey paper Bristow et al. (2006). The analysis of ILC schemes is firmly outside standard (or 1D) control theory — although it is still has a significant role to play in certain cases of practical interest. Instead, ILC must be seen (as one approach) in the context of fixed-point problems or, more precisely, repetitive processes (see the references in Rogers and Owens (1992)) which are a distinct class of 2D systems where information propagation in one of the two independent directions only occurs over a finite duration.

In ILC, a major objective is to achieve convergence of the trial-to-trial error and often this has been treated as the only objective. In fact, it is possible that enforcing fast convergence could lead to unsatisfactory along the trial performance. In this paper, the problem is addressed by first showing that ILC schemes can be designed for a class of discrete linear systems by, in effect, extending techniques developed for 2D systems using the framework of linear repetitive processes. This allows us to use the strong concept of stability along the pass (or trial) for these processes in an ILC setting as a possible means of dealing with poor/unacceptable transients in the along the trial dynamics. The results developed here give control law design algorithms which can be implemented via LMIs. Finally, the resulting control laws are experimentally validated on a gantry robot executing a pick and place operation where the plant models used for design are obtained by frequency response tests.

The symbols $\Gamma \succ 0$, respectively $\Gamma \prec 0$, are used in this paper to denote a symmetric positive definite, respectively negative, definite matrix.

2. BACKGROUND AND INITIAL ANALYSIS

The plants considered in this paper are assumed to be differential linear time-invariant systems described by the state-space triple $\{A, B, C\}$ which in an ILC setting is written as

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + Bu_k(t), 0 \leq t \leq \alpha \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{1}$$

where on trial k, $x_k(t) \in \mathbb{R}^n$ is the state vector, $y_k(t) \in \mathbb{R}^m$ is the output vector, $u_k(t) \in \mathbb{R}^r$ is the vector of control inputs, and the trial length $\alpha < \infty$. If the signal to be tracked is denoted by $r(t)$ then $e_k(t) = r(t) - y_k(t)$ is the error on trial k. The most basic requirement then is to force the error to convergence in k. This, however, cannot always be addressed independently of the dynamics along the trial as the following analysis demonstrates.

Consider the case where on trial k+1 the control input is calculated using

$$u_{k+1}(t) := \sum_{j=1}^{M} \alpha_j u_{k+1-j}(t) + \sum_{j=1}^{M} (K_j e_{k+1-j}(t) + (K_0 e_{k+1})) \tag{2}$$

In addition to the 'memory' M, the design parameters in this control law are the static scalars α_j , $1 \leq j \leq M$, the linear operator K_0 which describes the current pass error contribution, and the linear operator K_j , $1 \leq j \leq M$,

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which describes the contribution from the error on pass $k+1-j$.

It is now routine to show that convergence of the error here holds if, and only if, all roots of

$$z^M - \alpha_1 z^{M-1} - \ldots - \alpha_{M-1} z - \alpha_M = 0 \tag{3}$$

have modulus strictly less than unity. Also the error dynamics on trial k+1 here can be written in convolution form as

$$e_{k+1}(t) = r(t) - (Gu_{k+1})(t), 0 \leq t \leq \alpha$$

Suppose also that (3) holds. Then the closed-loop error dynamics converge (in the norm topology of $L_p[0,T]$) to

$$e_\infty = (I + GK_{\text{eff}})^{-1} r \tag{4}$$

where the so-called effective controller K_{eff} is given by

$$K_{\text{eff}} := \frac{K}{1-\beta}$$

and

$$\beta := \sum_{i=1}^{M} \alpha_i, \quad K = \sum_{i=1}^{M} K_i$$

The result here counter-intuitive in the sense that stability is largely independent of the plant and the controllers used. This is a direct result of the fact that the trial duration α is finite and over such an interval a linear system can only produce a bounded output irrespective of its stability properties. Hence even if the error sequence generated is guaranteed to converge to a limit, this terminal error may be unstable and/or possibly worse than the first trial error, i.e. the use of ILC has produced no improvement in performance.

We have the following (see Owens et al. (2000) for the details).

(1) Convergence is predicted to be 'rapid' if λ_e is small and will be geometric in form, converging approximately with λ_e^k, where $\lambda_e \in (\max|\mu_i|, 1)$ and μ_i, $1 \leq i \leq N$, is a solution of (3).
(2) The limit error is nonzero but is usefully described by a (1D linear systems) unity negative feedback system with effective controller K_{eff} defined above. If $\max_i(|\mu_i|) \to 0+$ then the limit error is essentially the first learning iterate, i.e. use of ILC has little benefit and will simply lead to the normal large errors encountered in simple feedback loops. There is hence pressure to let $\max_i |\mu_i|$ be close to unity when K_{eff} is a high gain controller which will lead (roughly speaking) to small limit errors.
(3) Zero limit error can only be achieved if $\sum_{i=1}^{N} \alpha_i = 1$. (This situation — again see Owens et al. (2000) for the details — is reminiscent of classical control where the inclusion of an integrator (on the stability boundary) in the controller results in zero steady state (limit) error in response to constant reference signals.)

There is a conflict in the above conclusions which has implications on the systems and control structure from both the theoretical and practical points of view. In particular, consider for ease of presentation the case when $K_i = 0$; $1 \leq i \leq M$. Then small learning errors will require high effective gain yet GK_0 should be stable under such gains.

To guarantee an acceptable (i.e. stable (as the most basic requirement)) limit error and acceptable along the trial transients, a stronger form of stability must be used. Here we consider the use of so-called stability along the trial (or pass) from repetitive process theory. In effect, this demands convergence of the error sequence with a uniform bound on the along the trial dynamics. We also work in the discrete domain and so assume that the along the pass dynamics have been sampled at a uniform rate T_s seconds to produce a discrete-state space model of the form (where for notational simplicity the dependence on T_s is omitted from the variable descriptions)

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + Bu_k(p), p = 0, 1, \ldots, \alpha - 1 \\ y_k(p) &= Cx_k(p) \end{aligned} \tag{5}$$

Consider now the so-called discrete linear repetitive processes described by the following state-space model over $p = 0, 1, \ldots, \alpha - 1, k \geq 1$

$$\begin{aligned} x_k(p+1) &= \hat{A}x_k(p) + \hat{B}u_k(p) + \hat{B}_0 y_{k-1}(p) \\ y_k(p) &= \hat{C}x_k(p) + \hat{D}u_k(p) + \hat{D}_0 y_{k-1}(p) \end{aligned} \tag{6}$$

where $x_k(p) \in \mathbb{R}^n$, $u_k(p) \in \mathbb{R}^r$, $y_k(p) \in \mathbb{R}^m$ are the state, input and pass profile vectors respectively. Also rewrite the state equation of the process model in the form

$$x_k(p) = Ax_k(p-1) + Bu_k(p-1) \tag{7}$$

and introduce

$$\begin{aligned} \eta_{k+1}(p+1) &= x_{k+1}(p) - x_k(p) \\ \Delta u_{k+1}(p) &= u_{k+1}(p) - u_k(p) \end{aligned} \tag{8}$$

Then we have

$$\eta_{k+1}(p+1) = A\eta_{k+1}(p) + B\Delta u_{k+1}(p-1) \tag{9}$$

Consider also a control law of the form

$$\Delta u_{k+1}(p) = K_1 \eta_{k+1}(p+1) + K_2 e_k(p+1) \tag{10}$$

and hence

$$\eta_{k+1}(p+1) = (A + BK_1)\eta_{k+1}(p) + BK_2 e_k(p) \tag{11}$$

Also $e_{k+1}(p) - e_k(p) = y_k(p) - y_{k+1}(p)$ and we then obtain

$$\begin{aligned} e_{k+1}(p) - e_k(p) &= CA(x_k(p-1) - x_{k+1}(p-1)) \\ &\quad + CB(u_k(p-1) - u_{k+1}(p-1)) \end{aligned} \tag{12}$$

Using (8) we now obtain

$$e_{k+1}(p) - e_k(p) = -CA\eta_{k+1}(p) - CB\Delta u_{k+1}(p-1)$$

or, utilizing (10),

$$\begin{aligned} e_{k+1}(p) &= -C(A + BK_1)\eta_{k+1}(p) \\ &\quad - C\eta_{k+1}(p) + (I - CBK_2)e_k(p) \end{aligned} \tag{13}$$

Also introduce

$$\begin{aligned} \hat{A} &= A + BK_1 \\ \hat{B}_0 &= BK_2 \\ \hat{C} &= -C(A + BK_1) \\ \hat{D}_0 &= I - CBK_2 \end{aligned} \tag{14}$$

Then clearly (11) and (13) can be written as

$$\begin{aligned} \eta_{k+1}(p+1) &= \hat{A}\eta_{k+1}(p) + \hat{B}_0 e_k(p) \\ e_{k+1}(p) &= \hat{C}\eta_{k+1}(p) + \hat{D}_0 e_k(p) \end{aligned} \tag{15}$$

which is of the form (6) and hence the repetitive process stability theory can be applied to this ILC control scheme. In particular, stability along the trial is equivalent to uniform bounded input bounded output stability (defined in terms of the norm on the underlying function space), i.e.

independent of the trial length,and hence we can (potentially) achieve trial-trial error convergence with acceptable along the trial dynamics.

The stability theory for linear repetitive processes is critically dependent on the structure of the boundary conditions and, in particular, the state initial vector sequence Here we assume that

$$\eta_{k+1}(0) = 0, \; k \geq 0$$

Note 1. It is also possible to give a 2D discrete linear systems representation of the ILC scheme considered here, as first proposed in Kurek and Zaremba (1993). The work here progresses beyond this, and other work on ILC using 2D model descriptions, by use of a stronger form of stability, control law design and (uniquely for such ILC laws) experimental verification.

3. ANALYSIS

The following result gives stability along the trial under control action together with formulas for control law design.

Theorem 1. The ILC scheme of (15) is stable along the trial if there exist compatibly dimensioned matrices $X_1 \succ 0$, $X_2 \succ 0$, R_1 and R_2 such that the following LMI is feasible

$$\begin{bmatrix} -X_1 & 0 & X_1A^T + R_1^T B^T & -X_1A^TC^T - R_1^T B^T C^T \\ 0 & -X_2 & R_2^T B^T & X_2 - R_2^T B^T C^T \\ AX_1 + BR_1 & BR_2 & -X_1 & 0 \\ -CAX_1 - CBR_1 & X_2 - CBR_2 & 0 & -X_2 \end{bmatrix} \prec 0 \quad (16)$$

If (16) holds, the control law matrices K_1 and K_2 can be computed using

$$\begin{aligned} K_1 &= R_1 X_1^{-1} \\ K_2 &= R_2 X_2^{-1} \end{aligned} \quad (17)$$

Proof.
Numerous conditions for stability along the trial of discrete linear repetitive processes of the form (15) exist, e.g. Rogers and Owens (1992); Rogers et al. (2007) but here we use the co-called 2D Lyapunov equation approach (see Rogers et al. (2007) Chapter 6) and hence (15) is stable along the trial if there exists $P = \text{diag}\{P_1, P_2\} \succ 0$ such that

$$\Phi^T P \Phi - P \prec 0 \quad (18)$$

where

$$\Phi = \begin{bmatrix} \hat{A} & \hat{B}_0 \\ \hat{C} & \hat{D}_0 \end{bmatrix} \quad (19)$$

An obvious application of the Schur's complement formula to (18) yields

$$\begin{bmatrix} -P_1 & 0 & \hat{A}^T & \hat{C}^T \\ 0 & -P_2 & \hat{B}_0^T & \hat{D}_0^T \\ \hat{A} & \hat{B}_0 & -P_1^{-1} & 0 \\ \hat{C} & \hat{D}_0 & 0 & -P_2^{-1} \end{bmatrix} \prec 0 \quad (20)$$

Now introduce

$$\begin{aligned} X_1 &= P_1^{-1} \\ X_2 &= P_2^{-1} \end{aligned} \quad (21)$$

and pre- and post multiply (20) by

$$\text{diag}\,\{\, X_1, \; X_2, \; I, \; I \,\} \quad (22)$$

to obtain

$$\begin{bmatrix} -X_1 & 0 & X_1\hat{A}^T & X_1\hat{C}^T \\ 0 & -X_2 & X_2\hat{B}_0^T & X_2\hat{D}_0^T \\ \hat{A}X_1 & \hat{B}_0X_2 & -X_1 & 0 \\ \hat{C}X_1 & \hat{D}_0X_2 & 0 & -X_2 \end{bmatrix} \prec 0 \quad (23)$$

Now use (14) to obtain (after some routine manipulations)

$$\begin{bmatrix} -X_1 & 0 & X_1A^T + X_1K_1^T B^T & -X_1A^TC^T - X_1K_1^T B^T C^T \\ 0 & -X_2 & X_2K_2^T B^T & X_2 - X_2K_2^T B^T C^T \\ AX_1 + BK_1X_1 & BK_2X_2 & -X_1 & 0 \\ -CAX_1 - CBK_1X_1 & X_2 - CBK_2X_2 & 0 & -X_2 \end{bmatrix} \prec 0 \quad (24)$$

Finally, let

$$\begin{aligned} R_1 &= K_1 X_1 \\ R_2 &= K_2 X_2 \end{aligned} \quad (25)$$

to obtain the required LMI of (16) and the control law matrices which define (17) can be calculated from (25). This completes the proof.

In practical applications, it is often beneficial (or indeed essential) to bound the entries (above or below) in the control law matrices. In the ILC setting, there could well be cases where it is beneficial to keep the entries in the control law matrix K_2 as large as possible. Note, however, that direct manipulation of the entries in K_2 is difficult to achieve in an LMI setting and hence other approaches must be employed. As an example in this latter category is described next drawing on the work of Siljak and Stipanovic (2000) where it was first proposed (in an non ILC setting)). The basic result is that if L and $k_l > 0$ are real scalars subject to the constraint

$$L^2 < k_l \quad (26)$$

then in LMI terms this can be written as

$$\begin{bmatrix} -k_l & L^T \\ L & -1 \end{bmatrix} \prec 0 \quad (27)$$

This operation can also be applied in the matrix case — the scalar L^2 is replaced by the matrix $L^T L$, k_l by $k_l I$, where I is an identity matrix of compatible dimensions, and less than is replaced by a negative definite constraint.

4. EXPERIMENTS

Other work, e.g. Ratcliffe et al. (2006) has used a gantry robot facility to experimentally verify ILC designs. Figure 1 shows this experimental facility where the robot head performs a pick and place task and is similar to systems which can be found in many industrial applications. These include food canning, bottle filling or automotive assembly, all of which require accurate tracking control, each time the operation is performed, with a minimum level of error in order to maximize production rates. This is an obvious general area for application of ILC.

Each axis of the gantry robot has been modelled based on frequency response tests where, since the axes are orthogonal, it is assumed that there is minimal interaction between them. Here we first consider the X-axis (the one parallel to the conveyor in Figure 1) and frequency response tests (via the Bode gain and phase plots in Figure 2) result in a 7th order continuous-time transfer-function as an adequate model of the dynamics on which to based control systems design. This has then been sampled at $T_s = 0.01$ seconds to yield the following z-transfer function approximation of the dynamics

$$G(z) = \frac{0.00051745(z+0.5823)(z-0.3014)}{(z-1)(z^2-0.07057z+0.009459)} \cdot \frac{(z^2-0.09718z+0.008969)(z^2-0.2046z+0.7846)}{(z^2+0.3149z+0.1024)(z^2-0.7757z+0.5403)} \tag{28}$$

and hence in the state-space model used for design (where the subscript x is to distinguish that it is this axis we are considering)

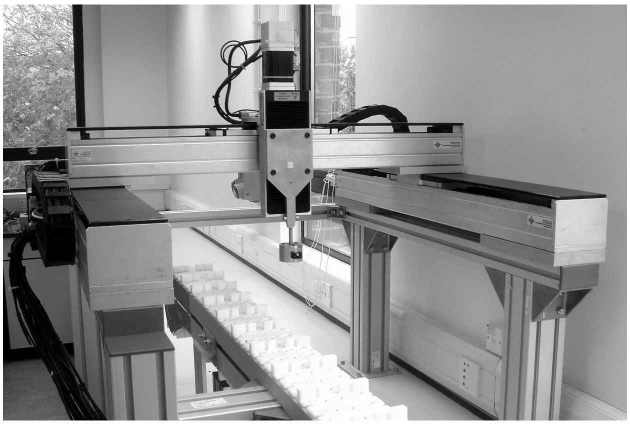

Fig. 1. The multi-axes gantry robot.

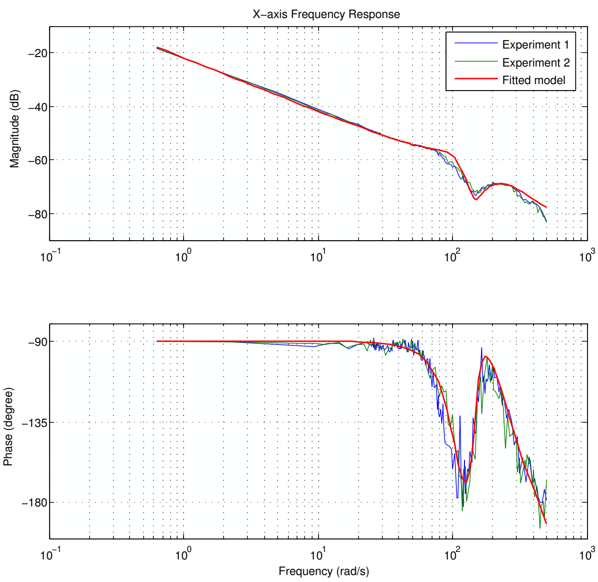

Fig. 2. Frequency response test results and fitted model

$$A_x = \begin{bmatrix} 2.41 & -0.86 & 0.85 & -0.59 & 0.30 & -0.19 & 0.32 \\ 4.00 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.00 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.00 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \end{bmatrix}$$

$$B_x = [\,0.0313\ 0\ 0\ 0\ 0\ 0\ 0\,]^T$$

$$C_x = [\,0.0095\ -0.0023\ 0.0048\ -0.0027\ 0.0029\ -0.0011\ 0.0029\,]$$

At this stage, Theorem 1 can be used to undertake control law design and note that the LMI setting actually produces a family of such designs. As one example, we consider the case when the following additional LMI constraints are imposed (where for this particular case X_2 is a scalar)

$$X_2 < 1 \times 10^{-4} \tag{29}$$

$$X_1 \prec 1 \times 10^{-2} \tag{30}$$

$$R_1 \prec 1 \times 10^{-2} \tag{31}$$

The control law matrices of (10) for this data are given by

$$K_1 = [7.3451\ -2.7245\ 0.1499\ 7.6707\ 2.7540\ -3.6088\ -20.4519] \tag{32}$$

$$K_2 = 82.4119 \tag{33}$$

Suppose now that the trial length is $\alpha = 200$ and the reference signal is given by Figure 3.

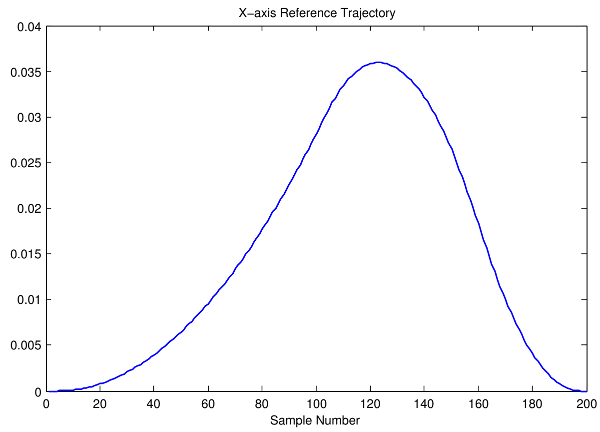

Fig. 3. Reference signal for X-axis

Figure 4(a) shows the outputs produced by the X-axis of the gantry for 20 trials and Figures 4(b) and 4(c) the corresponding control input and error dynamics. It can be seen that within this number of trials, the tracking error has been reduced to a very small value.

It is been reported (see for example Longman (2000)) that in ILC algorithms high frequency noise will build up as the number of trials increase and tracking of the reference signal then begins to diverge (one possible cause is numerical problems in both computation and measurement). One option to limit this is to employ a zero-phase low pass filter to remove such noise (and retain stability along the trial). Figure 5 shows a case where the trial error without filtering starts to diverge after (approximately) 100 trials but the

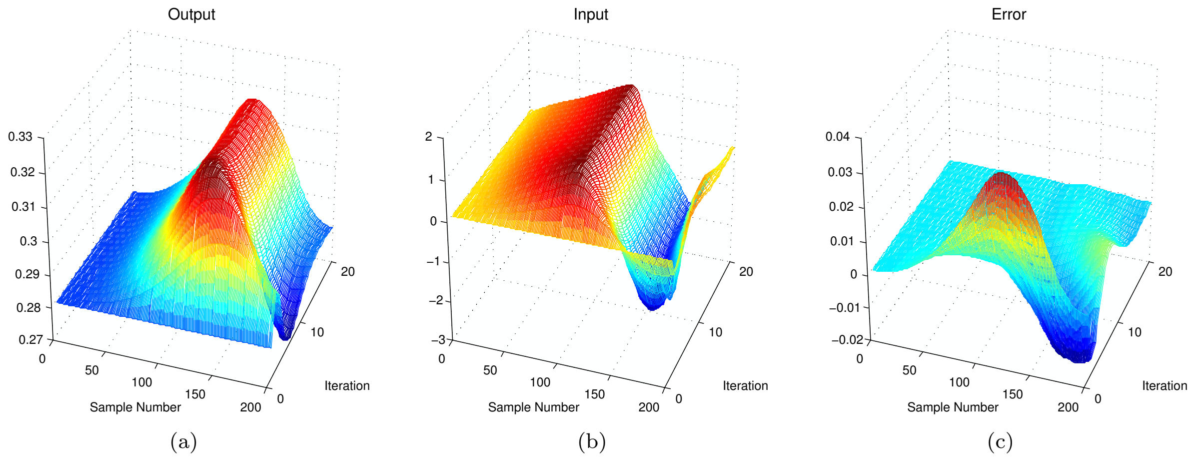

Fig. 4. (a) Evolution of the output of X-axis for the first 20 trials in one experiment, (b) Evolution of the control input, (c) Evolution of the error dynamics

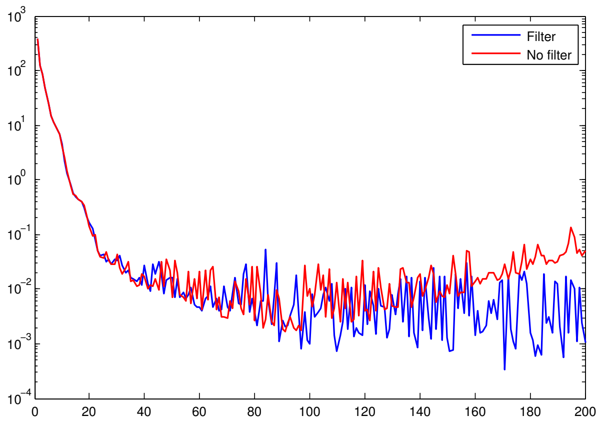

Fig. 5. The effect of filtering

addition of a filter of this type is able to maintain (this aspect of overall) performance. (Zero-phase filtering of the previous trial error is undertaken during the time when the gantry robot is resetting to start the next trial and relies on the fact that the previous trial error is fully available once this resetting commences. In the gantry robot this resetting time is 2 seconds.)

After the success of the initial test programme on the X-axis, the design exercise has been repeated for the Y (perpendicular to the X-axis in the same plane) and Z (perpendicular to the X-Y plane)-axes. Figure 6 shows the mean square error for all axes in comparison to those from a simulation study for each corresponding axis with the ILC control law applied. Moreover, the along the trial dynamics have proved to be less oscillatory than alternative designs where no stability constraint was placed on the along the trial dynamics (i.e. on the eigenvalues of the state matrix).

5. CONCLUSIONS

This paper has considered the design of ILC schemes in a 2D linear systems setting and, in particular, the theory of discrete linear repetitive processes. This releases a stability theory for application which demands uniformly bounded along the trial dynamics (whereas previous ap-

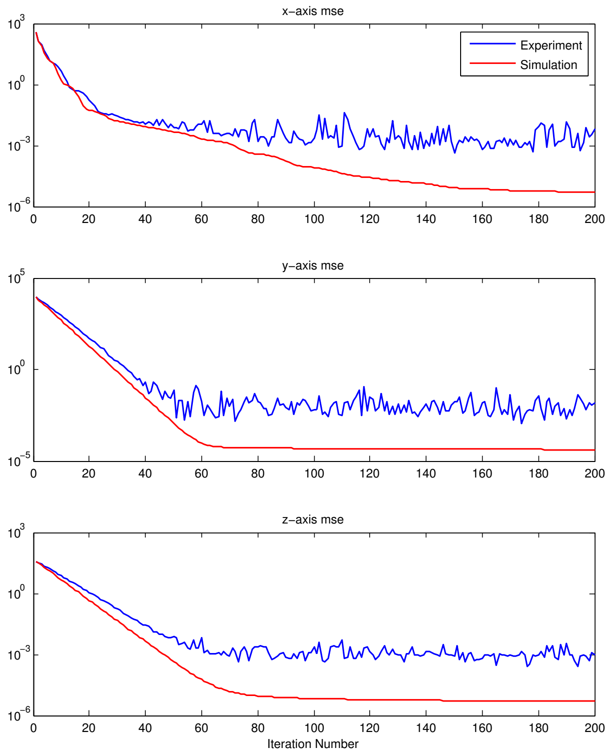

Fig. 6. Mean squared error for all axes

proaches only demand bounded dynamics over the finite trial length). Here we have shown that this approach leads to a stability condition expressed in terms of an LMI with immediate formulas for computing the control law matrices. This is a potentially powerful approach in this general area which also makes a significant step forward in the application of 2D linear systems theory. Uniquely (in the 2D systems approach) the designs have been experimentally validated on a gantry robot system whose basic task is to continually execute a pick and place operation.

The results here establish the basic feasibility of this approach in terms of both theory and experimentation. There is a significant degree of flexibility in the resulting design algorithm and current work is undertaking a detailed investigation of how this can be fully exploited. One aspect which clearly requires investigation is to attempt design without the need to use current trial state feedback (particularly as the control law in this paper actually uses the difference of the state vector on two successive trials). Another is to extend this analysis to other classes of ILC and seek ways to reduce the possible conservativeness arising from the use of sufficient, but not necessary, stability conditions. Moreover, the gantry robot system has been used in experimentally testing a wide range of other non LMI based ILC designs and hence it will also be possible in due course to compare relative performance (an essential item in terms of end users).

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