Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Numerical methods based controller design for mobile robots

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Abstract: This paper presents the design of four controllers for a mobile robot such that the system may follow a pre-established trajectory. To reach this aim, the kinematic model of a mobile robot is approximated using numerical methods. Then, from such approximation, the control actions to get a minimal tracking error are calculated. Both simulation and experimental results on a PIONEER 2DX mobile robot are presented, showing a good performance of the four proposed mobile robot controllers.

1. INTRODUCTION

The main problems found in mobile robot control is trajectory tracking. In general, the objective is that the mobile robot reaches the Cartesian position (x,y) with a pre-established orientation θ for each sampling period. These combined actions result in tracking the desired trajectory of the mobile robot. In order to achieve this objective, only two control variables are available: the linear and angular velocity of the robot, V and W respectively (Fig. 1). The use of path tracking in a navigation system is justified in structured workspaces as well as in partially structured workspaces where unexpected obstacles can be found during the navigation. In the first case, the reference trajectory can be set from a global trajectory planner. In the second case, the algorithms used to avoid obstacles re-plan the trajectory in order to avoid a collision; therefore, a new reference trajectory, which must be followed by the robot, is generated. Besides, there exist algorithms that express the reference trajectory of the mobile robot as function of a descriptor called r (F. Del Rio et al 2002) or s (called "virtual time") (S. Lee and J.H Park, 2003) whose derivative is function of the tracking error and the time t. For example, if the tracking error is large, the reference trajectory should wait for the mobile robot; on the other hand, if the tracking error is small, then the reference trajectory must tend to the original trajectory calculated by the global planner. Accordingly, the module of trajectory tracking will use the original path or the on line re-calculated path as reference to obtain the smallest error when the mobile robot follows the path (J. Normey-Rico et al, 2003). Therefore, the path tracking is always important independently from whether the reference trajectory has been generated by a trajectory global planner or a trajectory local planner.

Various control strategies have been proposed for tracking trajectory, some of which are based on either the kinematic or the dynamic models of the mobile robot (T. Lee,, et al, 2001), (K.D. Do, J. Pan, 2006), depending on the operative speed and the precision of the dynamic model. Different structures to control these systems have been developed as well. In (T. Tsuji, 1995), the authors use a time-varying feedback gain whose evolution can be modified through the parameters that determine the convergence time and the behaviour of the system. In (R. Fierro, F. Lewis, 1995), the controller proposed by (Y. Kanayama, 1990) is used. It generates the inputs to a velocity controller, making the position error asymptotically stable. Then, a controller to make the mobile robot velocity follow the reference velocity is designed. The work of (T. Fukao et al, 2000), extends the design proposed by (R. Fierro, F. Lewis, 1995) and considers that the model parameters are unknown. In (S. Kim, et al, 2000), an adaptive controller which takes into account the parametric uncertainties and the robot external perturbations, is proposed to guarantee perfect velocity tracking. The reference for velocity is obtained by using the controller proposed by (Y. Kanayama, 1990). In (D. Chwa, 2004) two controllers are designed. They are called position and heading controller. The former ensures the position tracking and the latter is activated when the tracking error is low enough and the tracking reference does not change its position. This reduces the error over the mobile robot orientation at the end of the path. In (H. Shim., 2004) the posture controller is designed in function of the posture error and in this way, the reference velocities are generated based on a set of specifications such as: i) if the distance to a reference posture is large enough, then the movement is quickly, and the speed is reduced as the robot approaches to the target; ii) the robot should take fewer amount of time to reach the desired posture. Later, the reference velocities go into a PID controller that generates the torque needed in function of the desired speed. In (S. Sun, 2004), a controller for trajectory tracking is designed using the kinematic model of the mobile robot and a transformation matrix. Such matrix is singular if the linear velocity of the mobile robot is zero; therefore, the effectiveness of this controller is only assured if the velocity is different from zero. Simulation results using linear velocity different from zero as initial condition are shown in this paper. In (S. Sun, 2005), a controller based on the error model of (Y. Kanayama, 1990), is proposed.

This controller is formed by two expressions where one or the other will be used depending on whether the angular velocity of the mobile robot is lower than a pre-established value . In this work, the control scheme presented in (R. Fierro, 1995; T. Fukao, 2000; S. Kim, 2000; H. Shim, 2004; D. Cruz et al, 2007) will be used; where first, a kinematic controller which generates the reference velocities to reach the desired goal is designed and second, the velocities obtained are used as input to the velocity controller. In our work a PID is used as velocity controller, on board the mobile robot PIONEER 2DX, to maintain the robot's translational and rotational speeds at desired values, the same as (H. Shim, 2004; D. Cruz et al, 2007). Besides, in our work it is not necessary to switch the controller as in (D. Chwa, 2004) in cases when position reference does not change and tracking error is small. Our purpose is that when this situation is detected, the desired orientation changes, calculating the control signal by using the same expression.

In this paper, the designed controller does not present the disadvantage of (S. Sun, 2004), where a linear velocity different from zero is necessary. Furthermore, our controller does not need to change the control expression when the angular velocity is lower than a pre-established value (D. Cruz, et al 2007).

We propose to use numerical methods, not only to simulate the evolution of the mobile robot, but also to find the control actions that allow going from the mobile robot's current state to the next one. The result is that four controllers are obtained. Each one of these proposals is used according to the available information. Two of these obtained controllers make use of the velocity used to generate the reference and the other two don't need it. The main contribution of this work is that the four controllers are obtained by the same design methodology and, complex calculations to get the control signal are not necessary. The simulation and experimental results are shown applied on a PIONEER 2DX in mobile robot which the error between the real and the desired trajectory is very small. The effectiveness and feasibility are then demonstrated in a practical sense through a set of experiments carried out for similar speed-range reported in others papers about trajectory tracking using laboratory equipment (J. Normey-Rico et al, 2001).

The paper is organized as follows: Section 2 presents the methodology to solve differential equations using numerical methods. Section 3 describes the kinematic model of the mobile robot, approximated through numerical methods. In addition, the formulation of the proposed control algorithm is obtained as well. Section 4 presents the simulations and experimental results using the proposed controller on a PIONEER 2DX mobile robot and the re-design of the controller. Conclusions are detailed in Section 5.

2. STATEMENT OF THE PROBLEM

Let us consider the following differential equation,

$$\dot{y} = f\left(y,u,t\right); y\ \left(0\right)\ = y_0 \tag{1}$$

Where *y* represents the output of the system to be controlled, *u* the control action, and *t*, the time. The values of $y(t)$ at discrete time $t = nTo$, where To is the sampling period, and $n \in \{0,1,2,3,\cdots\}$ will be denoted as y_n. Thus, when wishing to compute y_{n+1} by knowing y_n, (1) should be integrated over the time interval $nTo \le t \le (n+1)To$ as follows,

$$y_{n+1} = y_n + \int_{nTo}^{(n+1)To} f(y,u,t)dt \tag{2}$$

There are several numerical integration methods to calculate y_{n+1}. For instance, the Euler, and trapezoidal methods approach could be used (Eqs. (3) and (4) respectively).

$$y_{n+1} \cong y_n + To\, f(y_n, u_n, t_n) \tag{3}$$

$$y_{n+1} \cong y_n + \frac{To}{2}\ \left\{f(y_n, u_n, t_n) + f(y_{n+1}, u_{n+1}, t_{n+1})\right\} \tag{4}$$

Where y_{n+1} on the right-side member of (4) is not known and, therefore, can be estimated by (3). The use of numerical methods in the simulation of the system is based mainly on the possibility to determine state of the system at instant $n+1$ from the state, the control action, and other variables at instant n. So, y_{n+1} can be substituted by the desired trajectory and then the control action to make the output system evolve from the current value (y_n) to the desired one can be calculated. To accomplish this, it is necessary to solve a system of linear equations for each sampling period, as it can be seen in section 3.

This work proposes applying this approximation to the kinematic model of a mobile robot and, thus, obtain the control action that enables the robot to follow a pre-established trajectory during its navigation. The next section will analyse the kinematic model of the mobile robot and the design of the proposed controller.

3. METHODOLOGY FOR CONTROLLER DESIGN AND PROBLEM DEFINITION

A non-linear kinematic model for a mobile robot will be used as shown in Fig.1, represented by, (Campion, et al ,1996),

$$\begin{cases} \dot{x} &= V\cos\theta \\ \dot{y} &= V\sin\theta \\ \dot{\theta} &= W \end{cases} \tag{5}$$

where *V*: linear velocity of the mobile robot, *W*: angular velocity of the mobile robot, (x, y): Cartesian position, θ: orientation of the mobile robot, $\{R\}$: inertial frame and $\{R_C\}$: frame attached to the robot. Then, the aim is to find the values of V and W so that the mobile robot may follow a pre-established trajectory. We assume that the mobile robot is moving on a horizontal plane without slip. In order to classify and develop our work properly, we made some considerations about the geometric conditions of the trajectory followed by the mobile robot.

First Hypothesis: $|\theta_{n+1} - \theta_n| < \lambda$, being λ an angle small enough: Through Euler's approximation of the kinematic model of the mobile robot (5), the following set of equations is obtained,

$$\begin{cases} x_{n+1} \approx x_n + ToV_n \cos\theta_n \\ y_{n+1} \approx y_n + ToV_n \sin\theta_n \\ \theta_{n+1} \approx \theta_n + ToW_n \end{cases} \quad (6)$$

This can be expressed in vectorial form as,

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ \theta_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \\ \theta_n \end{bmatrix} + To \begin{bmatrix} \cos\theta_n & 0 \\ \sin\theta_n & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_n \\ W_n \end{bmatrix} \quad (7).$$

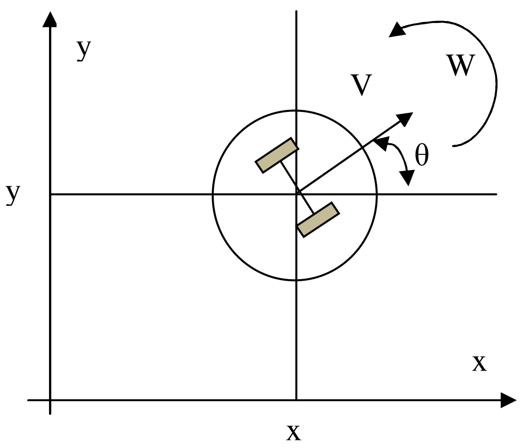


Fig. 1. Geometric description of the mobile robot.

If the desired trajectory $\left[x_{d_{n+1}}\ y_{d_{n+1}}\ \theta_{d_{n+1}}\right]^T$ is known, then $[x_{n+1}\ y_{n+1}\ \theta_{n+1}]^T$ in (7), can be substituted by $\left[x_{d_{n+1}}\ y_{d_{n+1}}\ \theta_{d_{n+1}}\right]^T$ and thus, it will be possible to calculate the control actions V_n, W_n necessary to make the mobile robot go from the current state $[x_n\ y_n\ \theta_n]^T$ to the desired one $\left[x_{d_{n+1}}\ y_{d_{n+1}}\ \theta_{d_{n+1}}\right]^T$. By defining,

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{d_{n+1}} - x_n \\ y_{d_{n+1}} - y_n \\ \theta_{d_{n+1}} - \theta_n \end{bmatrix}, \quad B = \begin{bmatrix} \cos\theta_n & 0 \\ \sin\theta_n & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

and then by replacing (8) into (7), the following equation is obtained,

$$B \begin{bmatrix} V_n \\ W_n \end{bmatrix} = \frac{1}{To} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} \quad (9)$$

The optimal solution of the equations system given by (9) is, (Strang, 1982),

$$B^T B \begin{bmatrix} V_n \\ W_n \end{bmatrix} = \frac{1}{To} B^T \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} V_n \\ W_n \end{bmatrix} = \begin{bmatrix} \frac{\Delta x}{To}\cos\theta_n + \frac{\Delta y}{To}\sin\theta_n \\ \frac{\Delta\theta}{To} \end{bmatrix} \quad (11)$$

where: V_n and W_n are the linear and angular velocities necessary to make the mobile robot go from the current state to the desired one. To find a closed solution for the system of equation (9), it is necessary that real constants a_1, a_2 exist such that,

$$a_1 \begin{bmatrix} \cos\theta_n \\ \sin\theta_n \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix}; a_1, a_2 \in \Re \quad (12)$$

where:

$$a_1 \begin{bmatrix} \cos\theta_n \\ \sin\theta_n \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \\ 0 \end{bmatrix} \Rightarrow \frac{\sin\theta_n}{\cos\theta_n} = \frac{\Delta y}{\Delta x} \quad (13)$$

So, the desired orientation is defined by,

$$\theta_{d_{n+1}} = a\tan\frac{y_{d_{n+1}} - y_n}{x_{d_{n+1}} - x_n} \quad (14)$$

where $\theta_{d_{n+1}}$ represents the necessary orientation at time *n+1*, to make the mobile robot tend to the reference trajectory. Then, the proposed controller for the mobile robot is given by,

$$\begin{bmatrix} V_n \\ W_n \end{bmatrix} = \begin{bmatrix} kv\left(\frac{\Delta x}{To}\cos\left(\theta_{d_{n+1}}\right) + \frac{\Delta y}{To}\sin\left(\theta_{d_{n+1}}\right)\right) \\ kw\frac{\Delta\theta}{To} \end{bmatrix} \quad (15)$$

In (15), the value of $\theta_{d_{n+1}}$ is used instead of θ_{d_n} due to the values used to calculate control actions are the desired in next sample time; *kv*, *kw* are positive constants that allow adjusting the function of the proposed control system, besides satisfying $0 \le kv \le 1, 0 \le kw \le 1$. The next section will illustrate the simulation and experimental results of the control law obtained, under assumption of the use of this controller over simple and non exigent trajectories (in reference to the first hypothesis previously developed); then, the re design of the controller, by using the same methodology, will be exposed in cases more complex than the first one and its performance on a mobile robot will show the feasibility of the method.

4. RESULTS, DISCUSSION AND CONTROLLER REDESIGN

Simulation and experiments to test the proposed controller performance were carried out using a PIONEER 2DX mobile robot. The simulation software SAPHIRA of Active Media was also used (K. Konolige, 1998). Fig. 2 shows the Pioneer 2DX and the laboratory facilities where the experiments were carried out. The PIONEER 2DX mobile robot includes an estimation system based on odometry, which adds accumulative errors to the system. From this, updating the data through external sensors is necessary. This problem is separated from the strategy of trajectory tracking and it is not considered in this paper (J. Normey-Rico, 1999; J. Normey-Rico et al, 2001). The PIONEER 2DX has as a PID velocity controller, used to maintain the velocities of the mobile robot in the desired value (H. Shim, 2004; Cruz et al, 2007). In order to test the performance of the proposed controller, on a trajectory that satisfies the first hypothesis, a circumference of 600 mm radius was used as the desired one, with centre on the origin of the coordinate system. The starting point for the robot was the centre of the circumference, with an initial orientation $\theta = 0^o$. From this starting point, it evolves to the desired trajectory. The reference trajectory starts at $(600,0)mm$ and is generated at constant linear and angular velocities respectively known as $Vref$ and $Wref$. In the PIONEER 2DX mobile robot the value of the sample time To is 0.1 sec.

Fig. 2. Pioneer 2DX mobile robot and its environment.

A set of tests were developed in simulation and experimentally. A simulation using the SAPHIRA simulation software of Active Media (K. Konolige, 1998), for the mobile robot, was developed and the results are shown in Figure 3a), with $kv = kw = 1$ in (15), when $Vref$ is $100mm/\sec$. It can be noticed that the mobile robot follows the desired trajectory but in an oscillatory way. In order to correct this undesired behavior, the control actions can be calculated by the minimization of a quadratic index, in which not only the tracking error but also the square of state variables derivative has been considered, as seen in (16),

$$J = k_1^2\left[(x_{d_{n+1}} - x_{n+1})^2 + (y_{d_{n+1}} - y_{n+1})^2\right] + k_2^2\left(\dot{x}_n^{\,2} + \dot{y}_n^{\,2}\right) + k_3^2(\theta_{d_{n+1}} - \theta_{n+1})^2 + k_4^2\dot{\theta}_n \tag{16}$$

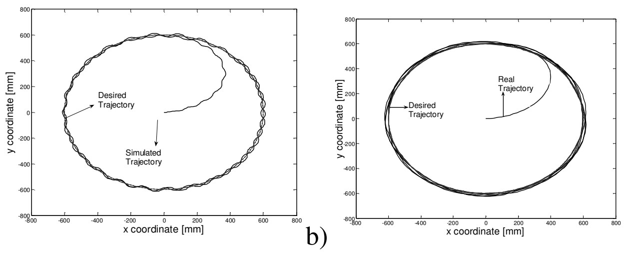
$$\frac{\partial J}{\partial V_n} = 0; \frac{\partial J}{\partial W_n} = 0 \qquad (17)\text{-}(18)$$

$$\begin{cases} V_n = \underbrace{\dfrac{k_1^2}{k_1^2 + \dfrac{k_2^2}{To^2}}}_{kv}\left(\dfrac{\Delta x}{To}\cos\theta_n + \dfrac{\Delta y}{To}\sin\theta_n\right) \\ W_n = \underbrace{\dfrac{k_3^2}{k_3^2 + \dfrac{k_4^2}{To^2}}}_{kw}\dfrac{\Delta\theta}{To} \end{cases} \tag{19}$$

If (15) and (19) are compared, then it can be seen that, to minimize the state variables variations, the constant values of kv and kw should be chosen less than 1, for that reason, we propose to reduce the values kv and kw to values kv = 0.2 and kw = 0.2. During the execution of the reference trajectory, at a random instant of time, certain values of (xd, yd) will be kept fixed. In this way, the proposed controller performance is monitored when a trajectory is to be followed by the mobile robot and then it is suddenly stopped at a certain point. From Fig.3b, experimental results on the mobile robot PIONEER 2DX can be analysed, with $Vref = 200mm/\sec$ and $Wref = 19.1\deg/\sec$. Fig. 3b shows the mobile robot following the reference trajectory without undesirable oscillations. The speed range used for testing the performance of the proposed controller is typical in the trajectory tracking papers referenced by the current bibliography (. Normey-Rico, J. Gomez-Ortega, E. Camacho,2001). Figs. 4a) show the time evolution of the real angular velocity, denoted as Wreal, of the mobile robot. It is important to remark that the absolute value of the difference between the desired and real trajectory, once the mobile robot has reached the geometric pre-defined path will be called error. In this way, Fig. 3b) shows that the mobile robot follows the desired trajectory with a maximum error of 20 mm, which is very small when compared to the distance between wheel axes (330 mm). However, linear and angular velocities present a considerable variation with respect to the reference value; it can be seen from Fig 4a) in reference to the angular velocity. To improve this issue, we propose considering in index J not only the error between the current and desired state, but also the difference between the real and reference linear and angular velocities, this is,

$$J = cv1^2\left\{(x_{d_{n+1}} - x_{n+1})^2 + (y_{d_{n+1}} - y_{n+1})^2\right\} + cw1^2\left(\theta_{d_{n+1}} - \theta_{n+1}\right)^2 + cv2^2(Vref - V_n)^2 + cw2^2(Wref - W_n)^2 + cv3^2\left(\dot{x}_n^{\,2} + \dot{y}_n^{\,2}\right) + cw3^2\dot{\theta}_n \tag{20}$$

where $cv1, cv2, cv3, cw1, cw2, cw3$, are constants that allow adjusting the control system response. By proceeding likewise,



a) b)

Fig.3a).Simulated and Desired Trajectory, $Vref = 100 mm/\sec.$, $kv = kw = 1$. b) Experimental results: Real and Desired Trajectory, $Vref = 200 mm/\sec.$ $Wref =$ 19.1deg./sec. $kv = 0.2$, $kw = 0.2$.

$$W_n = \frac{1}{cw1^2 + \frac{cw2^2}{To^2} + \frac{cw3^2}{To^2}} \left\{ cw1^2 \frac{\Delta\theta}{To} + \frac{cw2^2}{To^2} Wref \right\} \quad (22)$$

This can be expressed as,

$$V_n = \frac{kv^2}{kv1^2 + kv2^2} \left[kv1^2 \left[\frac{\Delta x}{To} \cos\theta_n + \frac{\Delta y}{To} \sin\theta_n \right] + \right. \quad (23)$$
$$\left. + kv2^2 Vref \right]$$

$$W_n = \frac{kw^2}{kw1^2 + kw2^2} \left\{ kw1^2 \frac{\Delta\theta}{To} + kw2^2 Wref \right\} \quad (24)$$

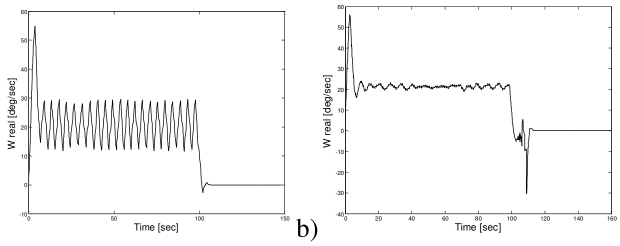
where $0 \le kv^2 \le 1$, $0 \le kw^2 \le 1$.

Besides, it can be noticed that the control actions depend on the linear and angular reference velocities. To test the performance of the new control law obtained, another experiment was carried out using the values for $kv1^2 = 1$, $kv2^2 = 3.5$, $kw1^2 = 1$, $kw2^2 = 1.1$, $kw^2 = 0.22$, $kv^2 = 0.24$ and the values of $Vref = 200 mm/\sec.$ and $Wref =$ 19.1deg./sec. Fig. 4b shows the time evolution of angular velocity when the controller given by (23) and (24) is used. The mobile robot follows the desired trajectory with a maximum error of 10 mm, which is very small considering the distance between the axes of the mobile robot (330mm). It probes the good performance of the controller. In addition, if figs 4a and 4b are compared, it can be seen that, the variation of the real angular velocity has been reduced considerably. A set of experiences was carried out at different references velocities and a summary of these tests is presented on Table 1, the most representative results of the experimental tests will be shown in the figures.

Another typical benchmark trajectory of reference, like a senoidal-type, was used to test the controller performance, in this case Fig. 5 shows the trajectory followed by the PIONEER 2DX mobile robot on the plane x-y, in case of the initial position of the mobile robot was (x = -4.0, y = 0) m. It can be seen from fig. 5 that the mobile robot tends to the desired trajectory and then follows it in a precise way. Fig. 6a and 6b show the time evolution of the linear and angular velocities by using a PID controller to maintain the velocities on the reference values; Fig. 6a) shows that the mobile robot goes at high linear velocity for mobile robotics. In Fig. 6b), we observe that the mobile robot is moving with a soft behavior without strong oscillations through the desired trajectory.

Table 1. Summary of the Errors in to the Trajectory and the Angular Velocity for the experimental test by the use of Controllers defined by (19) and (24)-(25).

Vref mm/sec	Max Error by (19)	MaxError by (23)-(24)	Max error W by (19) deg/sec	Max error W by (23)-(24) deg/sec
100	12mm	5mm	4.5	1.2
200	21mm	10mm	8	2
300	28mm	14mm	9.5	2.3



a) b)

Fig. 4a. Experimental results: Real angular velocity, Controller (15). At $Wref = 19.1 \deg/\sec$. b: Experimental results: Real angular velocity, Controller (24) and (25). At $Wref = 19.1 \deg/\sec$.

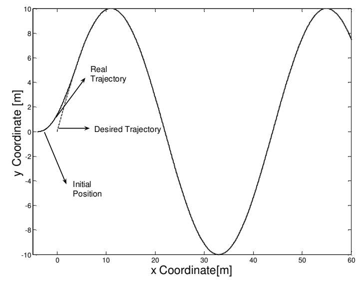
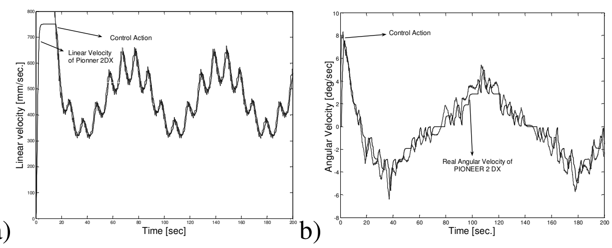


Fig. 5: Experimental results: Real and Desired Trajectory.

If a comparison between our experimental results and results recently published is made (for example K.D. Do, J. Pan,2006 which presents an algorithm based on the dynamic model of the mobile robot showing simulation results), we conclude that the control system proposed in this paper, presents a similar performance, working at same range of speeds. The maximum linear velocity was limited at 750 mm/sec. for safety conditions.



a) b)

Fig. 6: Experimental results. a)Control action (24) and real linear velocity of the mobile robot, b) Control action (25) and real angular velocity of the mobile robot

Second hypothesis: value of $\left|\theta_{n+1}-\theta_n\right|$ higher than the considered in first hypothesis. It mean, the presence of suddenly changes of angle in the desired trajectory followed by mobile robot: Now, if (6) is not valid - a case occurring when the desired trajectory suddenly changes its direction - it is sensible to expect a momentarily increase of the error and then a decrease. To visualise these effects, a box of 2200mm side is used as reference trajectory, which is generated at constant linear speed (Vref = 200 mm/sec.); the initial position of the mobile robot was (-100,-100)mm. The experimental results are shown in Fig. 7a which displays the trajectory followed by the mobile robot PIONEER 2DX on the x-y plane. It can also be noticed that, when the trajectory direction suddenly changes, the error increases, but it decreases afterwards, with a maximum error of about 100mm. Besides, the error is not too large when compared with the size of PIONEER 2DX, considering the demanding desired trajectory chosen. This trajectory-type is used to test the performance of the system, because it is a situation of worst case, where the error is acceptable since it is smaller than half the distance between the axes of the mobile robot. In other trajectory-types which satisfy the first hypothesis made, the performance will be better than in this case. However, a modification of the control algorithm is stated in order to reduce the peak in the trajectory shown in fig. 7a.

If in addition to knowing both the position and orientation of the mobile robot, the linear and angular velocities in nTo are also known, a trapezoidal-type integration approach can be made (Eq.4). In this way, another controller for a mobile robot is obtained and, consequently, it can be expected that the system behavior be enhanced due to the use of a better numerical approach of Eq (5). By the use of a trapezoidal-type integration method,

$$x_{n+1}=x_n+\int_{nTo}^{(n+1)To} V\cos\theta dt \approx x_n+\frac{To}{2}\{V_n\cos\theta_n+V_{n+1}\cos\theta_{n+1}\}$$
$$y_{n+1}=y_n+\int_{nTo}^{(n+1)To} V\sin\theta dt \approx y_n+\frac{To}{2}\{V_n\sin\theta_n+V_{n+1}\sin\theta_{n+1}\} \qquad (25)$$

$$\theta_{n+1}=\theta_n+\int_{nTo}^{(n+1)To} W dt \approx \theta_n+\frac{To}{2}\{W_n+W_{n+1}\} \qquad (26)$$

where x_n, y_n, θ_n, V_n and W_n are, the Cartesian position, orientation, linear velocity and angular velocity at *nTo* respectively.

The aim is to find the values for θ_{n+1}, V_{n+1} and W_{n+1} so that the mobile robot goes from its current position (x_n, y_n) to (xd_{n+1}, yd_{n+1}). From Eqs. (25-26),

$$\begin{cases} V_{n+1}\cos\theta_{n+1}=\dfrac{2}{To}(xd_{n+1}-x_n)-V_n\cos\theta_n \\ V_{n+1}\sin\theta_{n+1}=\dfrac{2}{To}(yd_{n+1}-y_n)-V_n\sin\theta_n \end{cases} \qquad (27)$$

$$\frac{\sin\theta_{n+1}}{\cos\theta_{n+1}}=\tan\theta_{n+1}=\frac{\dfrac{2}{To}(yd_{n+1}-y_n)-V_n\sin\theta_n}{\dfrac{2}{To}(xd_{n+1}-x_n)-V_n\cos\theta_n} \qquad (28)$$

The value of θ_{n+1} is thus defined. As shown in (25-26), this is a three-equation with two-unknown system, and by proceeding likewise,

$$V_{n+1}=\frac{2}{To}\Delta x\cos\theta_{n+1}+\frac{2}{To}\Delta y\sin\theta_{n+1}-V_n\cos(\theta_{n+1}-\theta_n) \qquad (29)$$

$$W_{n+1}=\frac{2}{To}\Delta\theta-W_n \qquad (30)$$

the proposed controller will be,

$$V_{n+1}=kv\left[\frac{2}{To}\Delta x\cos\theta_{n+1}+\frac{2}{To}\Delta y\sin\theta_{n+1} -V_n\cos\left(\theta_{d_{n+1}}-\theta_n\right)\right] \qquad (31)$$

$$W_{n+1}=kw\left[\frac{2}{To}\Delta\theta-W_n\right] \qquad (32)$$

$$\tan\theta d_{n+1}=\frac{\dfrac{2}{To}(yd_{n+1}-y_n)-V_n\sin\theta_n}{\dfrac{2}{To}(xd_{n+1}-x_n)-V_n\cos\theta_n} \qquad (33)$$

where $0<kv\leq 1$, $0<kw\leq 1$ are the variables to adjust the system behaviour. From (31) and (32), we can observe that the control signals also depend on the current position, current orientation and the linear and angular velocity of the mobile robot. Figure 7b depicts an instance of the 2200 mm square-shaped reference trajectory followed by the PIONEER 2DX, generated with constant linear velocity of Vref = 200 mm/sec using the controller defined by (31)-(32) from the robot's initial position of (-100,-100)mm. If both figures 7a and b are compared, it can be seen that the performance of the controller improves. It means that the controller given by (31) and (32) shows a better performance than that of the controller of (23) and (24). This significant improvement, shown in Figs 7b, comes from using a better approximation of the system, which results in a controller which uses -in addition to the desired position and orientation- the real linear and angular velocities of the mobile robot. The speed range used to test the controller performance is typical in papers about trajectory tracking using laboratory equipment (Normey-Rico et al 2001, Dixon et al, 2004).

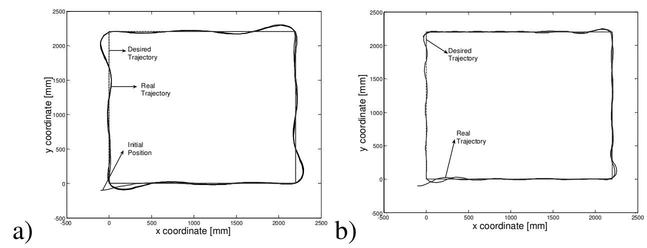


Fig. 7 Experimental results: a) Real and Desired Trajectory, $Vref=200mm/\sec$. By using (23)- (24) Controller. 7b) Trajectory followed by the mobile robot on the x-y plane Vref = 200 mm/sec. By using (31)-(32) controller

If the information about reference velocities is available, the previous control law can be modified by following the procedure indicated in (20)-(25), thus, the linear and angular reference velocities are incorporated into the controller expressions as,

$$V_{n+1} = \frac{kv1^2}{kv1^2 + kv2^2} *$$
$$\left[kv1^2 \left(\frac{2}{To} \Delta x \cos\theta_{n+1} + \frac{2}{To} \Delta y \sin\theta_{n+1} - V_n \cos\left(\theta_{n+1} - \theta_n\right) \right) \right. \quad (33)$$
$$\left. + kv2^2 Vref_{n+1} \right]$$

$$W_{n+1} = \frac{kw1^2}{kw1^2 + kw2^2} \left[kw1^2 \left(\frac{2}{To} \Delta\theta - W_n \right) + kw2^2 Wref_{n+1} \right] \quad (34)$$

The trajectory followed by the PIONEER 2DX mobile robot on the plane *x-y*, when the controller is described by (33)-(34), is shown in Figures 8. A circumference with radio 600 mm and linear velocity of *Vref* = 750 mm/sec was used as reference trajectory.

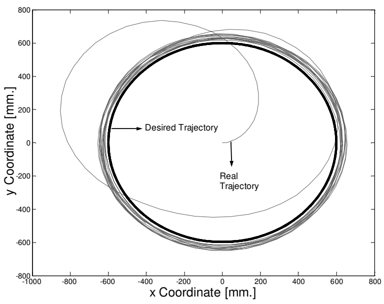
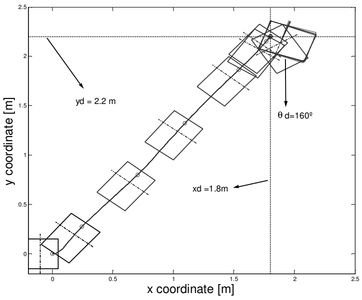


Fig. 8: Experimental results. Trajectory followed by the mobile robot on the x-y plane, Vref = 750 mm/sec. Using controller defined by (33)-(34)

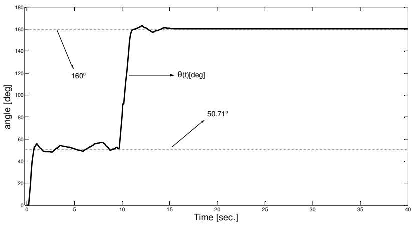
Figures 8, show that the error is very small for high velocities of the mobile robot. This result is obtained due to use of the proposed controller by (33)-(34) taking into account more information than the one in (31)-(32).Another important problem that has been studied previously (D. Chwa,2004), is reaching a point in the plane *x-y* and then, making a re orientation procedure with some desired angle of orientation established by trajectory planner. Figure 9a shows the path followed by the mobile robot in plane *x-y* when the experiment considered was the problem of positioning. In that case, the values for the position and orientation were $xd = 1.8m,\ yd = 2.2m$ and $\theta d = 160^{\circ}$. In Figure 9b, the orientation of the mobile robot varying in function of time can be seen, where the initial values for position and orientation were, $\left(x, y, \theta\right) = \left(0m, 0m, 0^{\circ}\right)$ respectively. In case the positioning error is big, the orientation θd_{n+1} is calculated by using (32) and when the positioning error is small enough it is assumed that $\theta d_{n+1} = 160^{\circ}$. It means,

$$\theta_{d_{n+1}} = \begin{cases} a\tan\left(\dfrac{\frac{2}{To}\left(y_{d_{n+1}} - y_n\right) - V_n \sin\theta_n}{\frac{2}{To}\left(x_{d_{n+1}} - x_n\right) - V_n \cos\theta_n} \right) if \sqrt{\Delta x^2 + \Delta y^2} > \varepsilon \\ 160^{\circ} if \sqrt{\Delta x^2 + \Delta y^2} \le \varepsilon \end{cases}$$

Being Epsilon a significantly small value, for this case the value of Epsilon used was $\varepsilon = 0.01m$. It can be seen from figures 9a and 9b, how the mobile robot, defines an orientation to reach the point $(x, y) = (1.8, 2.2)m$ and when, it is close enough to its new desired orientation which is $\theta d_{n+1} = 160^{\circ}$.



a)



b)

Fig. 9: Experimental results: a) Trajectory followed by PIONEER 2DX in x-y plane b) Time evolution of $\theta(t)$

5. CONCLUSIONS

In this work, four control law have been proposed for the trajectory tracking of mobile robots. Each one of these controllers is used according to the available information. The first proposal is used only if the desired position is available (15), the second one is applied when the desired position and desired velocity are available ((24) and (25)), the third one is used when the position, orientation, linear velocity and angular velocity are available (31), and finally, the fourth one is applied when the information used in the third one plus the linear and angular reference velocities are available (33).The above control structures can be designed and implemented without great difficulty, because standard algebraic-numerical techniques are used. Simulation and experimental results of the developed controllers on a PIONEER 2DX mobile robot have been also addressed. Through the analysis of these experiments, it can be concluded that the trajectory error between the desired and the real trajectory of the mobile robot is very small. Also, the task of reaching a new reference point and then, making a new orientation was considered. In this case, it can be seen that this goal was completely and efficiently reached without difficult calculations. From the experimental results, we conclude that the proposed methodology is quite simple for selecting the parameters of the controller in order to achieve a good performance of the system during the navigation of the mobile robot.

The proposed methodology for the controller design can be applied to other types of systems. The required precision of the proposed numerical method for the system approximation is smaller than the one needed to simulate the behaviour of the system. This is because, when the states for the feedback are available, in each sampling time, any difference from accumulative errors is corrected (e.g. rounding errors). Thus, the approach is used to find the best way to go from one state to the next one, according to the availability of the system model. The controller design was also stated as a minimization of a quadratic index, which is a simple problem, and allows considering other trajectory properties, such as $Vref$ and $Wref$.

ACKNOWLEDGEMENTS

This work was partially supported by the National Council for Scientific (CONICET), Argentina and the German Service for Academic Exchange (DAAD – Deutscher Akademischer Austausch Dients).

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